Circuit Analysis for Analog Designers
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Lecture - 56
Weak nonlinearity in electronic circuits, second-order intermodulation distortion

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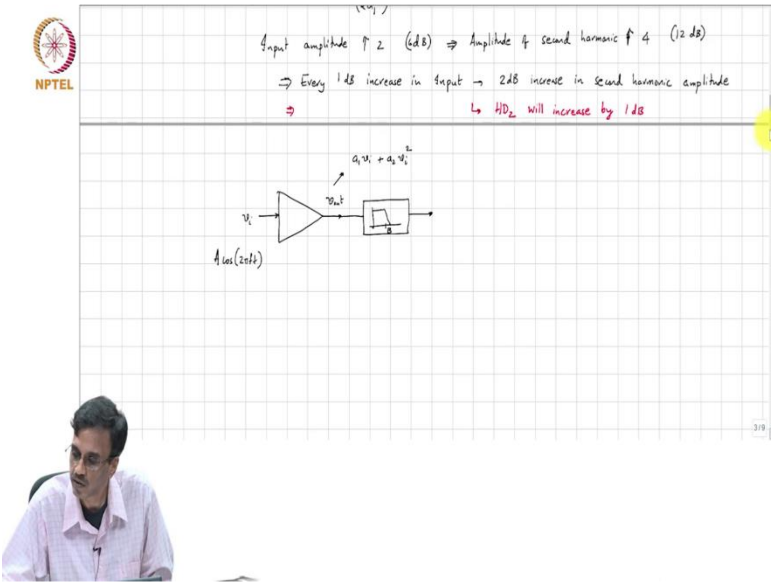

So, why do we need to worry about or when do we need to worry about, it is a second order nonlinearity or why is this problematic. So, you know so, let us say let me ask you a question. So, let us say one has an amplifier right. So, again this is weakly non-linear right. And so, somebody argues that well, if I put in an input f here.

So, let us say I put in a cos 2 pi f t, ($A\cos(2\pi f t)$) and the output obviously, contains distortion second harmonic. So, he says why do not I put a filter here, which will remove the second hormone. So, let us say the signal bandwidth is limited to B right. And that is your that is your band of interest and so, you know the person says you know why do you worry about non-linearity right; let I mean let this be as non-linear as it wants right.

I am just going to simply put a filter and remove the higher order harmonics right. If I remove the if I put an input tone less than B, the second harmonic will be you know will be around $2B$. So, I am going to I mean I have a filter anyway. So, I am going to remove the;Yeah, I am going to remove the second harmonic right. So, I mean if I just put this whole thing in a black box and give it to you, I mean well you put in a tone at f and the output looks clean right.

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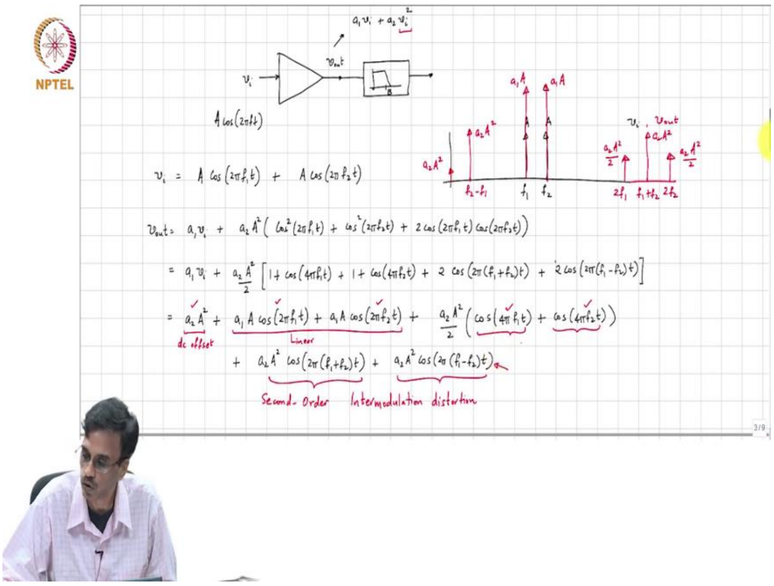

But unfortunately that is.

