Circuit Analysis for Analog Designers
Prof. Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 58
Third-order intermodulation distortion

Now, it turns out I mean and just like yesterday when we saw that there was, 2 tones when you put in a 2 tone input they would interact with each other and cause inter modulation. Well, the same thing should be expected even in the third or I mean third or non-linearity case it turns out that things are a lot more serious here. As the inter modulation products actually fall very close to the input tones as I will as we will discuss going forward.

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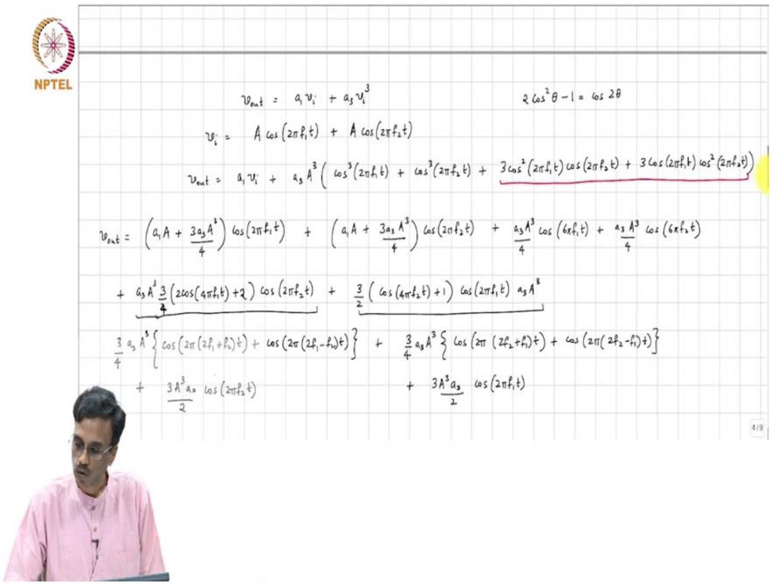

So, again v out is a 1 v i plus a 3 v i cube and v i is let us say A cos 2 pi f 1 t plus a cos 2 pi f 2 t alright and.

$$v_{out} = a_1 v_i + a_3 v_i^3$$

$$v_i = A\cos(2\pi f_1 t) + A\cos(2\pi f_2 t)$$

So, v out therefore, is a 1 times v i where v i is what we just wrote above plus a 3 times A cube times you know cos 2 pi f 1 t plus cos 2 pi f 2 t the whole cube. And what does the cube look like. So, its basically cos cube 2 pi f 1 t plus cos cube 2 pi f 2 t plus 3 cos square 2 pi f 1 t Cos 2 pi f 2 t plus 3 Plus 3 cos 2 cos 2 pi f 1 t Cos square 2 pi f 2 t, right.

$$v_{out} = a_1 v_i + a_3 A^3 (\cos^3(2\pi f_1 t) + \cos^3(2\pi f_2 t) + 3\cos^2(2\pi f_1 t)\cos(2\pi f_2 t) + 3\cos(2\pi f_1 t)\cos^2(2\pi f_2 t))$$

Before we go out and you know expand all this stuff let us figure out what we should expect to see we should expect to see the usual gain compression terms, right. We should also expect to see third harmonic distortion correct, ok. And what is that? And we should also now also expect based on our experience with the second order non-linearity we should also expect to see, that the 2 terms will interact with each other through that third order term and these are the cross terms, right where the 2 tones interact with each other through the non-linearity. So, let us get started. So, v out is a 1 A plus 3 a 3 A cube by 4 times cos 2 pi f 1 t plus. Similarly, a 1 A plus 3 a 3 A cube by 4 times cos 2 pi f 2 t, right Plus a 3 A cube by 4 cos 6 pi f 1 t Plus a 3 A cube by 4 6 pi f 2 t. So, these are all terms that we are familiar with, right. Gain compression plus third harmonic distortion of the individual tones right.

$$v_{out} = \left(a_1 A + \frac{3a_3 A^3}{4}\right)\cos(2\pi f_1 t) + \left(a_1 A + \frac{3a_3 A^3}{4}\right)\cos(2\pi f_2 t) + \frac{a_3 A^3}{4}\cos(6\pi f_1 t) + \frac{a_3 A^3}{4}\cos(6\pi f_2 t) + \cdots$$

And as you might imagine, in the second order case the distortion terms were typically not important. Because they can be easily filtered away, right. But what you cannot get rid off is basically the inter modulation terms and that is what we are going to see next. So, remember that cos square theta 2 cos square theta minus 1 is cos 2 theta, right. So, you basically have 3 by 2 times.

