**Circuit Analysis for Analog Designers**
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**Lecture - 61**
**Current - injection analysis of distortion in a negative feedback system**

Good evening, everybody, welcome to advanced electrical networks, this is lecture 51. In the last class, we were looking at; good evening, everybody and welcome to advanced electrical networks, this is lecture 51.

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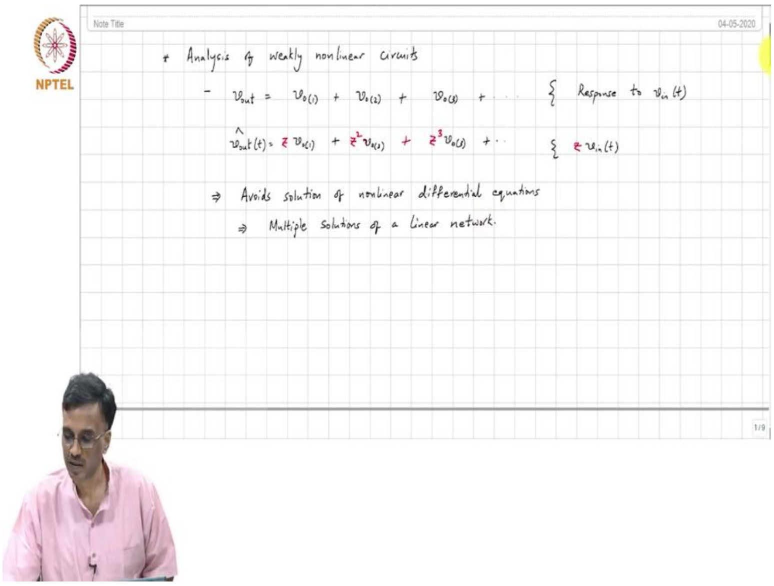


In the last class, we were investigating the analysis of weekly non-linear circuits. And we saw that we could write the output v out as the sum of a linear term a second order term, where yeah. So, it is s terms due to the second order and v o 3, which basically is due to terms of both including both the second and third order and so on.

$$v_{out} = v_{o_{(1)}} + v_{o_{(2)}} + v_{o_{(3)}} + \cdots$$

And if the input is scaled by so; if this is the output in response to v in of t. Now, if v in of t is scaled by a number z then v out of t will be also changed. But the linear term will be scaled by z.

This will get scaled by, the second order term gets scaled by z square and the third order term will get scaled by z cube, right etcetera.

$$\widehat{v_{out}}(t) = Zv_{o_{(1)}} + Z^2 v_{o_{(2)}} + Z^3 v_{o_{(3)}} + \cdots$$

Then we saw that basically this approach, where you split up the output into something which scales as z, something which scales as z square and something that scales as z cube is basically avoiding solution of a non-linear differential equation.

And instead replaces it with multiple solutions of a linear network, ok. So, today let us take some simple examples and see that convince ourselves that this indeed works.

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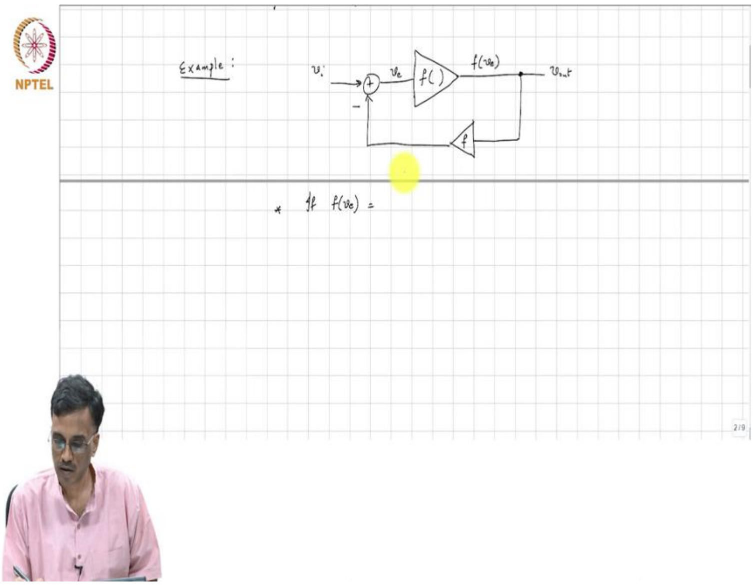


So, we will take a block diagram representation of a feedback system. So, this is v i, this is v out alright and well the this, is the error and this is some function of the $v_e$, right.

