Circuit Analysis for Analog Designers
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Lecture - 59
Weak nonlinearities in circuits: Intuition behind the method of current injection

So, today let us analyze if we find out ways if we can simplify the analysis of circuits that are weakly nonlinear right and as usual.

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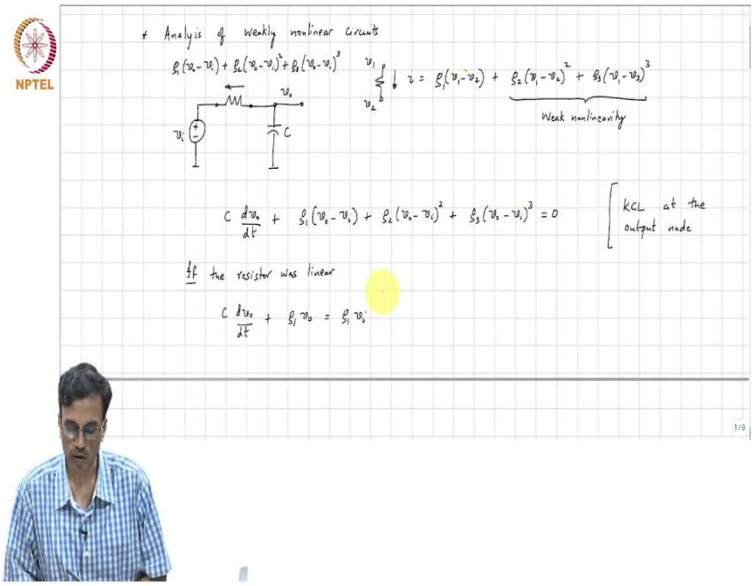

So, as usual we will it makes sense to work with a simple example rather than to complicate our lives by you know drowning ourselves in notation.

So, I am going to take a very simple example, let us say our good old friend the first order RC network. Of course, if the resistance was linear, and then all of you would laugh at it and say you know there is nothing to do right. Now, the question we are going to ask is what happens if the resistance is nonlinear?

So, let us say this is v out and the conductance is basically, there are series v 1 and v 2 and the current i for this conductance here is given by so, G times v 1 minus v 2 plus let me call it G 1 time v 1 minus v 2 plus G 2 times v 1 minus v 2 the whole square plus G 3 times v 1 minus v 3 the whole cube right.

$$i = g_1(v_1 - v_2) + g_2(v_1 - v_2)^2 + g_3(v_1 - v_3)^3$$

And we hope that we will be operating in a range that the higher order terms can be neglected right and the next thing, we also hope is that weak nonlinearity holds that basically means that the strength of these two terms is much smaller than the strength of the linear term.

Right. So, well we know for sure that let us call this therefore, this is G of the function G.

So, this would this current therefore, would be G 1 times v o minus v i plus G 2 times v o minus v i the whole square plus G 3 times v o minus v i the whole cube right.

$$g_1(v_o - vi) + g_2(v_o - v_i)^2 + g_3(v_o - v_i)^3$$

Well, the only thing we know is how to write Kirchhoff's laws. So, we will do that. So, the Kirchhoff's law is telling us that C times d vo by d t plus G 1 times v o minus v i.

Plus no not that. You have take into account the entire current to the resistant not nearly the linear term. Right plus G 2 times v o minus v i the whole square plus G 3 times v o minus v i the whole cube must be equal to 0. So, this is nothing but KCL at the output node alright.

$$C\frac{dv_o}{dt} + g_1(v_o - v_i) + g_2(v_o - v_i)^2 + g_3(v_o - v_i)^3 = 0$$

Now, one thing that I would like to point out is the following. If the resistor was perfectly linear what would we have got?