Cos square 2 pi f 1 t is nothing but cos 4 pi f 1 t Plus 1 Into cos 2 pi f 2 t plus of course, the other term were we interchange f 1 and f 2 cos 4 pi f 2 t plus 1 times cos 2 pi f 1 t, right. And we will simplify this 1 and the other 1 just simply follows. So, this is nothing but a sorry we have forgotten an a 3 A cube a 3 A cube a 3 A cube.

$$v_{out} = \left(a_1 A + \frac{3a_3A^3}{4}\right)\cos(2\pi f_1 t) + \left(a_1 A + \frac{3a_3A^3}{4}\right)\cos(2\pi f_2 t) + \frac{a_3A^3}{4}\cos(6\pi f_1 t)$$
$$+ \frac{a_3A^3}{4}\cos(6\pi f_2 t) + a_3A^3\frac{3}{2}(\cos(4\pi f_1 t) + 1)\cos(2\pi f_2 t)$$
$$+ \frac{3}{2}(\cos(4\pi f_2 t) + 1)\cos(2\pi f_1 t)\, a_3A^3$$

Just a 3 A cube right ok. So, now we will multiply this by 4 and take that stuff in there. And, this is basically a 3 A cube times 3 by 4 times 2 cos a cos b is cos of 2 pi times 2 f 1 plus f 2 times t plus. Cos 2 pi times 2 f 1 minus f 2 times t, right. Plus what do we see here? a 3 A cube times 3 by 2 cos.

That is what that is what I am saying. So, if I this is nothing but plus 3 A cube times a 3 by 2 Cos 2 pi f 2 t right.

$$a_3A^3\frac{3}{2}(\cos(4\pi f_1 t) + 1)\cos(2\pi f_2 t) \times 4 \Rightarrow a_3A^3\frac{3}{4}\ (2\cos(4\pi f_1 t) + 2)\cos(2\pi f_2 t)$$

$$\Rightarrow \frac{3}{4}a_3A^3\{\cos(2\pi(2f_1 + f_2)t) + \cos(2\pi(2f_1 - f_2)t)\} + \frac{3A^3a_3}{2}\cos(2\pi f_2 t)$$

And you know this we can add off to that to the fundamental later, right and so, what else. So, if you do the same thing here this will simplify to 3 by 4 a 3 A cube times Cos 2 pi Into 2 f 2 plus f 1 into t Plus cos 2 pi 2 f 2 minus f 1 into t, right? Plus 3 A cube A 3 by 2 times Cos 2 pi f 1 t right.

$$\frac{3}{2}(\cos(4\pi f_2 t) + 1)\cos(2\pi f_1 t)\, a_3A^3 \times 4\ \Rightarrow$$

$$\Rightarrow \frac{3}{4}a_3A^3\{\cos(2\pi(2f_2 + f_1)t) + \cos(2\pi(2f_2 - f_1)t)\} + \frac{3A^3a_3}{2}\cos(2\pi f_1 t)$$

$$v_{out} = \left(a_1A + \frac{3a_3A^3}{4}\right)\cos(2\pi f_1 t) + \left(a_1A + \frac{3a_3A^3}{4}\right)\cos(2\pi f_2 t) + \frac{a_3A^3}{4}\cos(6\pi f_1 t)$$
$$+ \frac{a_3A^3}{4}\cos(6\pi f_2 t)$$
$$+ \frac{3}{4}a_3A^3\{\cos(2\pi(2f_1 + f_2)t) + \cos(2\pi(2f_1 - f_2)t)\}$$
$$+ \frac{3A^3a_3}{2}\cos(2\pi f_2 t)$$
$$+ \frac{3}{4}a_3A^3\{\cos(2\pi(2f_2 + f_1)t) + \cos(2\pi(2f_2 - f_1)t)\}$$
$$+ \frac{3A^3a_3}{2}\cos(2\pi f_1 t)$$

So, the fundamental terms you know the fundamental frequency amplitudes are approximately a 1 A and for both the tones are a 1 A. So, the input tones.