So, if f of v e is simply or let me call that G, because you have already used f for the feedback factor.

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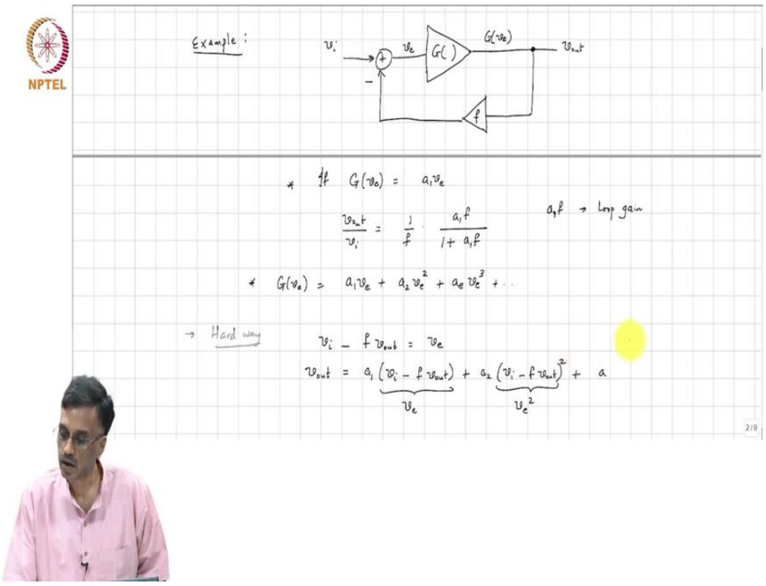


So, right if G of v e is nothing but a 0 v e; then this is nothing but a linear feedback system and v out by v i is nothing but 1 over f times A 0 f by 1 plus A 0 f as a very well-known formula and this a 0 f is the loop gain and you know you do the math and right.

$$If\ G(v_e) = a_0 v_e$$

$$\frac{v_{out}}{v_i} = \frac{1}{f}\frac{a_0 f}{1 + a_0 f}$$

And as you know a 0 f tends to infinity, the closed loop transfer function becomes independent of the loop gain and only depends on the feedback factor, alright.

Now, the question is, what happens when G of v e is non-linear? So, it is let us say it is a 0 v e plus; actually, I am sorry just let me change the notation a little bit, let me make that a 1. So, this becomes a 1 and this becomes a 1.

$$If\ G(v_e) = a_1 v_e$$

$$\frac{v_{out}}{v_i} = \frac{1}{f}\frac{a_1 f}{1 + a_1 f}$$

So, the next question is what happens when you have G of v e is a 1 v e plus a 2 v e square plus a 3 v e cube, right? And typically, there will also be higher order terms, but we will neglect the stuff beyond the third order, ok.

$$G(v_e) = a_1 v_e + a_2 v_e^2 + a_3 v_e^3 + \cdots$$

So, the hard way is what is the hard way? I mean remember here there is no memory of any sort, correct.

So, everything is ah memory less. So, well you just only get; you do not get non-linear differential equations, you get just non-linear equations, right. So, what would you do? Well v i minus f times v out equals v e and v e equals sorry and v out equals A 1 times v e which is v i minus f v out, right. So, this is v e plus a 2 times v i minus f v out the whole square, this is v e square plus a 3 times v i minus.

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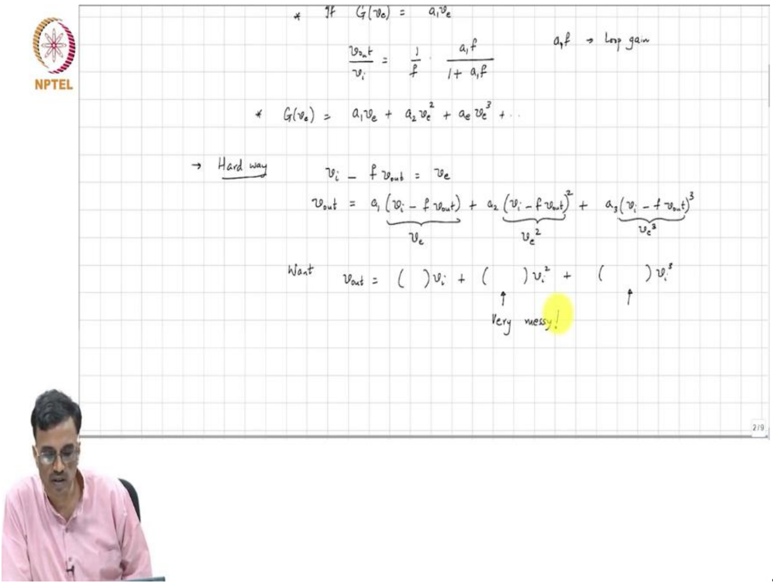


