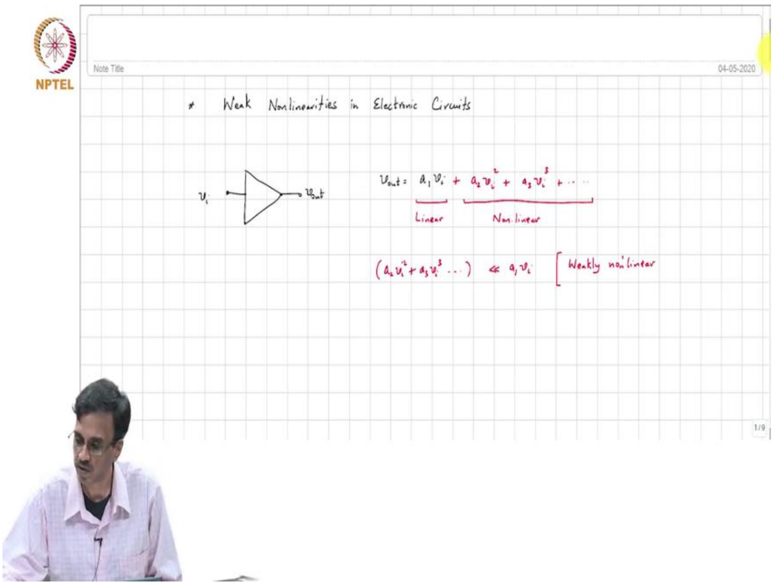
Circuit Analysis for Analog Designers
Prof. Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 55
Weak nonlinearity in electronic circuits, second-order harmonic distortion, HD2 and IM2

(Refer Slide Time: 00:21)



Today we will start off on a new topic and that is namely weak nonlinearities in electronic circuits, right. So, so far, we have basically spent most of the semester on talking about circuits that are linear right. First, we talked about circuits which are both linear and time invariant. Then we talked about circuits that are linear, but varying in time and specifically those that vary periodically with time right, but in reality, it turns out that you know most circuits are essentially non-linear and you know with lot of design effort they still turn out to be within quotes non-linear. But the nonlinearity is very small if and you know the hope is that the nonlinearity is sufficiently small that it can be ignored right. But in many cases, you know even this weak nonlinearity causes problems.

And so today we are going to start studying about weak nonlinearities in circuits and for the time being we will stick to circuits that are supposedly linear and time invariant ok. And we will try to understand you know what nonlinearities do to circuits, right. So, let me take a simple example. Let us say you have an amplifier here and ideally; we want this amplifier to be linear. So, v out is supposed to be some gain a 0 times v i right.

$$v_{out} = a_0 v_i$$

In practice however, there is going to be a 1 v i square plus a 2 v i sorry let me make this a 1; a 1 v i plus you will have a 2 v i square plus a 3 v i cube and so on right.

$$v_{out} = a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \cdots$$

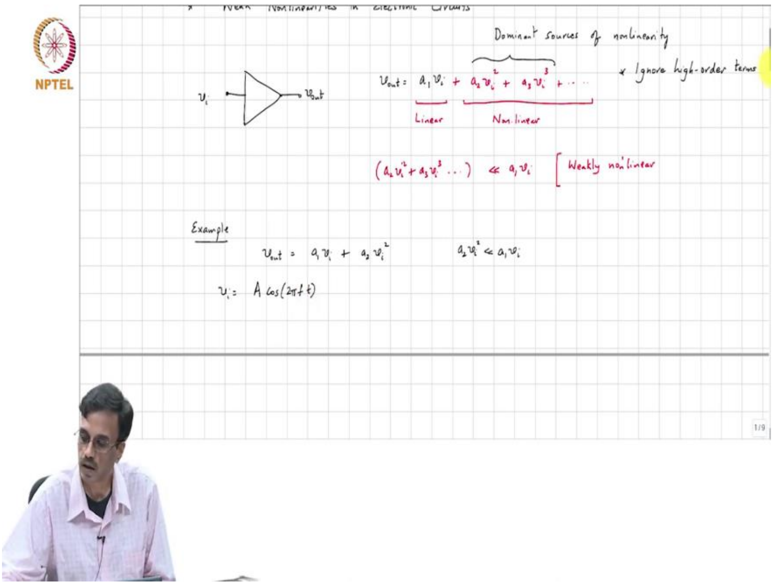
So, $(a_1 v_i)$ this is the within quotes linear term and $(a_2 v_i^2 + a_3 v_i^3 + \cdots)$ these are all non-linear terms. Now, let us see you know what it means to say weakly non-linear. So, when you say weakly non-linear terms, it basically means that this a 2 v i square plus a 3 v i cube etcetera the sum of all these is much much smaller than the linear term which is a 1 v i right.

$$(a_2 v_i^2 + a_3 v_i^3 + \cdots) \ll a_1 v_i$$

And in most work, I mean unless you want the circuit to be deliberately non-linear in most linear most circuit design techniques basically, we will try to ensure that the strength of the non-linear terms is very small compared to the strength of the linear term.

There are of course, there are deliberately non-linear circuits where the nonlinearity is needed for the circuit to function. But at this point we are not concerned about though you know such circuits. What we are more interested in is well the circuit was supposed to be linear now there are some small non-linear terms. What does the, what happens to the output, right. And we will start with the simplest things first I mean I am sure you have seen this in some context or the other.

