Circuit Analysis for Analog Designers
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Lecture - 08
Inter-reciprocity in linear time-invariant networks (Contd)

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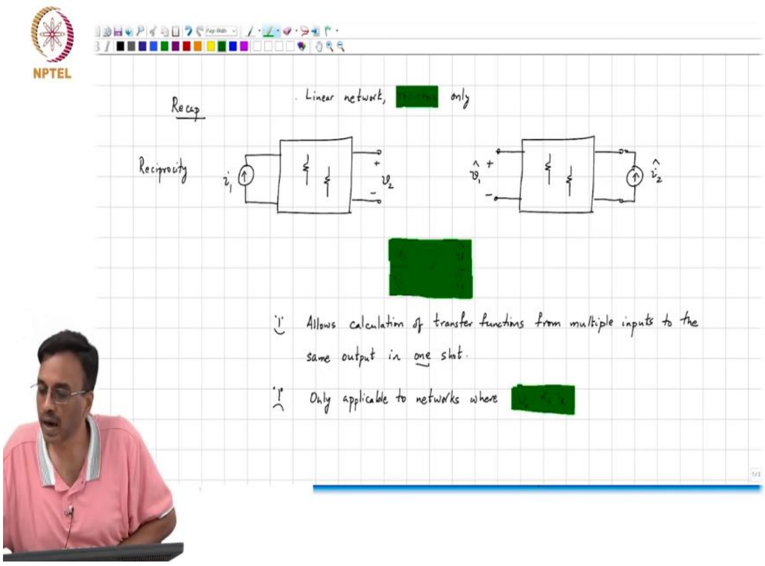

Alright. Good morning everybody welcome this is advance electrical networks lecture 4. A quick recap of what we were doing the last time around. We saw that if you had a network with only bilateral elements namely things like resistors, inductors and capacitors where you can relate either the instantaneous voltage and instantaneous current by resistance or the voltage phaser with the current phaser by an impedance.

Then you can within course interchange the location of the excitation and the response as illustrated in this picture and we saw last time that you can use Tellegen's theorem to show that v 2 by i 1 is the same as v 1 hat by i 2 hat in the network on the right.

$$\frac{v_2}{i_1} = \frac{\hat{v}_1}{\hat{i}_2}$$

Now, so, apart from you know the mathematical curiosity of this and its elegance it is got a very practical application and that application is the following. Many times a particularly you will appreciate this when we start doing noise calculations many times it turns out that you are interested in transfer functions from multiple inputs to the same output ok.

Now, that you know the simple minded way of solving such a problem would be the first instinct as many of you pointed out in the last class would be to use network analysis to find the transfer functions from one source to the output one at a time using super position correct.

It turns out as we again discussed in the last class that there is a cleverer way of doing this, if you exploit the fact that the network only consists of elements where you can relate the branch currents to the branch voltages you know in this fashion and that is the following.

So, you excite the output port right and then look at the voltages or currents you know through the independent sources inside the network. And this way you solve the network only once, but in one short we are able to get all the transfer functions that we want without having to evaluate the network multiple times ok.

Now, that is you know one of the advantages of or one of the practical uses of reciprocity; well there is a still an issue and that is that you know while this is great you know most practical networks that we deal with as a you know analog circuit designers are network set consist of controlled sources right. Voltage controlled voltage sources, voltage controlled current sources CCVS and VCCS.

And clearly there every branch voltage cannot be expressed as a linear function of its branch current because the branch voltage could be dependent on you know the voltage across some other branch if it is a voltage control voltage source and so on right. So, this is a great seemingly useful idea unfortunately its applicability seems to be limited in practice simply because you know we do not deal all the time with networks where every branch voltage is related to its branch current.

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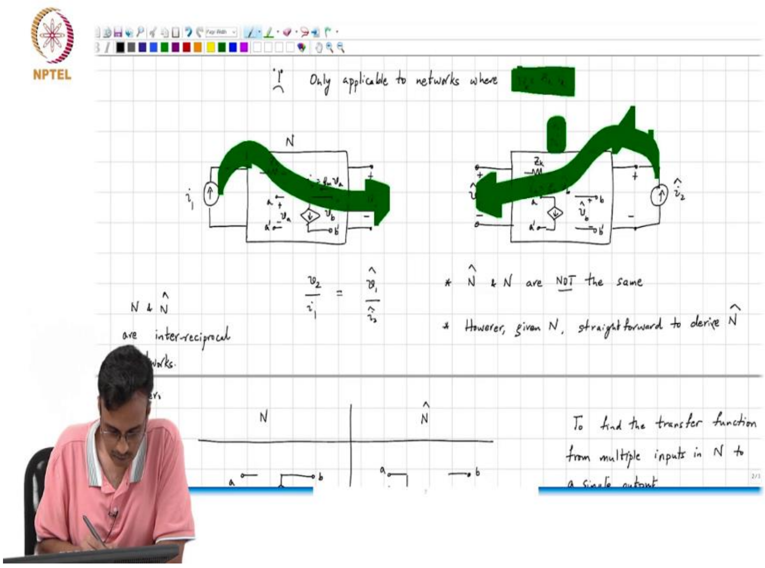

