

Circuit Analysis for Analog Designers
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Lecture - 52
Course summary and recap

Good afternoon, everybody and welcome to advanced electrical networks this is lecture 52 and the last lecture of the course. So what I am going to do today is simply an overview of what we have done in this course so far.

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- * Circuit equations \rightarrow MNA formulation
 - \rightarrow Tellegen's Theorem
 - \rightarrow Reciprocity and interreciprocity in LTI networks
 - \rightarrow Adjoint network
- * Filters
 - \rightarrow Approximation (Butterworth Magnitude)
 - \rightarrow Cascade of biquads realization of high-order lowpass filters
 - \rightarrow Opamp-RC integrators and biquads
 - \hookrightarrow Parasitic insensitive

So, we started off learning about reviewing KCL and KVL and writing circuit equations using the MNA formulation. And it turns out that the modified nodal analysis formulation is what is used in spice to set up all the circuit equations and then, they are solved numerically alright. And then we also saw Tellegan's theorem and we saw reciprocity and inter reciprocity in linear time invariant networks.

And the key conclusion there was that the concept of inter reciprocity which we have also seen during our study of LPTV systems is extremely important.

Extremely useful in noise calculations and a basically in situations where you need to find a multiple transfer functions.

When you have multiple inputs and a single output, right. And another thing that was is a slight departure from what is done normally is we in earlier classes you have probably learnt that I mean you learned to prove reciprocity using Tellegan's theorem right, but actually that is only a way of proving it but, a more intuitive way of understanding is to simply using the MNA formulation that we have seen and the I mean the transfer function basically when you transpose the MNA matrix you know it turns out that you can interchange the input and output ports.

And this also automatically leads to the adjoint network. And the adjoint network basically has the MNA matrix of the adjoint is simply the transpose of the MNA matrix of the original network and now you know why the name adjoint makes sense because the transpose of the matrix is often referred to as the adjoint. And then we learnt about we decided to do something useful with all this circuit equation formulation.

So, we learnt about filters as useful signal processing blocks that we know as the name suggests basically select a certain band of frequencies and reject others and we saw in this we saw magnitude approximation we were exposed to one flavor of approximation, so it is Butterworth magnitude approximation. We saw the intuition behind why high order filter where which tries to approximate a equal response needs poles with higher and higher quality factor right.

Then, we saw the realization as a cascade of biquads, realization of high order, low pass filters then while the cascade of biquads was only seen with low pass filters it turns out that its also a common thing to do with band pass filters where they all become a cascade of resonators instead. Once we did that, we a basically try to figure out how to realize a biquad using op amp RC or active RC integrators and biquads the op amp RC integrator is a very is a very versatile and a useful building block.

The utility of this is that this the op amp RC is integrator is parasitic insensitive and so it is and thanks to the op amp the integrator is also extremely linear. Then, we saw that it is not merely enough to just make biquads and cascade them we also need to figure out biquad ordering as well as the need for dynamic range scaling right.

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NPTEL

- Op-amp RC integrators and biquads
 - ↳ Parasitic insensitive
- Biquad ordering, dynamic range scaling
- Non-idealities in biquads → finite opamp gain bandwidth product
 - Q-enhancement
- * Noise in linear time-invariant circuits
 - Noise spectral density
 - Noise through a transfer function
 - Noise in passive RLC networks
 - Bode's Noise Theorem $\frac{2kT}{16L} (G_1 - G_2)$

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Then we saw what happens with non-idealities is in biquads and basically the effect of finite op amp gain bandwidth. There we saw that if the op amp you know has a finite bandwidth, then well the integrator instead of you know being ideal basically has with it some delay.

And when you put two integrators inside a loop, then each integrator has got a delay and you have feedback loop. So, when you introduce delay into a feedback loop then.

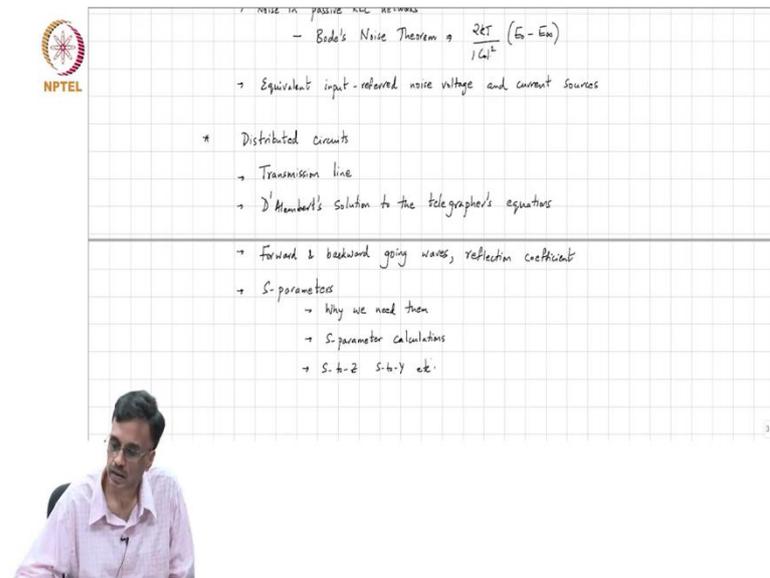
It the degree of stability decreases in other words the quality factor of the poles becomes higher, right. So, we saw that there is going to be yeah Q enhancement, alright.