C d vo by d t Plus G 1 times v o equals G 1 times v i. Right, because the G 2 and the G 3 terms should be 0 right.

$$C\frac{dv_o}{dt} + g_1 v_o = g_1 v_i$$

One observation that I would like to draw your attention to is the following. If this the resistive of linear;

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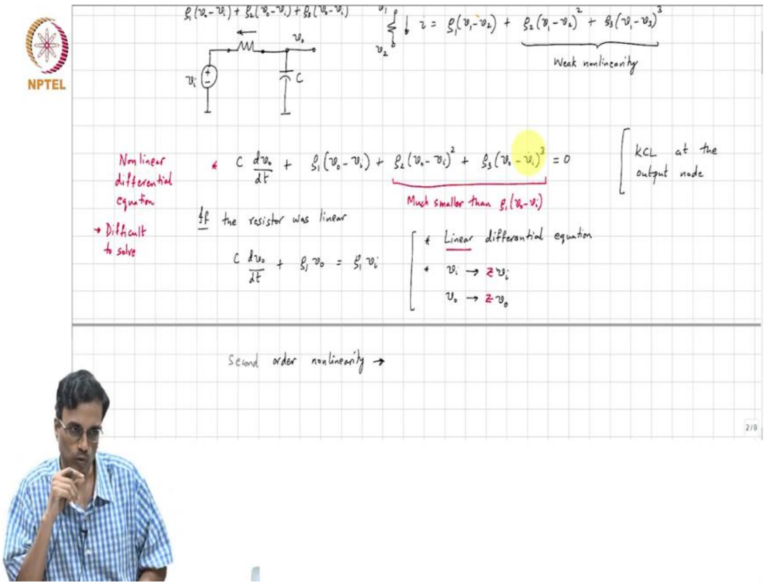

What all do we notice here, this is what kind of differential equation?

It is first order differential equation, because it is one capacitor, but what else what kind of differential equation is it?

It is a linear differential equation right with v i being the input alright. Now, suppose v i is increased to a number z, z is some number right. It is not to be confused with the z transformer anything; it is just simply a number. If I scale v i by a number is z right, what comment can you make? Let us say when v i was the input, let us say v o was the output, now if I scale the input by a number z. What comment can we make about the output?

Very simple the output will also scale by a number. Sorry, this must be z time v alright.

$$v_i \rightarrow Zv_i$$

$$v_o \rightarrow Zv_o$$

Now, based on our experience yesterday when we looked at you know memory less nonlinearities, we saw that whenever you have second order nonlinearity apart from the fundamental you have second harmonic term, which is basically arising because of something multiplying with itself. When you have third harmonic when you have third nonlinearity you get some I mean something proportional to I mean the by itself twice.

So, basically you get error terms which are proportional to the cube of the input amplitude. So, if you have second order nonlinearity we saw a non error at the output or whatever nonlinear terms which are proportional to the square of the input amplitude of the input signal and likewise, if we have a cubic nonlinearity. They would also contain something which is proportional to the cube of the input amplitude right.

Now, let us bear that in mind. The next thing that I would like to draw your attention to is this fact here, I mean this is of course, a linear differential equation as you have pointed out. What comment can we make about this equation?

$$C\frac{dv_o}{dt} + g_1(v_o - v_i) + g_2(v_o - v_i)^2 + g_3(v_o - v_i)^3 = 0$$

This is a nonlinear differential equation right. And in general, nonlinear differential equations are difficult to solve. You might argue that here maybe we can, but more importantly you know when you have a more complicated network you now not only have nonlinear differential equations at every node, because the KCL KVL equations now become nonlinear differential equations and they are all coupled.

Correct, because there is going to be v i whatever v o of this node in one equation then it is going to be appear in some other equation right. And so, therefore in general you will have coupled nonlinear differential equations which might be virtually impossible to solve analytically right.

But the if we are if we know that we still have not I mean of course, a general nonlinear differential equation is very difficult to solve, but what is it that we have not exploited yet. I mean we understand that this is a nonlinear differential equation and therefore, it might be difficult to solve; but we have not exploited something that we know about the problem and what is that.

I mean can G 2 and G 3 be arbitrary. No. So, in other words, what I am asking you is what aspect of the problem have we not used as yet? What information about the problem have we not used as yet?

So, what we have not used is the fact that this part, $(g_2(v_o - v_i)^2 + g_3(v_o - v_i)^3)$, we have not used the assumption of weak nonlinearity. Right. So, we have therefore, we have not used the fact that these nonlinear components are much smaller than G 1 into v o minus v i ok, $(g_1(v_o - v_i))$.

