Circuit Analysis for Analog Designers
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Lecture - 57
Gain compression and third-order harmonic distortion

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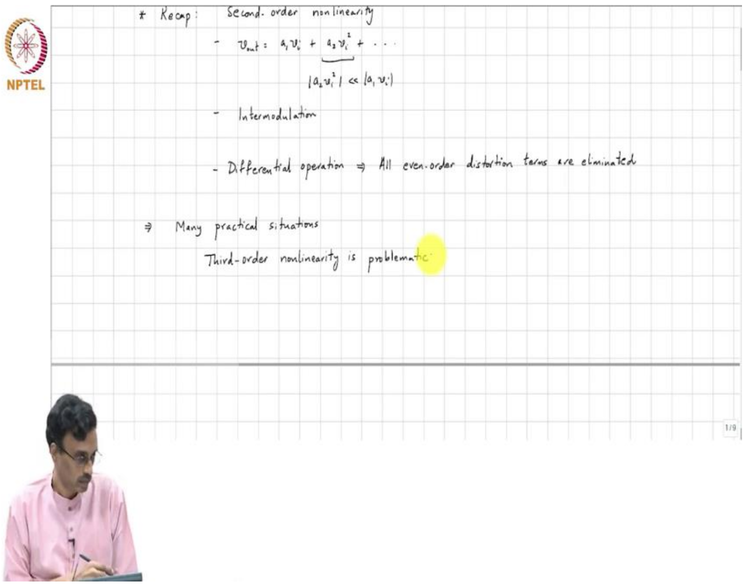

A quick recap of what we saw yesterday, we talked about Second order non-linearity and remember that we are only concerned about weak non-linearity. So, v out is sum a 1 v i plus a 2 v i square plus higher order terms.

$$v_{out} = a_1 v_i + a_2 v_i^2 + \cdots$$

Yesterday we considered only the case second order non-linearity, and when we said weakly non-linear it basically means that a 2 v i square in magnitude is much much smaller than a 1 v i in magnitude right.

$$|a_2 v_i^2| \ll |a_1 v_i|$$

So, of and we saw the phenomenon of inter modulation, where two different tones basically get multiplied with each other. So, if you have two tones of at frequencies f 1 and f 2, you not only see the harmonic distortion components namely at two f 1 and two f 2.

But you also see f 1 plus f 2 and f 1 minus f 2 in addition to some d c offset right. And then we finally, looked at fully or differential circuits, and one of the important consequences of differential operation is that all even order distortion terms are element.

So, in many practical systems, it is only the third harmonic or third order non-linearity its very problematic ok, and that is what we are going to see today.

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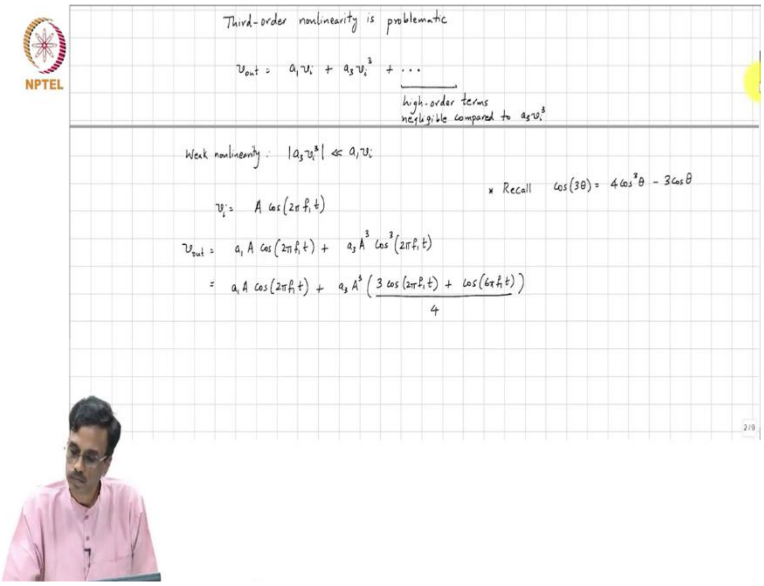

In other words, v out is a 1 v i plus a 3 v i cube plus higher order terms.

$$v_{out} = a_1 v_i + a_3 v_i^3 + \cdots$$

We assume that the higher order terms negligible compared to a 3 v i cube and weak non-linearity basically means that, a 3 v i cube is much much smaller than a 1 v i alright.

$$|a_3 v_i^3| \ll |a_1 v_i|$$

So, as usual let us see what happens when you put in a single tone first.

