

Circuit Analysis for Analog Designers
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Lecture - 09
Inter reciprocity in network with ideal operational amplifiers

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Only applicable to networks where [redacted]

$N \hat{N}$ are inter-reciprocal networks.

Inter-reciprocity

$\frac{v_2}{i_1} = \frac{\hat{v}_1}{\hat{i}_2}$

* \hat{N} & N are NOT the same

* However, given N , straight forward to derive \hat{N}

To find the transfer function from multiple inputs in N to a single output

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To find the transfer function from multiple inputs in N to a single output

* Derive \hat{N} from N

* Excite the output of \hat{N} - Measure current/voltage through/across the input sources.

Inter-reciprocal network \equiv Adjoint network

Doubt: Nonideal controlled sources

So, a question that was asked is ok. So, my controlled source is I know I mean let us say the controlled source is not ideal. So, let us say I have voltage controlled current source

like this. So, this is in N. So, let us call this R_{in} . This is a this is a prime, b, b prime g m v a ok. So, any comments on what I should do in the inter reciprocal network.

Very good right. Remember this is still an ideal controlled source correct. So, only you need to flip the controlled source and leave all the other non idealities. I mean a non ideal controlled source is simply an ideal controlled source with some resistors you know here and there right. So, all that you need to do is to flip. So, this will be R_{in} . This will be g m times v b hat and this will be v b, is that clear.

So, ok so the inter reciprocal network therefore, you know is a way of exploiting reciprocity that works in all practical situations and how many of you used spice before or some form of spice a big portion right. So, how many of you are familiar with noise analysis before? Do not be shy I mean it is ok if you have done it before right.

So, you know you know that when you if you have not done it you will get to learn it in this course, but those of you are familiar with noise analysis you know that you can click or you can find the contributions from individual noise sources to the output right. And that entails you know finding the transfer functions from individual noise sources to the output and that is all done. It is not you know it is not any more expensive to do that you just simply found using the inter reciprocal network.

And when we go a little further I will show you that we will see together that when we write down the metrices for, I mean when circuits are all as you know on a computer all solved using metrics techniques. It just turns out to be a trivial transformation of that metrics right of the circuit metrices. And therefore, this is routinely used in spice every time you run a spice simulation noise analysis it is used.

Some of you may also have been be aware of a less known feature of spice namely transfer function analysis right. How many of you are aware of the AC analysis? Everybody is aware and what does the AC analysis do? You have how many inputs and how many outputs. You have one input and you have you can click on any node and find the frequency response from that one input to the output correct ok. The transfer function analysis is a is the same thing done upside down.

You have you choose the output port and you can click on any independent source inside the network and it will show you the frequency response from that independent source to

the single output and that is done again using the inter reciprocal right. And there is also another name for the inter reciprocal network inter reciprocal network and that is it is also called the adjoint network right. And there is a good reason for this as again we will see few lectures down the line right. At this point these are just two terms that you need to be familiar with, is that clear people? Very good ok.

So, let me you know you have done enough you know derivation let us kind of see this in action right, and a small example I mean and a slight extension of this as you know and you will be immediately able to see the point. For example, I mean when you do a analog circuits course you know what is the other element that you keep seeing all the time?

You have the four controlled sources you know you know say resistors, capacitors, inductors there is something we spend a whole course deriving, the ideal op amp right. So, in N let us say you had an ideal op amp alright, and if you have an ideal op amp what comment can we make about the properties of an ideal op amp?

Again is this are all the yeah of course, but in when you embed the ideal op amp inside a circuit what comment can we make about the terminal voltages and currents? So, basically yes anybody else. If you have an ideal op amp right, embedded inside a circuit what you do to analyze when you analyze the circuit.

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The slide contains the following content:

- NPTEL Logo** (top left)
- Handwritten Equations:**

$$v_a = v_{a'}$$

$$i_a = 0$$
- Handwritten Note:** "if dc negative feedback around the opamp"
- Circuit Diagrams:**
 - An ideal op amp symbol with inputs a and a' and output b .
 - An internal model of the op amp showing a dependent voltage source Nv_a and a dependent current source Ni_a .
 - A feedback configuration where the output b is connected to the inverting input a' through a resistor R_f .

So, what constraints does that the ideal op amp put? v_a must be equal to $v_{a'}$ ok.

$$v_a = v_{a'}$$

And if i_a is equal to 0 and what comment can we make about v_b ?

$$i_a = 0$$

Well v_b is dictated by the circuit right ok. And under what conditions are these valid? The most important thing to realize is that this is negative feedback or rather DC negative feedback around the op amp alright ok. So, in N if you had an ideal op amp what comment can you make about what you should do in the adjoint network? I mean what kind of I mean what kind of controlled source can you think of an op amp has?

Actually, it is not necessary to think of it as a voltage-controlled voltage source with infinite gain. You can think about it as a voltage controlled current source with an infinite trans conductance or a current controlled voltage source with infinite trans impedance or a current controlled current source with infinite gain right. As the controlling parameter tends to infinity right.

I mean and what is the intuition behind any of those assumptions? Remember if you have a controlled source which is embedded inside a circuit and the circuit is working all voltages and currents must be finite correct. So, if this controlling parameter happens to be infinite ok, and the voltage between b and b' is finite and μ is infinite what comment can you make about v_a ? Should be 0 ok.

Likewise, if the controlling parameter z is infinite and the current passing through the branch I mean the voltage between b and b' is finite what comment can you make about v_a and $v_{a'}$ the voltage between a and a' ? The voltage between a and a' in the current controlled voltage source is anyway 0, what comment can you make about the current? Is also 0 right.