So, that is one aspect. The next thing that I would like to draw your attention to is the following. So, when we saw, when there was second order nonlinearity right; in addition to the of course, a linear term what we see yesterday, if we increase the input amplitude by 1 d v, the within quotes the fundamental goes up by 1 d v; but the second harmony goes up by 2 d v. So, in the 2 d v is coming, because of the x square dependence.

On the input amplitude. So, therefore using the same intuition therefore; let us say our input here in the you know in the system was increased by a factor was changed by a factor z night ok. So, if that z was a very small number, in other words if the input was multiplied by a number z where that z is a very small number, then what terms do you think you can neglect you can forget about in the to find the output?

Yeah. So, if the input is tending to 0, then you can forget about the nonlinear term and you just use the linear differential equation, which you can solve easily correct. Now, if the input is very small, in other words if that z is very small right, and you increase it by a factor of 2. What do you think will happen to the output?

It will be; so, if the input is very small. So, that the weakly nonlinear terms can be neglected altogether; then you increase the input by a factor of 2; the output will simply increase by that same factor.

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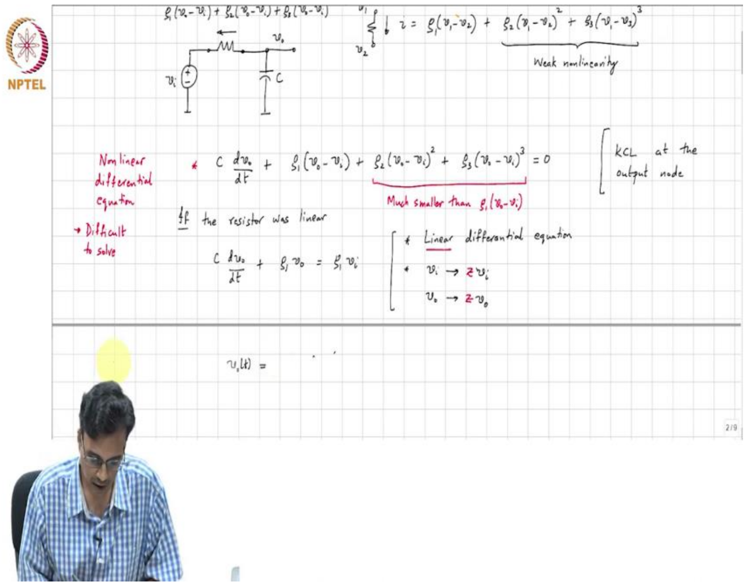

Right. So, in other words the output voltage v o of t right can be, so, let us say.

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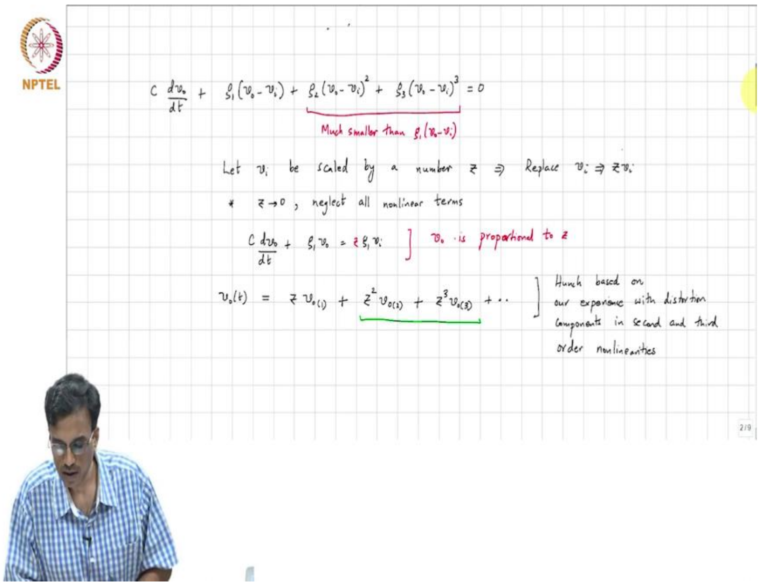

$$C\frac{dv_o}{dt} + g_1(v_o - v_i) + g_2(v_o - v_i)^2 + g_3(v_o - v_i)^3 = 0$$

Again, let me copy and paste this. Let v i be scaled by a number z right. So, in other words you replace v i in the above equation with z times v i right, where z simply a scalar it is a number. So, ok. So, as z tends to 0 he can neglect nonlinear terms.