So, one way around it as we saw last time was to recognize that the problem with reciprocity as we saw it occurred only as far as the this is i_1 this is v_2 and this is the network N. And as far as the branches where you have, I believe we call this a and a prime and b and b prime, this is v_b prime and this is i b, I mean sorry this is v_a and v_b, this is i_b which happens to be $g_m \times v_a$, ok this is r k or z k whatever.

So, when we did the, did it try to relate the port currents of when we wrote Tellegen's theorem we found that well it all works out except for the contribution of this controlled source and we said well you know we found within codes a hack right. We said, well it seems pretty straightforward to fix that problem and that is well we just move the controlled source this way. So, this is $\hat{v}_b$ and this current $\hat{\imath}_a$ is. What we see last time? g m times v b hat right.

$$\hat{\imath}_a = g_m \, \hat{v}_b$$

If we did this then you know all the terms other than the port voltages and port currents when you write Tellegen's theorem there simply cancel out and we see that like with a network with bilateral elements only we again have v 2 by i 1 is v 1 hat by i 2 hat right.

$$\frac{v_2}{i_1} = \frac{\hat{v}_1}{\hat{\imath}_2}$$

So, this network as you can see can you comment on whether this network and this network are the same? Are N and N hat the same? They are not the same because we have taken that controlled source and flip the controlled and the controlling ports right. That seems intuitively reasonable because in this in the network N you know signal flows from signal flows like this, right. And we have some transfer from v_2 to i_1 and if you want the same transfer from $\hat{\imath}_2$ to $\hat{v}_1$ it seems reasonable that the controlled source points the other right.

So, I am going to just right down. So, please note that $\widehat{N}$ and N are not the same; however, given N you know, what comment can you make about $\widehat{N}$ I mean is it like rocket science to find $\widehat{N}$ or what? It is just a matter of flipping the controlled and controlling ports in the in N to arrive at, is that clear people? Alright. Now, if there were multiple voltage controlled current sources what comment can you make about what I am going to do to derive $\widehat{N}$?

Very good. So, if you have multiple voltage controlled current sources what you want to do is simply flip the orientation of all the voltage controlled current sources to derive this $\widehat{N}$. And another aspect that I like to point out is that if g m is set to 0 right; whether you have N or whether you have $\widehat{N}$ please note that the port impedances at between a, a prime and b, b prime remain the same is that clear. So, if you set g_m to 0 what comment can we make about the impedance between a and a prime?

It is an open circuit and likewise between b and b prime it is also an open circuit. What comment can we make about $\widehat{N}$ if we set g_m to 0? It is the same is that clear very nice.

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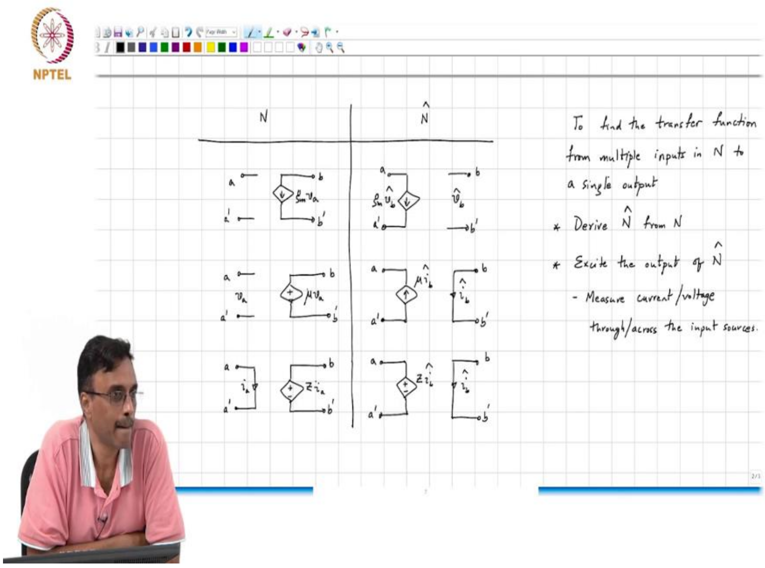

So, therefore, in N if we had a voltage controlled current source. So, this is a, this is a prime, this is b, this is b prime, this is $g_m v_a$, then in $\hat{N}$ we have a, a prime, b, b prime, v_b prime, this is $g_m \ \hat{v}_b$, alright, ok.