Then we moved on and we understood the we understood that there is a limit to the peak input signal that you can put into the filter, right. So that is on the positive side the maximum you know rather that is the largest input signal that you can put in. Well, the smallest input signal you can put in is basically limited by noise, so then we went about studying noise in linear time invariant circuits and we understood the concept of noise spectral density, we understood the concept of noise I mean how to calculate, noise through a transfer function that is given a noise spectral density.

What would happen if this is processed by a linear time invariant, right. Then, what did we say then we learnt about noise in passive RLC networks and learnt that if you are only interested in the total integrated noise, you can do it very easily using what is called the

Bode's noise theorem and it can be seen to be $\frac{2kT}{1 \text{ col}^2} (E_0 - E_\infty)$.

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Then we also saw the notion of equivalent input referred noise voltage and current sources. In other words, if you have a complicated network with a lot of noise sources as far as the behavior with respect to the ports is concerned you can.

You can represent, you can combine all those noise sources inside there could be any number of them into one equivalent input noise voltage source and another noise current source and we saw how to calculate those voltage and current sources. And in general, the noise voltage and the noise current source will be correlated. Then, we went off and started looking at so far, we only looked at circuits that are lumped in the sense that we saw R L and C.

Then, we looked at distributed networks and the element that we learnt about was the transmission line, right. So, we saw D'Alembert solution to the wave equation or in our case these are called the telegrapher's equations, the concept of forward and backward going waves and reflections. Then, we saw then we learned about this naturally led to this S parameters and here we saw why we need them.

And why the old set of parameters named is z y h and g why they are not adequate in practice, because all those parameters need to be measured with some port being either open or short.

And in a practical measurement set up whenever you have an open or a short. It depends on the length of the cable and so on and then more many times its entirely possible that the system becomes starts to oscillate. S parameters on the other hand basically work by terminating the transmission line. So, there is a real practical reason why these S parameters are there. Then we saw how to make simple S parameter calculations ok and you know and you know the transformations between S to Z and S to Y etcetera.

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NPTEL

- S-h-z S-h-y etc.
- * Linear Periodically Time Varying Circuits
 - Zadeh's Expansion
 - Harmonic transfer functions
 - MNA equations for LPTV networks →
 - Reciprocity and inter-reciprocity in LPTV networks
 - * time-reversal for the time-varying elements
 - N-path principle
 - Chopping in amplifiers

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Then, we moved on to linear periodically time varying circuits. And this basically involves so we saw this are the expansion and learnt about the notion of harmonic transfer functions. Then, we learnt how to write the MNA equations for LPTV networks in the frequency domain, right. Where now you have the idea is very straightforward. Earlier in a time invariant case you just had one phaser. Now, you have to keep track of, yeah you have to keep track of all frequencies of the form f plus k times f s right and but if you keep I mean the form of the equations is very similar to what you see in the linear time invariant case.

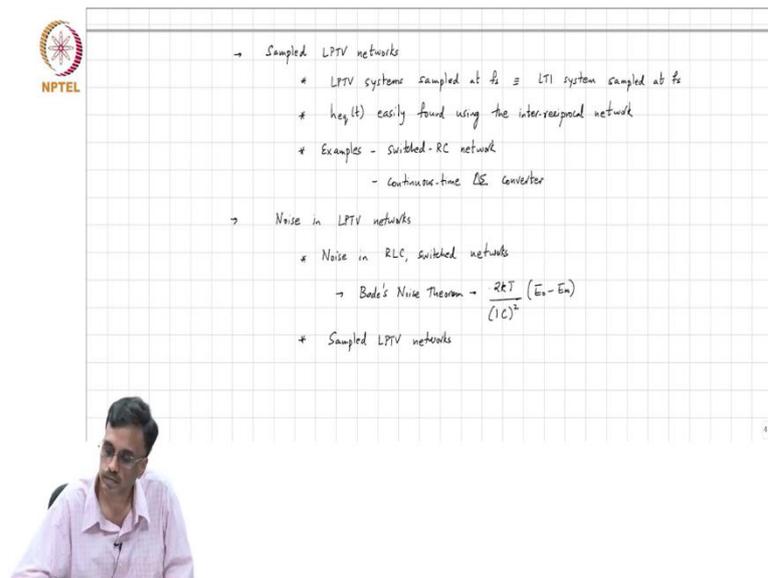
Only each element will be is a vector right, ok. And then, this automatically led to the concept of reciprocity and inter reciprocity in LPTV networks and the key difference between the time invariant case and the time variant case is that the key concept is you had