Yeah of course, for input B by 2 for a low frequency there will be a problem right. But another important issue is what happens when you have 2 tones cos 2 pi f 1 t and cos 2 pi f 2 t right.

$$v_i = A\cos(2\pi f_1 t) + A\cos(2\pi f_2 t)$$

So, you have 2 in other words the input basically is a 2-tone input, both have amplitudes A and the frequencies are f 1 and f 2. So, this is v i. So, what would v out be? Well, it will be a times v i, which is uh a times that quantity which I am not going to write down again. Plus a 1 times v i sorry, plus a 2 times v i square which basically will be, I am going to remove a as a common factor a square times Cos square 2 pi f 1 t plus cos square 2 pi f 2 t plus 2 cos 2 pi f 1 t cos 2 pi f 2 t right.

$$v_{out} = a_1 v_i + a_2 A^2(\cos^2(2\pi f_1 t) + \cos^2(2\pi f_2 t) + 2\cos(2\pi f_1 t)\cos(2\pi f_2 t))$$

Which is a 1 v i plus a 2 A square times a 2 by 2 times 1 plus cos 4 pi f 1 t plus 1 plus cos 4 pi f 2 t plus 2 cos a cos b is nothing but cos of a plus b that is 2 pi f 1 plus f 2 into t Minus sorry, cos of a plus b plus cos of a minus b plus 2 cos of 2 pi f 1 minus f 2 into t right.

$$= a_1 v_i + \frac{a_2 A^2}{2}[1 + \cos(4\pi f_1 t) + 1 + \cos(4\pi f_2 t) + 2\cos(2\pi(f_1 + f_2)t) + 2\cos(2\pi(f_1 - f_2)t)]$$

So, this therefore, is if you take out all these terms. It is basically a 2 A square a DC offset and that makes sense right, because even though we have seen why it makes sense that you have a DC offset; plus a 1 A cos 2 pi f 1 t plus a 1 A cos 2 pi f 2 t plus. Plus a 2 A square by 2 times Cos 4 pi f 1 t plus cos 4 pi f 2 t correct, plus Yeah. So, where a 2 A square times cos 2 pi f 1 Plus f 2 times t plus a 2 A square cos 2 pi f 1 minus f 2 times t alright.

$$= a_2 A^2 + a_1 A\cos(2\pi f_1 t) + a_1 A\cos(2\pi f_2 t) + \frac{a_2 A^2}{2}(\cos(4\pi f_1 t) + \cos(4\pi f_2 t)) + a_2 A^2\cos(2\pi(f_1 + f_2)t) + a_2 A^2\cos(2\pi(f_1 - f_2)t)$$

So, let us go term by term and see if it makes sense. Well, $(a_2 A^2)$ this is DC offset, we have seen why this makes sense right. These are of course, $(a_1 A\cos(2\pi f_1 t) + a_1 A\cos(2\pi f_2 t))$ the linear terms and we see we know obviously, why these make sense right. What are these terms, $(\cos(4\pi f_1 t) + \cos(4\pi f_2 t))$? Yeah, that the second harmonic of the 2 terms right. So, if you have the input well, the output has a component at f 1 at f 2, where a 1 A. There is a DC offset which is a 2 A square there is a tone at Sorry, yeah this is 2 f 1 this is 2 f 2. And the amplitude. So, this is v out is in red. So, that is basically a 2. A square by 2. A square by 2 right and we also have f 1 plus f 2, where would that be.

Yeah, if f 1 and f 2 are close, well if . Yeah. So, if let us say let us say represented will be let us say right. This is f 1 plus f 2 this is Yeah, a 2 A square ok and these are I mean of course, these are second harmonics of the in 2 input tones. So, we understand where they are coming from and these 2 tones there is one at f 1 plus f 2 and there is one with f 1 minus f 2. So, this is f 1 minus f 2 is going to be f 2 minus f 1 and this must be a 2 A square sorry, we came we have to a 2 A square, a 2 A square right. So, what do these represent actually, $(a_2 A^2\cos(2\pi(f_1 + f_2)t) + a_2 A^2\cos(2\pi(f_1 - f_2)t))$?