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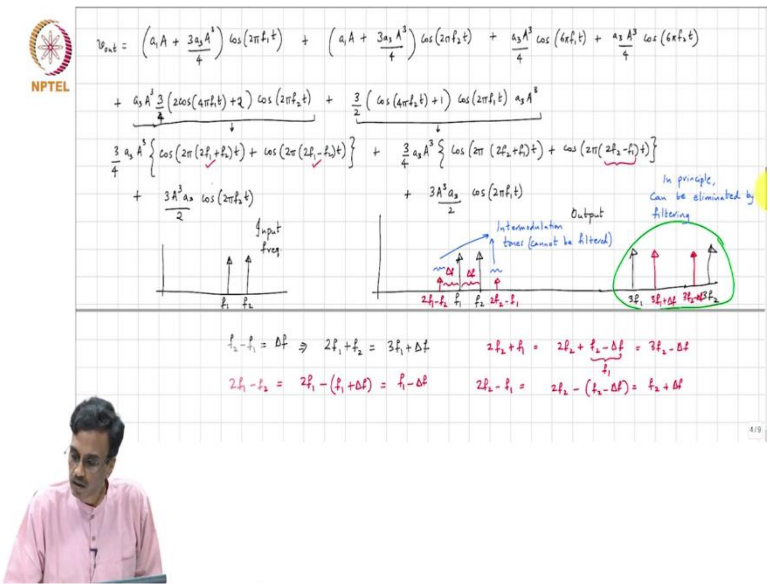

So, the input tones basically, if you plot on a picture the input tones. Let us say are f 1 and f 2 right. The output tones ok are what all are the output tone frequencies? So, f 1. So, there is f 1 f 2 there is harmonic distortion which is basically 3 f 1 3 f 2, right. There is 2 f 1 plus f 2, right which is basically, if you assume that f 2 minus f 1 you call that delta f, right. So, 2 f 1 plus f 2 is nothing but 3 f 1 plus delta f, correct. So, you will have 3 f 1 Plus delta f and if you have and 2 f 1 minus f 2 is nothing but 2 f 1 minus f 2 is nothing but 2 f 1 minus f 2 is nothing but f 1 plus delta f which therefore, is f 1 minus delta f, right.

$$f_2 - f_1 = \Delta f \;\Rightarrow 2f_1 + f_2 = 3f_1 + \Delta f$$

$$2f_1 - f_2 = 2f_1 - (f_1 + \Delta f) = f_1 - \Delta f$$

So, remember that this is delta f, correct. So, now, you have an inter mod tone which is here and this is 2 f 1 minus f 2, correct. So, we have seen this we have seen this. Now we have 2 f 2 plus f 1, right. And where does that so, 2 f 2 plus f 1 is nothing but? 2 f 2 plus f 2 minus delta f this is f 1, correct which is nothing but 3 f 2 Minus delta f.

$$2f_2 + f_1 = 2f_2 + f_2 - \Delta f = 3f_2 - \Delta f$$

So, that is going to be sitting here. That is 3 f 2 minus delta f, ok. And the next thing is that you have 2 f 2 minus f 1 2 f 2 minus f 1 is nothing but 2 f 2 minus f 2 minus delta f Which is f 2 Plus delta f, right.

$$2f_2 - f_1 = 2f_2 - (f_2 - \Delta f) = f_2 + \Delta f$$

So, these tones are all in the vicinity of I mean if f 1 and f 2 are close these tones are all in the vicinity of 3 times the average input frequency, right. And therefore, can in principle at least be easily written.