have to use time reversal for the time varying element time varying element, ok. And the next thing that we saw was we applied as an application of the theory that we studied, we looked at a few cases, we looked at N path, we looked at the N path principle which is simply a way of making sure that if you put N LPTV networks offset in phase by a fraction of the period.

Yeah, then it appears as if the, Yeah, the systems varying at a rate of n times f s. Even though in reality none of this I mean none of those systems are actually they are all varying at f s, but it is simply a way of combining the outputs of all these systems. So, that it effectively looks like a system that varies at n f s.

And then we also saw the application of reciprocity and inter reciprocity concepts in evaluation of in simplifying the analysis of chopped amplifiers.

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NPTEL

- Sampled LPTV networks
 - * LPTV systems sampled at $f_s \equiv$ LTI system sampled at f_s
 - * $h_{eq}(t)$ easily found using the inter-reciprocal network
 - * Examples - Switched-RC network
 - Continuous-time LC converter
- Noise in LPTV networks
 - * Noise in RLC, switched networks
 - Bode's Noise Theorem $\rightarrow \frac{2kT}{(1C)^2} (E_{in} - E_{out})$
 - * Sampled LPTV networks

Then, we saw, then we said that oh well a whole lot of practically useful networks are all sampled are not only LPTV, but also sampled right and the key concept here was that.

The LPTV System Sampled at f s. Yeah, I mean if the LPTV system is varying at f s and if its output is also sampled at f s this is equivalent to an LTI system whose output is sampled at, right. And the h equivalent of t which is the impulse response of the equivalent linear time invariance system can again be easily found using the inter reciprocal networks and we saw some examples right and again we saw that in the case of the switched RC

network we found the h equivalent of t, we also switched RC network and we also saw another very useful LPTV system namely the continuous time delta sigma converter, right.

So, for those of you taken an r f class or data converter class you have probably seen this before, but if you have not just treated these as LPTV systems that you need to study I mean that are illustrations of what we concepts that we study. Then with that then we saw noise in LPTV networks and specifically noise in RLC switched networks and perhaps quite surprisingly we find that the Bode's noise theorem still applies and that is basically the mean square noise is again $2 kT$ by 1 coulumb square times E_0 minus E infinity and this is exactly the same result that we got for an RLC network.

Then, of course, if your sample I mean with sampled LPTV networks its very straightforward, you just find the equivalent noise a transfer function from the noise source to the output that is an LTI system and you can find the corresponding noise spectral density and so on. So, that was basically all that we covered as far as the LTI the time varying circuits and systems was concerned.

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 * Noise in LPTV networks
→ Bode's Noise Theorem → $\frac{2kT}{(1C)^2} (E_0 - E_\infty)$
* Sampled LPTV networks

* Weakly nonlinear circuits
- HD_2 , HD_3 , HD_x
- Inter modulation distortion

- Method of current injection
→ Replace solution of a nonlinear differential equation with multiple solutions on a linear network
→ Example of distortion in a negative feedback loop

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Then we saw we learned about weakly non-linear circuits and we understand HD 2 HD 3 and basically HD x, right. We also understand the concepts of inter modulation distortion right and finally, we learnt about the method of current injection which is very useful technique to understand nonlinearity and analyze nonlinearity in a weakly non-linear

circuit and the basic idea is to replace the solution of a non-linear differential equation with multiple solutions on a linear network, alright.

And we saw an example of distortion and negative feedback right. And so, the background that you gained in this course basically applies in multiple situations you know whatever electronic system you are designing, right.

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NPTEL

- Intermodulation distortion
- Method of current injection
 - Replace solution of a nonlinear differential equation with multiple solutions on a linear network
 - Example of distortion in a negative feedback loop
- * Noise ✓
- * Distortion (or linearity) ✓
- * Distributed effects ✓
- * LPTV networks ✓
 - sampling ✓
 - mixing ✓

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So, in all electronic systems noise is a concern, distortion or linearity is a concern, right. In high frequency circuits, distributed effects are concern right and of course LPTV networks are there everywhere you use sampling or you use mixing ok. And by taking this course and studying all these topics, we have gained insight hopefully into all these aspects which therefore should keep you in good stead when whenever you are working with electronics.

Thank you very much and I will stop you.