F v out the whole cube, so this is v e the whole cube, ok.

$$v_i - f\, v_{out} = v_e$$

$$v_{out} = a_1\,(v_i - f\, v_{out}) + a_2(v_i - f\, v_{out})^2 + a_3(v_i - f v_{out})^3$$

$$v_i - f\, v_{out} = v_e$$

$$(v_i - f\, v_{out})^2 = v_e^2$$

$$(v_i - f\, v_{out})^3 = v_e^3$$

What we are interested in is; we want v out to be of the form something into v i right plus something into v i square plus something into V i cube right

$$v_{out} = (\ )v_i + (\ )v_i^2 + (\ )v_i^3$$

And of course, if a 2 and a 3 was 0; then it is very straight forward right, this you know. We can find it is a linear equation and you can find the unknown. But now because these v out square terms and the v out v i term and you know you have whole bunch of cross terms right; finding these terms right as very messy and right. And if you are not convinced, you can try it out on your own. You basically you as you can see to solve for v out, you have to solve a; I mean you basically end up having to solve a non-linear equation of the third order in v out and you know it is very painful, right. Now, we can do something simpler based on our knowledge of current injection.

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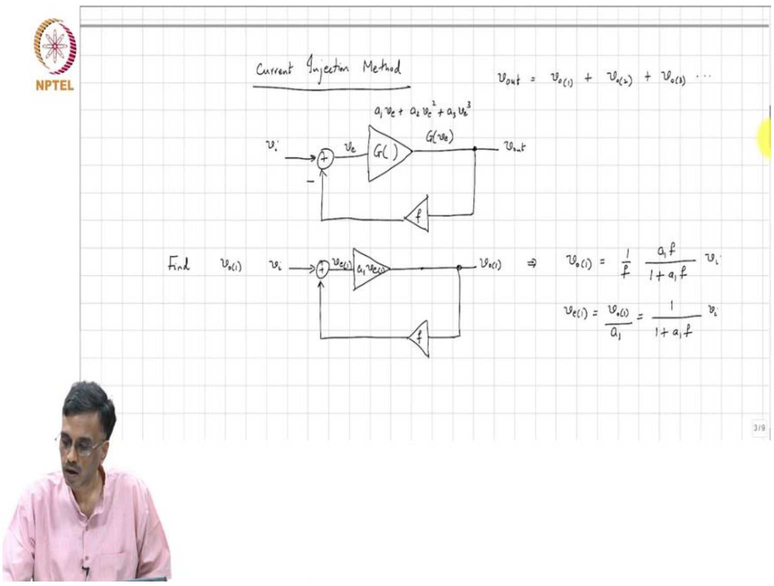


So, what is the premise; v out is nothing but v o 1 plus v o 2 plus v o 3 etcetera, correct. And what is and remember that G is nothing but a 1 v e plus a 2 v e square plus a 3 v e cube, alright.

$$v_{out} = v_{o_{(1)}} + v_{o_{(2)}} + v_{o_{(3)}} + \cdots$$

$$G(v_e) = a_1 v_e + a_2 v_e^2 + a_3 v_e^3 + \cdots$$

Now, finding v o 1, what do you do? So, to find v o 1 what we will do is. Consider only the linear term. So, this is the a 1 v e, right. So, this v e in this circuit is also v e 1 right; this is f and this is v o 1 and that is very straightforward, v o 1 is nothing but 1 by f times A 1 f by 1 plus a 1 f, ok.

$$v_{o_{(1)}} = \frac{1}{f}\frac{a_1 f}{1 + a_1 f}$$

And what is v e 1? V e the easier way to find out is, we know v o 1, that divided by a 1 is v e 1. And that is nothing but 1 by 1 plus a 1 f, ok. Oh sorry v i this times v i, correct. So, we found everything that there is in this network.