Yeah. So, what is happening is that the v square remembers, the root cause for all these tones is the v vi square term, where the input is getting multiplied with itself.

Right. So, of course, when f 1 gets multiplied with itself the you get 2 f 1. We understand that when f 2 gets multiplied with itself you get. Say you know 2 f 2, but what also happens is that f 1 cross mixes or multiplies with f 2. And you will therefore, when f 1 multiplies with f 2 you will get both the sum and the difference frequencies and this is what.

$(a_2 A^2 \cos(2\pi(f_1 + f_2)t) + a_2 A^2 \cos(2\pi(f_1 - f_2)t))$, This is this is often what is called Inter modulation distortion right. And because, this is coming because due to second order non-linearity, this is what is called second-order inter modulation distortion ok. So, most of the time it turns out that for example, in a scenario like this right; which do you think is more problematic, which of these 2 inter modulation distortion tones are more problematic.

Yeah. So, for example, if f 1 and f 2 are close by then the f 1 plus f 2 term will be a very high frequency and can often be removed by filtering. So, usually it turns out that this f 1 minus f 2 tone can be Problematic.

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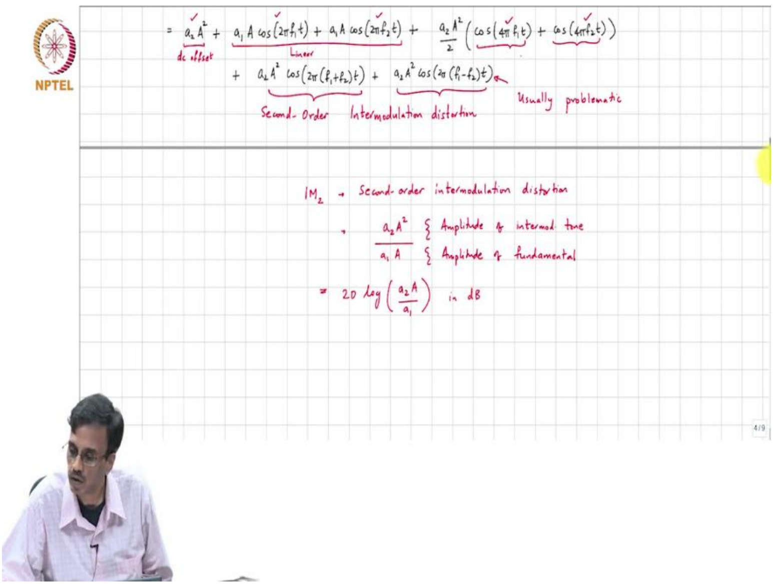

And so the problem ok and there is no way of getting rid of this by using a filter right. So, many times I mean so, 1 I remember that the second harmonic. So, it is often referred to as the just like how HD_2 refers to the ratio of the second harmonic distortion to the fundamental. IM_2 which is the second order inter modulation distortion right, is basically, yeah, it is basically the ratio of, Yeah, it is the ratio of it is a 2 A square by a 1 A, which is the amplitude of the inter mod tone to amplitude of the fundamental right.

$$IM_2 = \frac{a_2 A^2}{a_1 A}$$

And again, in dB 20 log a 2 A by a 1 in dB does make sense.