So, v i is a A cos 2 pi f 1 t and therefore, v out is nothing, but a 1 A cos 2 pi f 1 t plus a 3 times A cube cos cube 2 pi f 1 t and let us recall the following trigonometric identity cos 3 theta is nothing, but 4 cos cube theta minus 3 cos theta.

$$v_i = A\cos(2\pi f_1 t)$$

$$v_{out} = a_1 A \cos(2\pi f_1 t) + a_3 A^3 \cos^3(2\pi f_1 t)$$

$$Recall \qquad \cos(3\theta) = 4\cos^3\theta - 3\cos\theta$$

So, this is basically a 1 A cos 2 pi f 2 t plus a 3 A cube times 3 cos 2 pi f 1 t plus cos 2 Cos cube is nothing but? Now, we are trying to replace. Right. Yeah, plus cos 6 pi f 1 t divided by 4 alright.

$$v_{out} = a_1 A \cos(2\pi f_1 t) + a_3 A^3 \left(\frac{3\cos(2\pi f_1 t) + \cos(6\pi f_1 t)}{4}\right)$$

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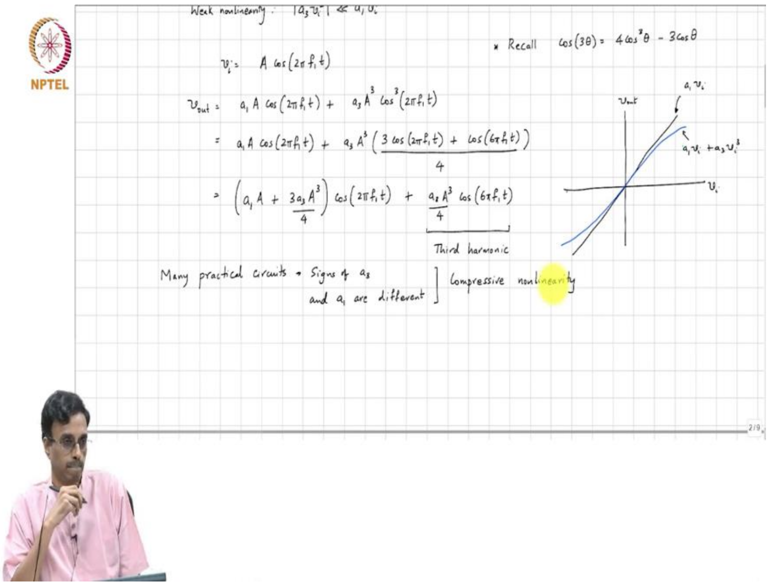

And so, this is nothing, but a 1 A plus 3 a 3 A cube by 4 times cos 2 pi f 1 t plus a 3 A cube by 4 cos 6 pi f 1 times t right.

$$= \left(a_1 A + \frac{3a_3A^3}{4}\right)\cos(2\pi f_1 t) + \frac{a_3A^3}{4}\cos(6\pi f_1 t)$$

So, this is basically the fundamental right, and $\left(\frac{a_3A^3}{4}\cos(6\pi f_1 t)\right)$, this is the third harmonic. And it turns out that many practical nonlinearities will have for example, if this is a perfectly linear you would have seen a characteristic like that right. So, this is v i and this is v out for instance.

But in practice what happens is that, many practical high order transfer curves which have only odd harmonic distortion behave like that. So, what comment can you make? So, this is basically a 1 v i right and this is a 1 v I Plus a 3 v i cube, $(a_1 v_i + a_3 v_i^3)$. So, what comment can you make about the sin of a 3 versus sin of a 1?

Student: Both will be same, like both will be the same quadrant.

Student: I said like if we if we obtain.

No man, is v i positive or negative there?

Student: That in the first quadrant positive.

Positive yeah. So, if this is a v i plus a 3 v i cube. And is that smaller than a 1 v i or larger than a 1 v i the blue curve? Smaller. So, if that has to be smaller what comment can you make about this?

Yeah. So, basically in many practical circuits, the sign of and a 1 are different right, and such a non-linearity is called a what is called a compressive non-linearity. Does it make sense?