$$v_{e_{(1)}} = \frac{v_{o_{(1)}}}{a_1} = \frac{1}{1 + a_1 f} v_i$$

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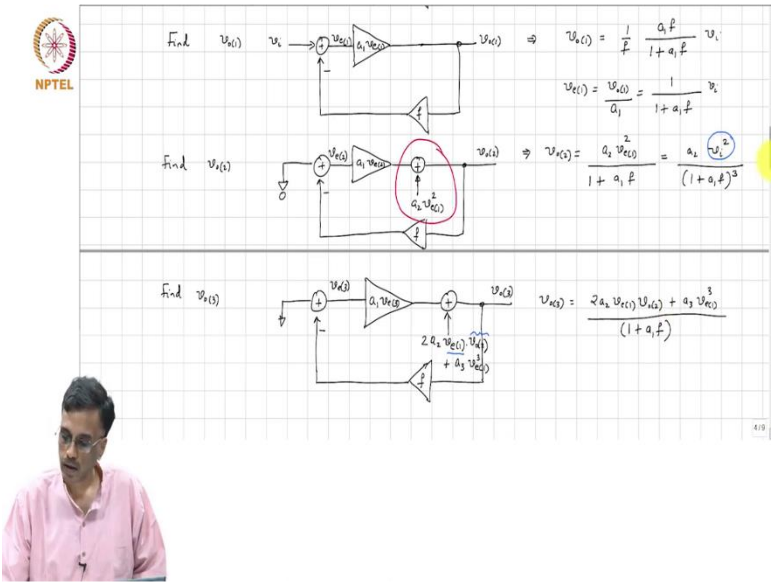


Now, the next thing is find v o 2; what you do now? Well we set the input to 0. And what should we inject? We have this is going to be v e 2. This is going to be a 1 times v e 2 right and we have to inject the. We have to inject, what are we supposed to do?

Student: We have to inject the current of a 2 times v e v often, we have to get the sum of current that is proportional to the v of 0 1 minus v e.

So, basically you inject v e 1. So, that is basically inject a 2 v e 1 square. You understand. So and this is v o 2, correct. And so, what is v o 2? What is it, I mean this is a linear feedback loop and you are adding some error here.

So, what is the transfer function from there to the output? Yeah. So, this is nothing but a 2 v e 1 Square By 1 plus a 1 f, right. And what is that? Well, a 2 as v e 1 is nothing but V i by 1 plus a f A 1 f. So, you are to square it. We get 1 by 1 plus a 1 f the whole cube, alright.

$$v_{o_{(2)}} = \frac{a_2 \, v_{e_{(1)}}^2}{1 + a_1 f} = \frac{a_2 v_i^2}{(1 + a_1 f)^3}$$

And as we expect if you double the amplitude of v I, this V out 2 will go up by a factor of 4, ok.

Now, what we say was the next thing is, find v o 3, alright. So, what do we do? How do we figure this out?

Student: V out will be injecting the (Refer Time: 16:20) another current source like where two current sources and the.

This is v e 3, this is a 1 times v e 3 and we had injected something here, right. And what was the stuff that we were supposed to inject?

Student: That a 2, a 2 times a 2 times v error 1 into v error 2. Like we have to inject 2 more current one is a 3 times v error 2 volt to volt q r will come Then the current a 2 times v error of 1 times v error of 2.

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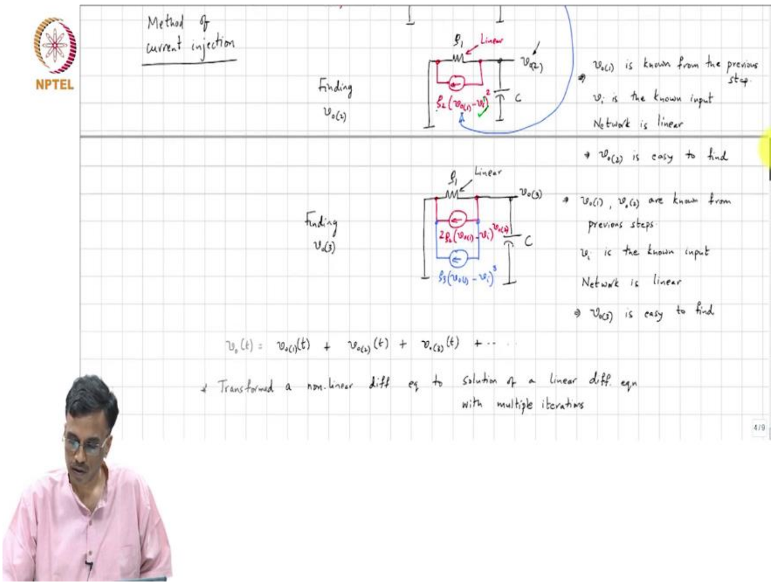