Yeah, in principle can be eliminated by filtering ok. Alright and unfortunately these tones they lie very close to f 1 and f 2. So, these are inter modulation tones and cannot be filtered, right. And these are the most important tones that 1 has to worry about, when 1 is talking about in a lot of systems this can be as I will illustrate going forward they can be really problematic.

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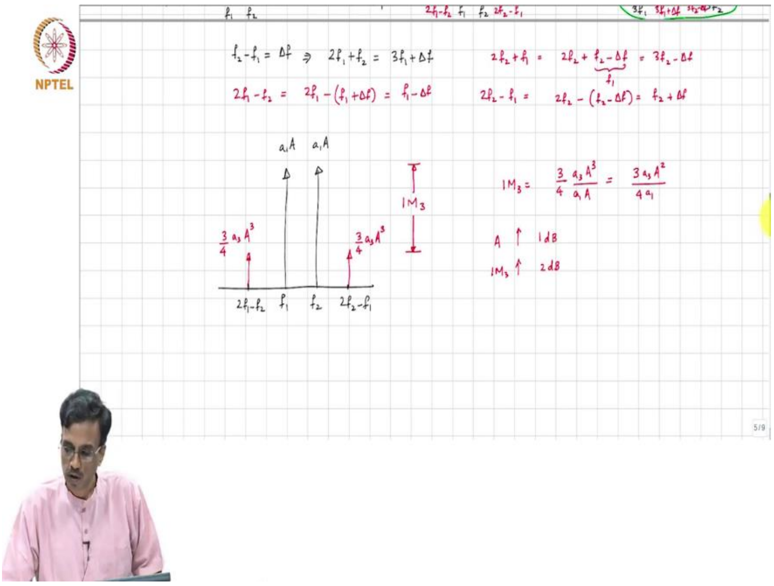

So, I am going to, let us just focus on the tones which are the inter modulation tones that are next to the input. So, this is f 1, f 2 this is 2 f 1 minus f 2 and this is 2 f 2 minus f 1 the amplitude of the fundamental is approximately a 1 A. So, where we assume that you know you are operating sufficiently linearly. So, that compression is not an issue and these are inter modulation tones, whose amplitude is 3 by 4 a 3 A cube this is also 3 by 4 a 3 A cube, alright.

And this ratio is called inter modulation distortion. Because a third non order non-linearity. So, this is called I M 3. So, I M 3 therefore, is nothing but 3 by 4 a 3 A cube divided by a 1 A, which is basically 3 a 3 A square by 4 a 1

$$IM_3 = \frac{3}{4}\frac{a_3 A^3}{a_1 A} = \frac{3a_3 A^2}{4a_1}$$

And if A increases by 1 dB I M 3 will increase by? 2dB

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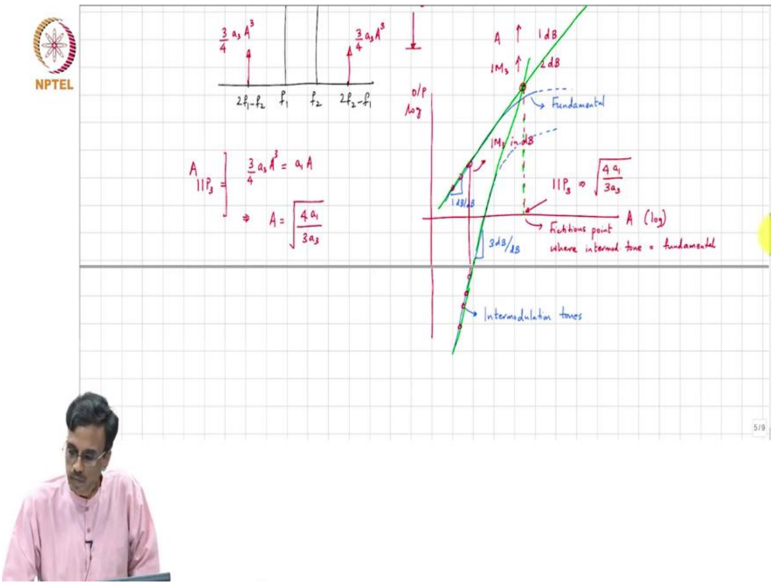

By 2 dB, alright. So, if I plot the input amplitude on a log scale and the output amplitudes whatever they are, also on a log scale the fundamental amplitude will basically do yeah, something like this and then we will compress and do something like that and this slope is 1 dB per dB ok.