$$= 20 \log\left(\frac{a_2 A}{a_1}\right)\ in\ dB$$

So, every 1 dB increase in the input will cause a 1 dB increase in the IM_2

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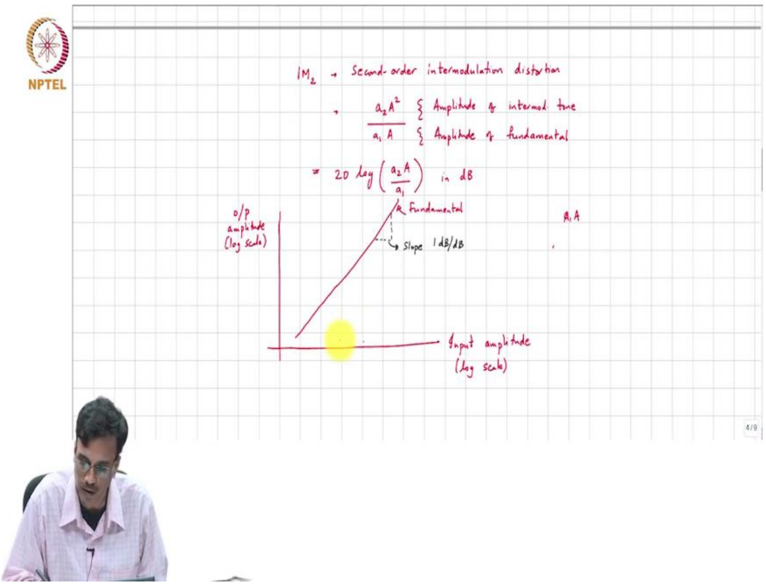

In the I M 2. So, often a way of measuring how linear the amplifier is, is to plot the input amplitude on a log scale and plot the output amplitude also on a log scale right; of both the on the I mean on the y axis. You know you can I mean you just show the output amplitude it can refer to the fundamental it can refer to the second order inter mod and so on. So, if you have an input amplitude which is basically varied in a log fashion the output will also vary log fashion. So, if you plot the fundamental right.

It is of course linear, but what is the slope? It is integral a 1 slope is a 1, sir. No. The fundamental output amplitude is nothing but, a 1 times A right, when you take the logarithm what happens. Multiplication of yeah.

That is not the slope. You are plotting log a versus the output amplitude. The output amplitude is a 1 times A. So, when you take the log this simply becomes 20 log a plus.You know whatever Plus 20 log capital A right and on the x axis it is 20 log A. So, you will get a straight line with the slope of. Slope is 1 right.

Yeah. So, the intercept will basically depend on the dc gain right. Yeah. So, this is the slope is; the slope is 1 dB per dB alright.

$$slope = 1dB/dB$$

Now, what comment can we make about the inter mod tone?

The strength of the inter mod tone will basically be a 2 A square and with what slope will it go up; as you keep increasing the input amplitude What will happen?

Yeah, the IM 2 I mean the second order inter modulation tone strength is proportional to A square. So, it will go up with a slope of 2 dB for every dB increase in the input. So, let us I think I should plot this maybe a little higher.

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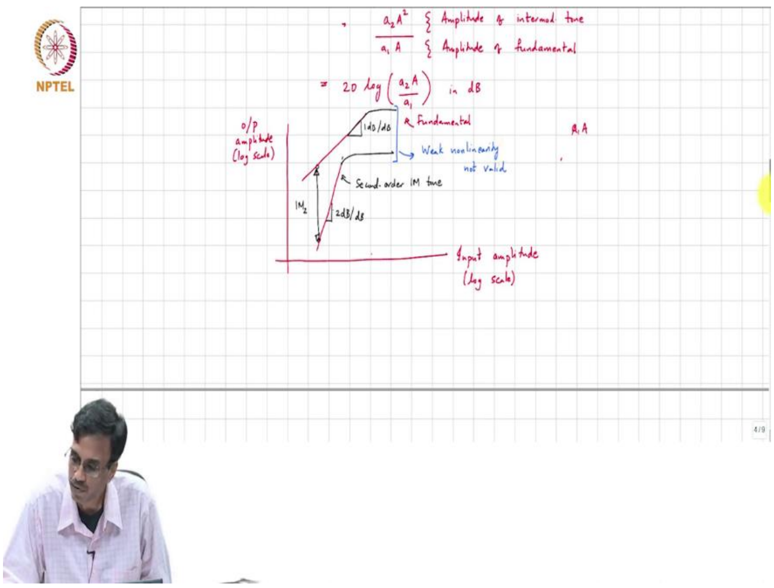

So, let us do this right, and this goes up by right. So, this slope is, this slope is 1 dB per dB and this is 2 dB per dB.