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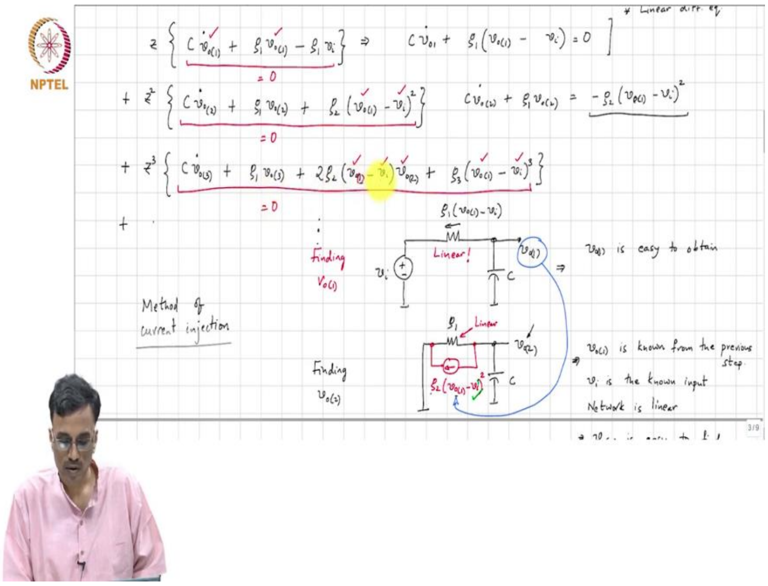


Let us go back to our notes. As you can see here, you have when we go back to our notes here, you can see that what did we do in the, second order term is straightforward right; we basically do it to G 2 times the all the second order current that is flowing the. In the third order case right, of course the cubic term is very easy to understand correct; but you also need to what you call.

You also need to have ah a second order dependence, right. And what is the second order dependence that we have? So, this is if you stare at this picture. So, you have a component which is v o 1 in the what you call in the third order network; this term is easy to understand correct, that simply the third order nonlinearity, right.

So, you also need to have v o 1 minus v I times V o 2. Right. So, that s basically what is this? This is in our context this corresponds to the voltage, this is this in our context this corresponds to V error 1 times v output 2, ok. So, and then there is a factor 2, that is the cross term, right. So, this should be 2 times G 2; actually this the there is yeah, this is fine, this must be 2 a 2 times V error 1 times v o 2 correct and plus A 3 times V error 1 the whole cube, alright, $\left(2a_2 v_{e_{(1)}} v_{o_{(2)}} + a_3 v_{e_{(1)}}^3\right)$.

A sanity check is that, v o 2 goes as, I mean remember that v o 3 must go as z; if you increase the input amplitude by a factor of 2, v o 3 must go as a factor of, As a by a factor of 8 correct and well v error 1 cube; v error 1 directly proportional to the input. So, therefore, if you double the input v error 1 the whole cube will go, v error 1 will double and v error 1 the whole cube will go up by factor of 8. V error 1 of course is directly; if you look at this term, v error 1 is directly proportional to the input, v o 2 on the other hand is proportional to the input Square and therefore, this whole product will go as By a factor of 8, right. So, that is the sanity check whenever you know ah are in doubt; you got to make sure that in the third order ah what you call ah to find the third order response, what you want to do is make sure that, if you double the input, the second order response must go by go as z square I mean as 4.