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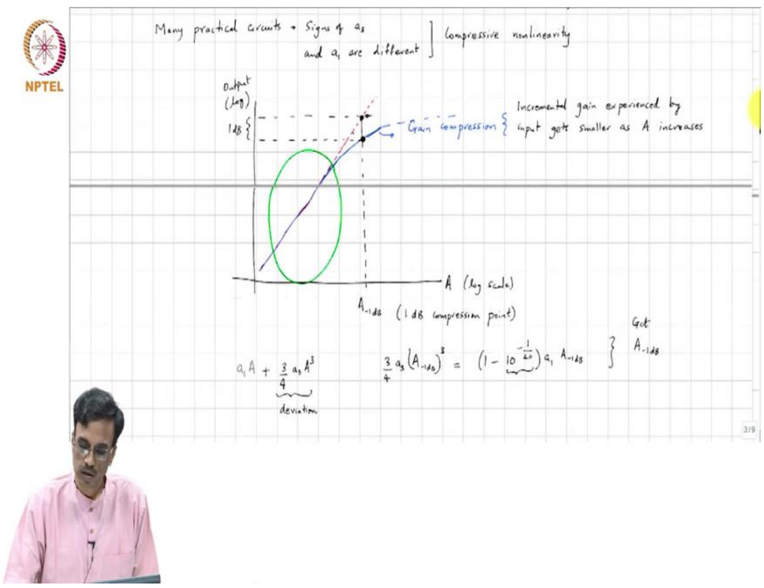

So, first things first. So, let us see what we can make what sense we can make of. So, if I plot the amplitude of v in on the log scale and let us first plot the amplitude of the fundamental right. So, what comment can you make about the amplitude of the if you draw the amplitude of the fundamental?

You will get a straight line, do you? No, think carefully, this is the amplitude of the fundamental. So, as you keep wearing a. Remember which is going to be the dominating term?

Yeah, a 1 a is going to be much larger than 3 a 3 A cube by 4. So, for small for very small amplitudes you basically will have a line which basically goes at with a slope of plus 1.

Correct? Now as you keep increasing the input amplitude what comment can you make? Remember that a 3 and a 1 have opposite signs.

No, as you keep increasing. So, if a 3 was 0 then this would keep continuing on forever. So, if a 3 is non zero what do you think will happen? As you keep increasing a at a certain point this a 3 times A cube will no longer be negligible compared to a 1 a. So, what you will actually see is a curve which does this right, and as you keep increasing amplitude, what will happen is that it will tend to start doing this right. Why? Because.

At these I mean beyond certain amplitude basically that 3 a 3 A cube by 4 cannot be neglected right. So, the fundamental amplitude itself is no longer a straight line with a slope of 1 d B per d B ok, technically even at very low amplitudes there is it is not a straight line right, but at those amplitudes that 3 a 3 times A cube is. So, by 4 is so small compared to a 1 a that for all practical purposes it looks like a straight line right. So, there therefore, this is a.

So, as you can see the effective gain as you know if you want to think about it that way, which is the slope of this characteristic, keeps decreasing as you keep increasing the input amplitude right. So, this is what is called gain compression. And what this means is that, the gain experienced gain experienced by the input gets smaller as input amplitude increases.

This is because of comparison nonlinearity correct of the opposite sign if a 3 had you know had the same sign then you would have an expansive output correct. So, one metric to basically see, what happens to the; what happens to you know or rather one metric to gauge how linear the amplifier is to look at you know when this. Yeah, when the deviation from the straight line reaches a certain value alright. If the if it was perfectly linear the blue curve will never deviate from that red from the red line which has a slope of 1 d B per d B, if the a 3 is finite then there will be a deviation. So, you know people just came up with one you know some metric they said well, let us kind of quantify for what input amplitude the output amplitude of the fundamental deviates from that straight line by 1 d B right.

And so, this is basically this is called the A minus 1 d B, where that minus 1 d B indicates that for this value of amplitude the amplitude of the output is basically smaller than what would be obtained from an ideal linear amplifier by 1 d B right. And how do you find that A minus 1 d B? Well, ideally you should see a 1 A you have you are seeing plus 3 by 4 sorry plus 3 a 3 A cube by 4 and therefore, the deviation is this is the deviation. So, 3 by 4 a 3 A minus 1 d B the whole cube must be. You know whatever right, 1 d B is 10 to the power so, this is 1 minus 10 to the power minus 1 by 20 right and times a 1 times A minus 1 d B the whole square sorry not whole square. So, this is what you would this is the linear output right and naturally.

$$a_1 A + \frac{3}{4} a_3 A^3$$

$$\frac{3}{4} a_3 (A_{-1dB})^3 = \left(1 - 10^{-\frac{1}{20}}\right) a_1 A_{-1dB}$$

Student: That is the linear.