So, this is nothing but this the Second order I M 2 right and this for a given amplitude. This difference is the IM 2 in dB correct. Now, one thing that you have to bear in mind is that this is nothing but, I mean this see all this is only valid for weak non-linearity where a 2 times A square is much Smaller than a 1 times A, but as you keep increasing the input amplitude it is not necessary that the I mean.

At least if you do the math here. Yeah. So, I mean if you go on increasing a 2 times A square can potentially become You know larger than a 1 A.

I mean just simply from the math right, but that at such a large amplitudes it will typically happen that you know the amplifier, which is weakly non-linear only over a certain range of input amplitude has now been pushed into a strongly non-linear region, in which case none of these application you know none of these approximations basically work. So, in a practical amplifier you know you should not expect that this curve keeps going up and up and up right.

Again, it will kind of do saturate like this right and likewise with the fundamental also. Just because it says a 1 A does not mean that we can put you know 100 volts into you know a small amplifier and expect the output to be 10 times 100 right ok. So, these are basically regions where weak non-linearity is no longer valid alright.

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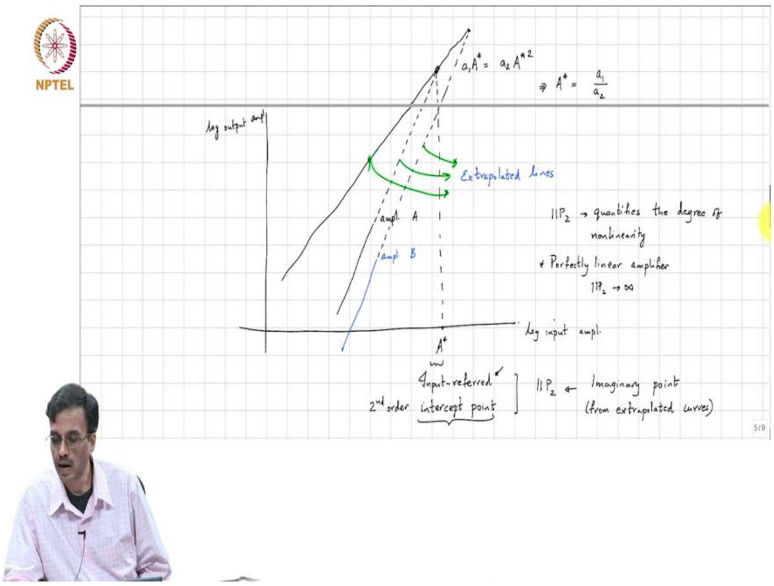

Now, let us say you had 2 amplifiers with the same gain. So, log input amplitude and this is log output amplitude right. And you had one amplifier, whose when I plotted the second order inter mod power you had something like this and you had. So, amplifier A and you had amplifier B, which of these is which of these amplifiers is more linear?

Student: Amp B is more (Refer Time: 22:54).

Amp B is more linear, because even though both of them have the same fundamental output amplitude the second order the intermod distortion is lower for amplifier B. So, one way to quantify the linearity of an amplifier is the following is to observe that you know if I produce this line or extrapolate this line. Yeah, at some point it will eventually intersects the, they are the for the fundamental curve and that happens when a 1 A becomes equal to. So, at this point of intersection a 1 A must be equal to Or let me call this a 1 A star must be equal to A 2 A star square.

$$a_1 A^* = a_2 {A^*}^2$$

So, this A star on a log scale. So, A star therefore, is equal to a 1 by a 2 ok.

$$A^* = \frac{a_1}{a_2}$$

So, A star is a 1 by a 2 and that therefore gives you. I mean if the amplifier is perfectly linear, what comment can you make about A star. Should be infinity. Right; ok. So, the more linear the amplifier is the higher the A star will be right a couple of things. So, like you can see amplifier B, which is more linear basically will these 2 curves will intersect at right.

Yeah. So, this is what is called the input referred second order intercept point alright. Why does it make sense to call this input referred well, we are this is the x axis, the x axis is the amplitude of the input right. Intercept point well, when the inter modulation curve intercepts or touches the fundamental the line is when a adds at that point and of second order, because we are talking about I M 2

Right. So, this is often called also IIP 2 right. Input referred inter I mean intercept point ok, IIP 2 for second order right. And so, this IIP 2 therefore, is basically it is an imaginary point. It does not mean that.