Now, the inter mod component will have a slope which is. So, this is the fundamental tone strength of the fundamental and this is the strength of the inter mod.

Student: Fundamental compression tone also.

Student: Fundamental tone fundamental.

Yeah, if the input amplitude is very large then you will start to see compression in the fundamental output also, right. And so, this the inter modulation tone the strength of each one basically will keep increasing with at 3 dB per dB, right and at a given amplitude what represents the I M 3. So, at any given amplitude this distance is basically I M 3 in dB, alright.

And so, and just like how you know we had IIP 2 which was a number that quantifies how linear the amplifier is. By the same token you know if the amplifier was perfectly linear then, this you know if you extrapolated this line.

Student: (Refer Time: 19:22).

Right. And this line these 2 lines will intersect at some fictitious point there, right? If the inter modulation distortion was lower, what would happen to the to that point?

No if the inter modulation distortion is lower then the point of intersection will move towards the right ok. So, this therefore, is a fictitious point which is. So, the key point is that this is a fictitious point where inter mod tone is the same as the fundamental, right. It does not mean that you go and put an amplitude of that value and you will get the same fundamental and the inter mod tone. It is obtained by measuring the amplifier or the non-linear block with very very small signals, right.

And measuring both the fundamental and the inter mod tones, right. You measure it for a few amplitudes. So, for example, let say maybe you measure a few amplitudes here, right. And then you will get a straight line for each one of these things, right. If you do not get a straight line that basically means that you are already in compression, right. So, or the system is not weakly non-linear.

And. So, therefore, you produce these lines and they will go and intersect at some point there and that is the that is this point is therefore, called the input referred intercept point for the third order inter modulation distortion, right. So, IIP 3 is when or rather A IIP 3 is when? 3 by 4 a 3 A cube equals a 1.

Which means A is 4 a 1 by 3 a 3 under root, right.

$$A_{IIP_3} = \} \frac{3}{4} a_3 A^3 = a_1 A$$

$$\Rightarrow A = \sqrt{\frac{4a_1}{3a_3}}$$

In other words, the moment you know the gain of the amplifier and you know the IIP 3 you will be able to go and figure out what that a 3 is. Which basically means that it will also be able to I mean remember that HD 3 the harmonic distortion and the inter modulation distortion are basically coming from that the same a 3.

So, if you know one you will be able to calculate the other and so on right. See one advantage of an inter mod measurement over a I mean, even if you want to measure harmonic distortion it always makes you know its easier from a measurement point of view to, what do you call its a more accurate measurement to measure the IIP 3 or rather not the IIP 3. But to measure the inter modulation distortion and then infer the harmonic distortion from inter mod rather than the other way around. Because if you have an amplifier which is producing you know a third harmonic.

By the time the third harmonic I mean the fundamental and the third harmonic go through different paths, right will have I mean they go through the same physical path, but the physical path may have some frequency dependence. So, the higher order harmonic may get attenuated by the time it goes into your measurement instrument, right.

On the other hand, with inter mod there is basically no there is no way you can make an error. Because if you take 2 closely spaced tones the inter mod tones are just next to the inputs and therefore, whatever happens to the fundamental the same thing will happen to the inter mod tone and therefore, when you measure you will be able to get an accurate picture of what that you know a 3 by a 1 is, right. Now, why is inter modulation so, important in a whole lot of systems particularly in RF.