And likewise with a voltage controlled current source if this current is finite and g_m is infinite ok. It is easy to see that well the difference between v_a and $v_{a'}$ is the voltage difference between a and a' is 0 correct. So, all that I would like to point out is that, it is not necessary for us to think of an ideal op amp as simply a voltage-controlled voltage source with infinite gain. You can think of any think of it as any one of those four controlled sources with that controlling parameter tending to infinite right.

Now, with that reasoning what comment can you make about the ideal op amp? If you want to replace the ideal op amp in the original network consisted of an ideal op amp, what could we replace it with in the adjoint?

Ideal op amp with what distinction? Remember an ideal op amp is more is slightly different from an ideal controlled source in the sense that the controlling ports terminals are both available right. So, this is v_a and let us say if I assume this to be a voltage-controlled voltage source this is μv_a . But, what comment can we make about the ideal op amp? Well from the picture you see that its only b which is accessible what is happening to a ?

I mean what is happening to b prime? b prime is internal to the op amp and is therefore, not accessible it is a ground, but it is internal to the op amp and not accessible. So, this is b alright. Now, if you want to form the adjoint if you had something like this, what would it if you I mean let us do it step by step. What comment can we make if we make this in the adjointed network how would this look like? It is a current controlled current source, the controlling port must have one terminal grounded correct.

So, this is therefore, i_b hat ok. Let me just change the signs to the ok and what must be the controlled port? This is a voltage-controlled voltage source for the original network. So, what did we say for the current controlled current source, what must be the sense of the current? Going up or coming down? Going up.

So, this is a prime and this is $\mu \hat{i}_b$, is that clear? And therefore, now we say well this is an ideal op amp. So, what must μ tend to? μ must tend to infinity right ok. So, μ must tend to infinity. So, even though the original circuit had the original op amp had both the input terminals accessible, but one output terminal not accessible. So, in the inter reciprocal network what else will what you see?

Well both the output terminals accessible, but only one input terminal is accessible. So, this is a and this is a prime and this is b alright.

$$\frac{v_{out}}{v_1, v_2, v_3}$$

So, what you suggest now that we do? We exploit reciprocity what you do? The voltage transfer functions in the original network now become voltage transfer functions in the original network become? Current transfer functions in the reciprocal or the inter reciprocal network right. So, what do you suggest that I do? Excite it with a current source i or let me call that \hat{i} alright. So, now, it is a standard circuit analysis where does all that \hat{i} flow? What is this potential by the way? It is the virtual ground of the op amp that is 0. So, where does \hat{i} flow?

Right. So, this is \hat{i} alright and what is this voltage? Come on people $-\hat{i} R_2$ correct and therefore, what comment can you make about this current? Yes what is that current? You know the voltage you I mean can somebody tell me what that current is. Remember all these voltage sources are I mean these are just names right. They are all 0 voltage sources because we need to find the currents through them correct ok. So, what is the current through R_1 ?

$-\hat{i} \frac{R_2}{R_1}$ ok and therefore, what comment can you make about v_3 by. So, v let us what comment can we make about v out by v 3 in the original network? It is simply the same as please think carefully and be careful about the signs s i g n s. What is the direction of the current through the voltage source v 3? So, v out by v 3 in the original network is what in the inter reciprocal network? What do we need to do? What is it?

I am not looking for the answer what is the input and what is the what is the output we are looking at? Yeah. The current into the positive terminal of v 3 is our output quantity right. So, that is the current flowing into the positive terminal of v 3 is minus \hat{i} . So, that is basically minus \hat{i} divided by \hat{i} which therefore, is minus 1 correct ok.

$$\frac{v_{out}}{v_3} = -\frac{\hat{i}}{\hat{i}} = -1$$

And likewise what comment can you make about v out by v 2 in the original network? What is the output we are looking at?

We need to find the current flowing through the positive terminal of v_2 in the inter reciprocal network. And that happens to be minus \hat{i} times R_2 by R_1 that divided by this must be minus R_2 by R_1 .

$$\frac{v_{out}}{v_2} = \frac{-\hat{i} (R_2/R_1)}{\hat{i}} = \frac{-R_2}{R_1}$$

And its evident that the current flowing through v_2 and the current flowing through v_1 are the same and therefore, this must also be equal to v_{out} by v_1 alright.

$$= \frac{v_{out}}{v_1}$$

So, in one shot I mean we have found all the three transfer functions right.

But, if we did not know this adjoint concept what we would probably have done is you know assume these two do not exist and what is the transfer function from v_1 to v_{out} minus R_2 by R_1 right and then you know if you open your eyes carefully and see then you would see that over the transfer function v_1 and v_2 are in series and therefore, there is no point in doing the analysis all over again. It is the transfer function from v_1 to v_{out} is the same as a transfer function from v_2 to v_{out} and you say oh well that is also minus R_2 by R_1 right.

From v_3 what is the transfer function from v_3 to v_{out} in the original network? Well, you assume v_1 and v_2 to be 0. This potential the virtual ground potential will be well you would say this is 0 ok and this would be this would be v_{out} will be minus v_{in} right and that is you know fortunately also what is predicted by the adjoint is that clear.

So, to summarize therefore, the concept of inter reciprocity is basically a profoundly important one and is used all the time in routinely used in circuit simulators.

How many of you are aware of this before? I mean I can understand the undergraduate is not seeing this, but our yeah. So, as you can imagine right even experienced students here have not seen this right and it is also quite strangely also not known to most circuit designers that you know you can use reciprocity with a small twist and you know it is not some exotic thing you know that is only there in the text books.

So, it is actually there routinely when you do spice on your circuits this what this is what is being used. In fact, you will see going forward that is actually as I was mentioning it is almost a trivial manipulation of the metrices that allow us to do this right, and as we will see going forward later in this course when we start talking about time varying circuits you will find that the same concept is also used to derive a whole bunch of quite interesting things right.