Yeah, it does not mean that you can apply A star and at that point the second harmonic will be the same, I mean second the second order inter modulation will be the same as the fundamental correct. So, this is you know a problem the I mean this is something that we born in mind and so, that is because we are these are generated from extrapolated curves right. So, these are all extrapolations. In fact, the same thing holds for the fundamental also. So, all these are extrapolated curves or extrapolated lines I would say ok and.

So, they are basically IIP_2 is a metric that quantifies the degree of nonlinearity right. In a perfectly linear amplifier IIP 2 is infinite right. And since, both IIP 2 and I mean the second order intermodulation and is coming, because of a 2 and likewise the second order harmonic distortion is also coming, because of a 2 right. Basically, there is a you know there is a relationship between the two of them.

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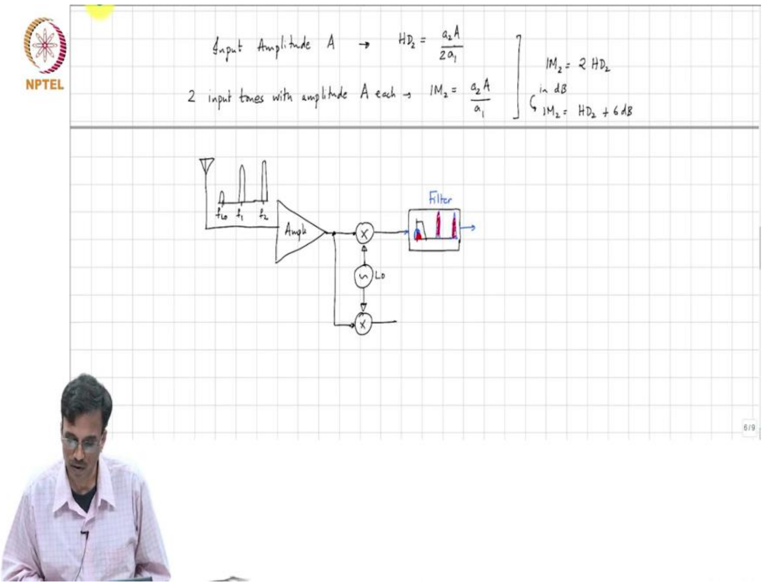

So, let us. So, if the input has an amplitude A, then the HD 2 is a 1 Capital A by sorry a 2 Capital A by a 1 right; as you can see from our earlier analysis.

$$A \to HD_2 = \frac{a_2 A}{2a_1}$$

So, I have (Refer Time: 30:07).

Right ok, if you have 2 input tones with amplitude A each IM 2 is nothing but, remember IM 2 is the ratio of the intermod tone to the amplitude of any one of those tones the input tones.

Right, I mean yeah one of the input frequencies, I mean we have to take the ratio with respect to the output amplitude. So, IM 2 as we can see is nothing but a 2 A square A square by a 2 A square by a 1 A, which is a 2 A by a 1 right.

$$IM_2 = \frac{a_2 A}{a_1}$$

And so, under these situations, we see that h IM 2 is basically twice HD 2,

$$IM_2 = 2HD_2$$

When they are all expressed as numbers and not in log scale right, otherwise in dB, IM 2 is HD 2 plus 60 ok alright.

$$in\ dB\ \ IM_2 = HD_2 + 6dB$$

So, when uh does I mean in a practical sense amplifier I mean in a practical system, where are examples where second order nonlinearity can be very problematic.

An example is again a RF receiver chain. So, let us say this is the front-end amplifier, this is the; this is the antenna and this goes into the first amplifier which is supposedly linear right. So, if I mean if you have 2 large 2 very large tones outside the signal band, what they can basically cause intermodal they can inter modulate to baseband and then that dc offset can potentially go and change the bias point of the operating point of the device in the amplifier.