Now, this is a voltage controlled current source. So, I am not going to sit, I mean here we did the derivation and found this you can do the same derivation for the other controlled sources there I will give that as a part of the homework. So, that you are convinced, but I am going to only put down the final result here. Let us say you know use our intuition to figure out what we are going to do, let us if the original network had a voltage controlled voltage source.

So, this is v_a, this is $\mu \, v_a$ what comment can we make about what we need to do in $\hat{N}$? This is a, this is a prime, this is b, this is b prime I mean we can derive it, but which I will leave for the homework, but what should you intuitively expect based on experience so far?

Very good. So, remember the controlled and the controlling ports must be interchanged. So, b, b prime must be the controlling port and remember another aspect is that if mu is 0 you want the port impedances to all be the same. So, if mu is 0 between b and b prime in N what comment can we make about the impedance we have? It is a short circuit. So, this must be the controlling port and must be a short circuit alright and. So, this must be a current controlled source right and what must be there between a and a prime in N hat.

Well, its impedance must be infinite when mu is 0. So, therefore, this must be current controlled current source. So, if this is i b hat this must be mu times alright. A common cause for confusion is the direction of the current. And what can we say about the direction of the current? How what is the good way of remembering what the direction is? Remember in the original network if you yank v a up. So, if v a increases right what happens what tends to happen to v b? v b will increase assuming mu is positive.

Now, we turn the source the other way and so, if v b increases right v a must tend to, must also tend to increase. So, v b increases i b that i b hat will tend to flow in that direction we have shown for that direction we must expect the voltage between a and a prime to increase; and how will voltage across a and a prime increase? When you push current into a, correct.

So, that is an easy way of remembering you know what direction you need to put the current source is that clear ok. So, now, without further or do let us write up the other cases. So, one is a I mean in fact, there is only one other case. What is that other case? Current controlled Voltage source, because the current controlled current source is you know is the same as the voltage-controlled voltage source right if you have a voltage-controlled voltage source we replace it with the current controlled current source in the opposite in the with controlled and controlling port flipped.

Likewise, if you had a current controlled current source, you would replace it with a voltage-controlled voltage source with the controlling and the controlled port flipped. If you have a current controlled current source so, this is i a, this is mu time i a, this is b and b prime. What should we do? This is a, a prime sorry this is the current controlled voltage source. So, this is say z times i a, this is b, this is b hat ok. Let us see what we should do; any thoughts? So, which is the controlled port and which is the controlling port?

B is the controlling port and what comment can we make about the what kind of controlling import is a is the voltage across b controlling? It is the current controlled port. So, this is i b hat and what comment can we make about the controlled port? It must be a voltage source; this is z times n alright ok. So, therefore, in summary if you have an arbitrary linear network now because the 4, I mean the 3 R L C passive elements plus all the 4 controlled sources right with these 4, you can form any linear network that you want correct.

Now, if you are interested in finding the transfer functions from multiple inputs inside the linear network to a single output right you do not need to despair all that you need to do is what? What is the to find the transfer function from multiple inputs in N to a single output.

What should you do? What is the first step? Derive $\widehat{N}$ from N correct. And how do you derive $\widehat{N}$ from N? Well, you simply look at this table right all the bilaterally elements namely the R L and C remain the same and all you need to do is flip the controlled sources as per this table right which you know how to derive ok and therefore, you have $\widehat{N}$. Then what you do?

So, excite the out, excite the output of which port of which network? Excite the output of $\widehat{N}$ right to the appropriate current voltage excitation that the situation warrants and right. Measure current or voltage right through or across the independence source. Does it make sense?

So, you know as we points out I mean you know it does not make sense to keep calling N hat, N hat, N hat right. So, it sufficiently seems to be sufficiently nifty and important to have a name right. So, N I mean you can see that the ports in N and N hat are reciprocal except that N hat is not the same as N, but can be derived from N. So, N and N hat are called inter reciprocal networks ok and this is called inter reciprocity.