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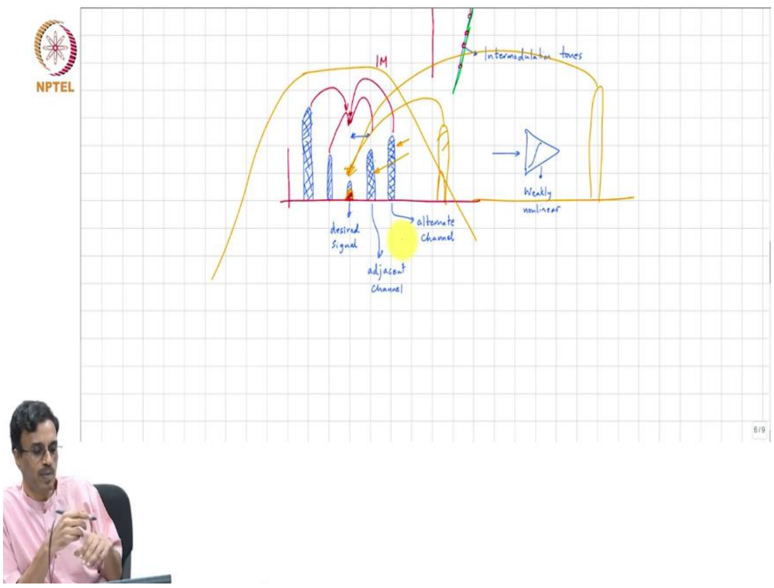

Well, it turns out that you know, in an RF input spectrum. You I mean, as you all know spectrum is very expensive. So, you will have you know very closely spaced channels and so, and the use case scenario in RF is that your desired signal is very small, right. And you have interferers that are unwanted ok and the spacing between. So, these are adjacent and alternate channels.

So, this is called the adjacent channel and this is called the alternate channel ok. And the spacing is you know is the spacing between channels is; obviously, called the channel spacing, alright. Now of course, the RF spectrum can be you know will have multiple channels because there are lots of customers, who are probably using the phone at the same time.

And in your cell, they better be your neighborhood they better all be using different channels. Otherwise, you know they will affect each other and therefore, right. So, the RF spectrum this is the representative RF spectrum and let us say this goes into front end amplifier. We have already seen earlier why you need a front-end amplifier. Because the input signal is so small that it has to be amplified before the next stage can make sense of it. And of course, this amplifier is made with active elements and therefore, this is you know at best its weakly non linear ok. Now this got and due to third non-linearity, what do you think can happen? Well.

These 2 the adjacent in the alternate will inter modulate, right and cause some junk right here ok. Likely and similarly these 2 will inter modulate and because you know another yeah signal which directly falls on the desired signal, right. So, our desired signal can get terribly corrupted by inter modulation distortion from alternate and adjacent channels. Not merely alternate and adjacent channels.

Let us say, you have you know one channel here which is very large, right and another channel which is here which is even larger and these 2 can also inter modulate and cause distortion, right. But in principle this those are easier to handle because you can put some kind of narrow band, I mean some kind of filter which can remove stuff which are very far away, right.

But the close in the alternate and adjacent channels you know you cannot remove by filtering, correct. So, the amplifiers and you know the signal chain has to be linear enough to be able to process such signals without causing sufficiently large inter modulation distortion. So, as to corrupt your signal completely, right and the point is that the desired signal is very small and the undesired signals can be several orders of magnitude larger, right?

And therefore, even a small amount of non-linearity can cause the. Remember the inter modulation distortion is proportional to you know.

If these 2 tones I mean the inter modulation distortion is proportional to the strength of the interfering tones which can be very large compared to your desired tone, right. So, you know. So, therefore, even a small amount of distortion I mean inter modulation distortion can potentially completely mess up your desired side, alright.

So, lot of this as you can see is just merely jargon there is nothing you know, I mean you have to be aware of the fact that you can have inter modulation distortion, right. And of course, there is a whole bunch of formulas which you can derive at any time. There is nothing what you call fundamentally difficult about conceptually difficult about inter modulation or you know basic non-linearities.

In the next class you know, we will basically move on to what happens when you have all these concepts apply to networks which are memory less, correct ok. So, now, the question is what happens when you have you know weakly non-linear say trans conductors and so on with capacitors and so on. I mean for instance we have seen filters, right where we have you know op amps and then capacitors and so on.

And of course, the op amps you know are made with voltage control current sources which are all transistors they are all non-linear. So, we have to figure out you know what happens with you know what happens when those are weakly non-linear and how they affect? How to analyze circuits like? We will continue in the next class.