And the third order response must go as 8. So, what do we see here now? So, v o 3 therefore, must be equal to 2 a 2 times v e 1 V o 2 Plus a 3 times v e 1 whole cube By 1 plus A 1 f ok.

$$v_{o_{(3)}} = \frac{2a_2 v_{e_{(1)}} v_{o_{(2)}} + a_3 v_{e_{(1)}}^3}{(1 + a_1 f)}$$

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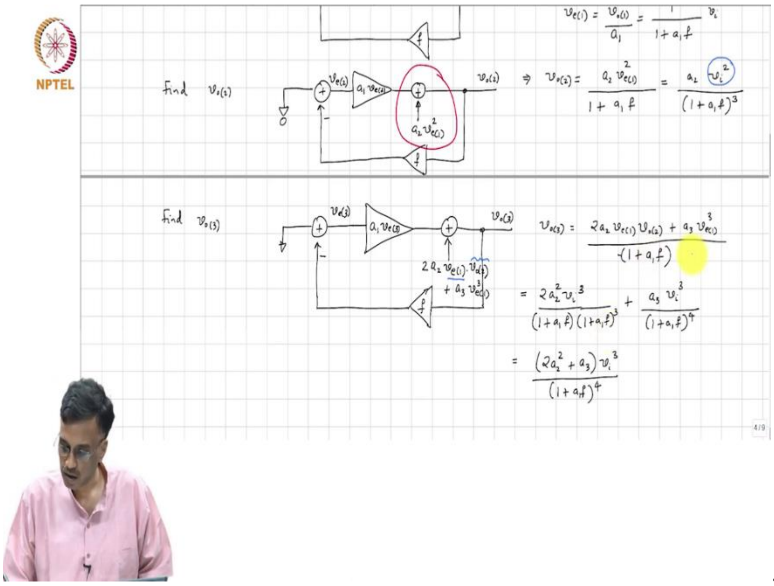


And which therefore, is nothing but 2 a 2; v 1 is already known that is nothing but V i by 1 plus A 1 f right; v o 2 is nothing but v i square. So, the whole thing becomes v i cube Times a 2 Right divided by A 1 f the whole cube. So, when you we can simplify that later plus a 3, v 1 cube is v i cube By 1 plus a 1 f the whole cube times; there is already 1 by 1 plus a 1 f, so that becomes the power 4. So, this is nothing but 2 a 2 square v i; 2 a 2 square plus a 3 times v i cube Divide by 1 plus a 1 f whole power 4, have I missing something.

Student: One factor will be there 1 plus a one f like if we take the first term already 1 plus a 1 f is there in the opposite (Refer Time: 23:56) and from the v error 1 to we will get 1 1 1 plus a 1 f.

Correct that is way I have V error 1 is basically.

Student: We get 1 1 plus a 1 because of v error.

Student: V output 2 we get.

V error 1 is 1 by v i by 1 plus a f is correct, v o 2 is cube.

Student: V o 2 you will get (Refer Time: 24:33) cube will get and another term will be there in the output by 1.

Oh yeah indeed that is right so.

Student: (Refer Time: 24:42)

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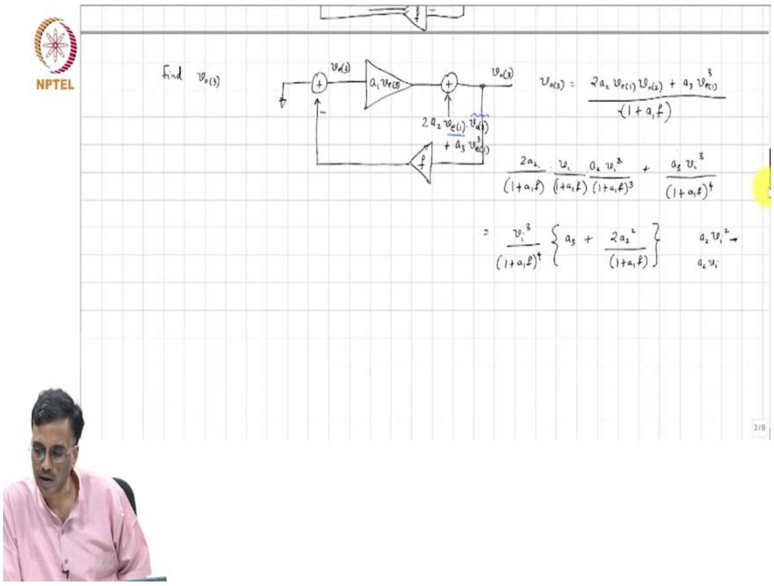

Just do this properly. So, this just becomes 2 a 2; v error 1 is nothing but 2 a 2 by 1 plus a 1 f times v error 1 is V i by 1 plus a 1 f times v o 2 is nothing but a 2 v i square By 1 plus a 1 f the whole cube plus A 3 times v I Cube divided by 1 plus A 1 f whole power 4, right.