Yeah. So, this would basically be the this is basically that guy there correct? So, a 1 times A minus 1 d B is that chap. Do you understand?

This is A minus 1 d B correct? So, this is on the red line there, would have been you know a 1 times A minus 1 d B and we want the blue curve at that point is 1 d B lower.

Correct, it has to be 1 d B lower. So, 1 d B lower is basically. Yeah, 10 to the power minus 1 by 20.

Correct? So, the difference between the two is basically a 1 times A minus 1 d B times 1 minus 10 to the power minus 1 by 20 is 3 by 4 a A 3 you know A 1 d B whole cube. So, you can go and do the math, and then this will give you the formula for A minus 1 d B right.

And; obviously, as the A minus 1 d B or the 1 d B this is often also called the 1 d B compression point right. And all that this means is that the 1 d B compression point is much higher than the amplifier can handle a larger input amplitude before it goes into the individual.

Yeah, right a before it starts to you know before non-linearity starts to have deleterious effects on the output. Now of course, the question is what will happen if you go on increasing the input amplitude beyond the 1 d B compression point? Right well then you know I mean it is very likely that the amplifier is already not merely in the weakly non-linear region anymore.

And then you know it can do all sorts of things right. So, we do not know what. And we would like typically you would like to operate you know somewhere in that region, where compression is not significant ok. So, the next thing that we will see is there is also a harmonic distortion term.

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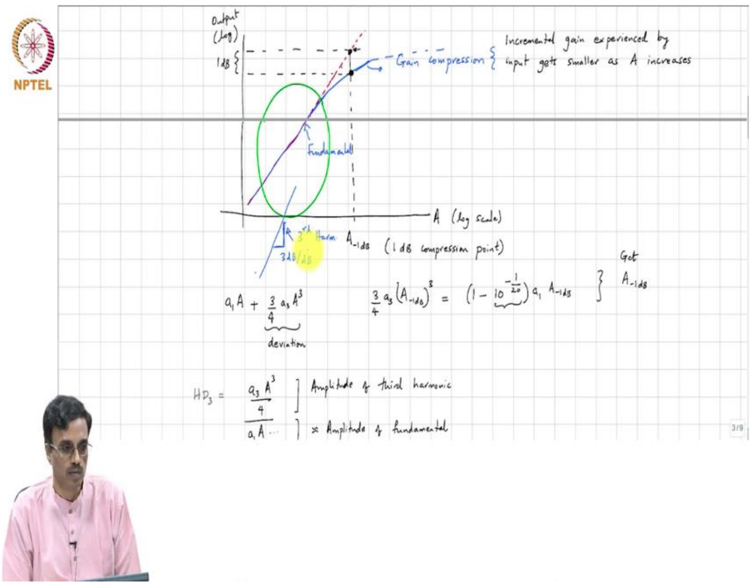

So, as we did yesterday H D 3 it simply the ratio of the amplitude of the third harmonic to the fundamental and that that as you can see is, 3 a 3 times A cube by 4 this is the Amplitude of the third harmonic divided by Fundamental a 1.

a 1 A and there is actually also another term there, but.

You know that is so, small hopefully that. Right is so small that can be neglected. And therefore, H D 3 a simply a 3 A square by 4 a 1 right.

$$HD_3 = \frac{\frac{a_3 A^3}{4}}{a_1 A \ldots}$$

$$HD_3 = \frac{a_3 A^2}{4 a_1}$$

And yesterday we saw that the second the amplitude of the second harmonic component was proportional to a square, today we see that the amplitude of the third harmonic is proportional to a cube and that makes sense because v i cube basically.

Means that you are multiplying the input with itself like thrice and therefore, you get A cube right. And since H D 3 is I mean the amplitude of the third harmonic is proportional to A cube it follows that the ratio of the third harmonic to the fundamental becomes you know a goes up as A square right. So, 1 d B increase in input amplitude a will lead to will result in 2 d B increase in H D 2, H D 3 I am sorry.

Alright and. So, therefore, you know if I plot the strength of the third harmonic, well that will basically go up like this right and that slope will be this is the fundamental and this is the third harmonic and this slope is 3 d B per every d B increase in the input. Does that makes sense?