Right, of course, you know it is also entirely possible that if you put 2 large tones the amplifier gets pushed out of it is, You know weakly non-linear region and get can get saturated right. So, large inputs can have all sorts of; all sorts of problems right. And so, let us say we have a small desired signal here.

And 2 large order band interferers right. So, this is f_{lo} and this is f_1 and this is f_2 correct. So, how will the receiver work? Well let us assume that the amplifier does not saturate. The next thing is that you have some kind of mixing operation right. And so this is what is called a direct conversion receiver. So, you mix the output of the amplifier with both sine and cosine to convert them to base band and then you are supposed to have a filter whose job is to eliminate everything other than the desired channel, let us say right. So, the desired channel is basically a small signal which is sitting here and then because you multiplied the f l o with f l o.

So, you have baseband to baseband, but then you have these 2 large signals here right and these 2 can inter modulate using, I mean through second order non-linearity of the filter. Remember, the filter is made with active elements right. Active elements are fundamentally non-linear, you do your best to make them as linear as possible, but if you have 2 large signals for instance they these 2 will through second and through IM 2 will basically you know can either cause if these are 2 very closely spaced tones. They will basically cause something here which can be confused with your desired signal right ok. So, that is what you need to be aware of with regard to second order distortion. Fortunately, in in most electronic systems today, there is a fairly straightforward way of eliminating or reducing second order distortion significantly.

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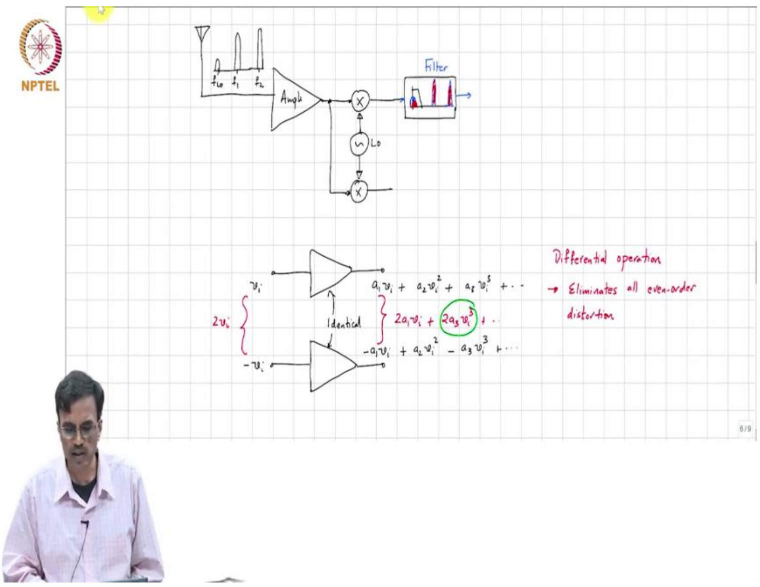

And that is through the following within quotes trick if you will. So, we know that this is we let us say you have an amplifier with a 1 v i plus a 2 v i square plus a 3 v i cube and so on right.

$$a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \cdots$$

And as I said the higher order terms are usually negligible in the region of interest. Now, let us say we took another amplifier identical amplifier.

So, both of them are identical. And you apply minus v i here. So, this will be minus a 1 v i plus a 2 v i square minus a 3 v i cube and so on alright.

$$-a_1 v_i + a_2 v_i^2 - a_3 v_i^3 + \cdots$$

So, if you look at the difference between the 2 inputs, its $2v_i$ and the difference between the 2 outputs is 2 a 1 v i plus 2 a 3 v i cube and so on right.

$$2a_1 v_i + 2a_3 v_i^3 + \cdots$$

So, this is what is called differential operation ok and this eliminates all even order distortion ok. So, you have to only worry now about odd order distortion right of which is the most dominant one. - $(2a_3 v_i^3)$

Yeah, the third order distortion is the third non-linearity is the dominant component of non-linearity in many practical circuits which all operate differentially alright.

So, in the next class we will learn about third order non-linearity ok. And these are all at this point we are all dealing with circuits without memory, once we have memory, we will have to figure something else out and we will do that as we go along.