$$= \frac{2a_2}{(1+a_1f)}\frac{v_i}{(1+a_1f)}\frac{a_2v_i^2}{(1+a_1f)^3} + \frac{a_3v_i^3}{(1+a_1f)^4}$$

So, basically this is nothing but v i cube by 1 plus a 1 f whole power 4 times 1 plus sorry a 3 plus 2 a 2 square By 1 plus a 1 f, ok.

$$= \frac{v_i^3}{(1+a_1f)^4}\left\{a_3 + \frac{2a_3^2}{(1+a_1f)}\right\}$$

I has checked this some dimensional inconsistencies were ok a 1, a 2 square; I am missing something, hold on see a 2 v square has got dimensions of voltage, a 3 v cube is got dimension of voltage. So, that is all fine. So, when we come here what we, have I miss something here.

A 3 v i cube is fine, a 2 this a f v 1 v i 1, v o 2 is a 2 v i square is voltage. So, a 2 v i square is voltage, v i is voltage. So, this into, sorry hold on a 2 v i square is volt has got dimensions this is a voltage, correct.

A 2 v I Is yeah a 2 v i is dimension. It is correct, a 2 v i is dimension. So, we are ok, alright ok.

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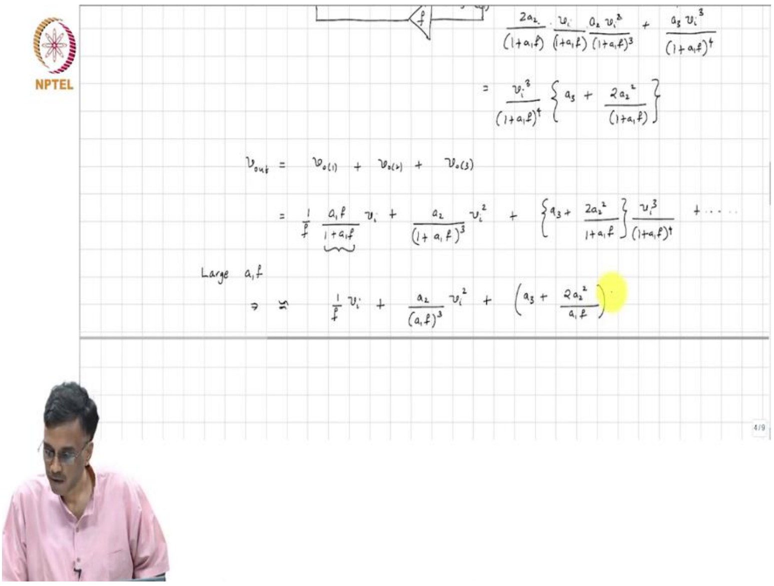

So, therefore, v out is nothing but the linear term which is v o 1 plus v o 2 plus v o 3 which is. This is nothing but a 1 f by I am sorry. This is a o a 1 by f times a 1 f by 1 plus a 1 f times v i plus A 2 by A 1 f the whole cube times v i square plus a 3 plus 2 a square by 1 plus a 1 f Times v i cube by 1 plus a 1 f Whole power 4 and so on, ok.

$$v_{out} = v_{o_{(1)}} + v_{o_{(2)}} + v_{o_{(3)}}$$

$$= \frac{1}{f}\frac{a_1 f}{1 + a_1 f} v_i + \frac{a_2}{(1 + a_1 f)^3} v_i^2 + \left\{a_3 + \frac{2a_3^2}{1 + a_1 f}\right\} \frac{v_i^3}{(1 + a_1 f)^4} + \cdots$$

So, for large loop gain, correct. What do you think will happen to this? This is approximately. No, no no. A 1 f is very large. What common can you make about a 1 f by 1 plus a 1 f? That is 1, right. So, this is basically approximately 1 over f times v i correct plus a 2 by a 1 f the whole cube times v i square plus A 3 plus 2 a 2 square By a 1 f Times V i cube by A 1 f whole power 4.

For Large $a_1 f$

$$\Rightarrow \approx \frac{1}{f}\, v_i + \frac{a_2}{(a_1 f)^3}\, v_i^2 + \left(a_3 + \frac{2a_3^2}{a_1 f}\right) \frac{v_i^3}{(a_1 f)^4}$$