And therefore, C times d vo by d t plus G 1 times v o is equal to G 1 times v i z one times. Sorry, I should have z here time z here does make sense right.

$$C\frac{dv_o}{dt} + g_1 v_o = Zg_1 v_i$$

Now, as you keep increasing z, what comment can you make about v o? It is a linear differential equation. So, as z is very small, if you keep increasing the input; If you increase input by so, the output v o is proportional to z ok, alright. Now, let us say z starts becoming larger and larger. So, that the nonlinear terms can no longer be neglected right.

So, based on our experience with second and third order nonlinearities, what comment can you make if you go on increasing the input amplitude due to second order nonlinearity, you should apart from the linear term you should also expect a component that is proportional to. So, as you go on increasing z, where the weekly nonlinear terms are can no longer be neglected. We saw with second order nonlinearity; we saw that apart from the fundamental there is also a distortion term whose strength is proportional to Yeah, G two is ok, but more importantly as you keep increasing the in-input amplitude, this the strength of that distortion term is proportional to the input amplitude squared.

Do you understand that? Right. So, if you go on increasing the input amplitude I mean input amplitude therefore, the output must consist of a linear term correct, something which is proportional to. Simply, that v i is constant the only thing I am changing is I am changing z. That the proportionality constant right. So, v o of t therefore, must contain a term which must be proportional to z, because otherwise you know.

You not get the linear term at all right and we will call that the v o 1 right.

$$v_o(t) = Zv_{o_{(1)}} + \cdots$$

Please notice that that 1 in the lower bracket basically, indicates that a it is proportional to z power 1 ok alright. As you keep increasing z, in other words if the input amplitude goes on increasing the you will get, because there is second order nonlinearity you should expect to see.

You should expect to see the output changing with respect to z square. So, you will have some other component which is proportional to.

Ok, and because you have third order on nonlinearity Z cube alright.

$$v_o(t) = Zv_{o_{(1)}} + Z^2 v_{o_{(2)}} + Z^3 v_{o_{(3)}} + \cdots$$

So, the question is I mean. So, this is basically a hunch based on our experience with distortion currents are distortion components in second and third order nonlinearities alright. So, in other words the intuition is the following, when the for a given v i if z is very small right, then the output will simply grow and proportional to z.

If z is very small and that makes intuitive sense, because if z is very small this z square and z cube and all these things become very small in relation to z. As you keep increasing z right, the nonlinear terms start to affect the output right, because of second order nonlinearity you should expect to see an error term which is proportional to z square, you should also expect to see a term which is proportional to z cube and all other higher order nonlinearity right.

So, now, our question is and you know this is you know consistent with what we observed yesterday right. When we put in a fundamental into a second order into a amplifier with a second order nonlinearity, you get a fundamental term which is proportional to z you get a aecond order term right, which is proportional to z square right ok. Because, remember I mean and this has got nothing to do with do not get confused as to I mean do not think that the error only needs to be at the second harmonic right. For example, when we saw gain compression in a third order amplifier, I mean in an amplifier with third order nonlinearity. We saw that the fundamental also gets you know affected by the gain compression term that gain compression term is proportional to a cube Correct.

So, in other words if you put in an input sine wave it does not mean that v 0 2 and v 0 3 etcetera. Only consist of second and third harmonics they can consist of; They can consist of even the input tool you understand right. It is just that v 02 is simply quantifies that error, which scales as z square and v o 3 I mean that I mean basically is you know quantifies the error that scales us as z cube alright. So, now, the question is you know how do you find v 01 v 02 and v 03 and all these higher terms?