(Refer Slide Time: 05:02)



So, and because the non-linear terms are very small, typically it is only these terms which are dominant sources of nonlinearity and the higher order terms are ignored ok at least for hand analysis. Well in and if you have compute of course, you know you can take as many terms as you want. Now, the first manifestation of nonlinearity is it is that creates stones which should otherwise not be around.

So, let us say let us say v out is a 1 v i plus a 2 v i square right all the higher order terms are either too small or nonexistent or can be so, small that they can be neglected.

$$v_{out} = a_1 v_i + a_2 v_i^2$$

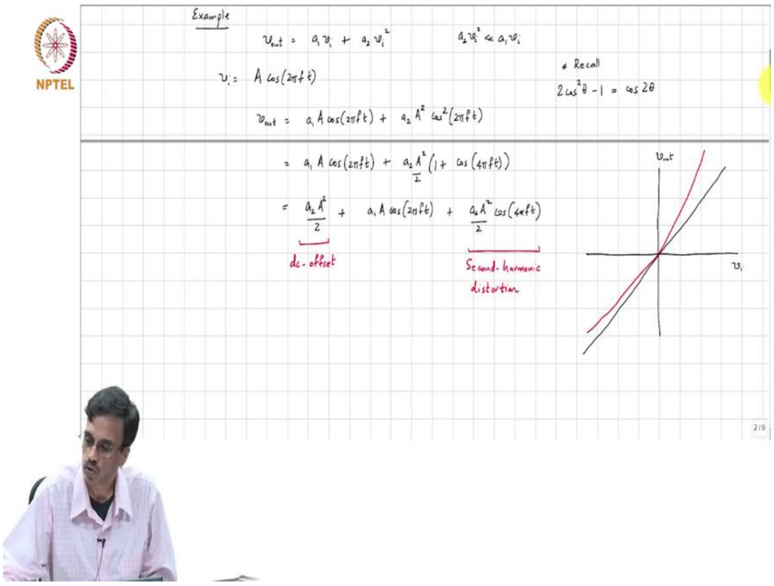
And a 2 v i square turns out to be less much smaller than a 1 v i ok.

$$a_2 v_i^2 \ll a_1 v_i$$

And the question is what happens when v i is a sine wave with amplitude capital A and then cos say 2 pi f t right.

$$v_i = A\cos(2\pi f t)$$

(Refer Slide Time: 06:43)



So, v out therefore, is a 1 A cos 2 pi f t that is simply the linear term plus a 2 A square cos square 2 pi f t and recall that 2 cos square theta minus 1 is cos 2 theta and therefore, this is nothing but a 1 A Cos 2 pi f t plus a 2 times A square By 2 times 1 plus Cos 4 pi f t alright,

$$v_{out} = a_1 A\cos(2\pi ft) + a_2 A^2\cos^2(2\pi ft)$$

$$Recall \quad 2\cos^2\theta - 1 = \cos 2\theta$$

$$= a_1 A\cos(2\pi ft) + a_2\frac{A^2}{2}\,(1 + \cos(4\pi ft))$$

which therefore, can be seen to be a 2 A square by 2 plus a 1 A cos 2 pi f t plus a 2 A square by 2 cos 4 pi f t ok.

$$= \frac{a_2 A^2}{2} + a_1 A\cos(2\pi ft) + \frac{a_2 A^2}{2}\cos(4\pi ft)$$

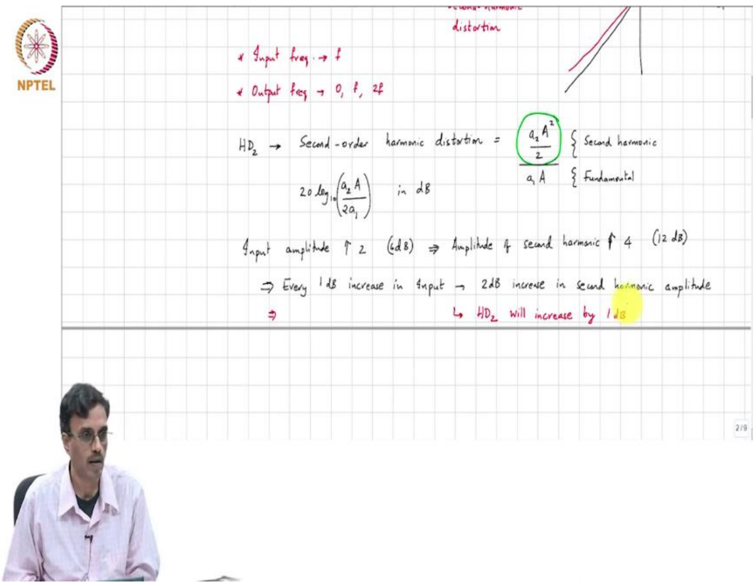
And so, as a consequence of nonlinearity we see that when you have second order non-linearity you say you say you see a dc offset even though the input was, The input is basically, You know clean right, it is a sinusoid with 0, Average value; however, when it passes through an even order nonlinearity right. I mean you basically start to see dc offset and the intuition is the following. The ideal linear output should have been the linear characteristic would have been that. So, this is v i and this is v out. If there is non-linearity what happens? Well, you probably get a curve say I do not know perhaps like that ok.

So, and so even though the input is a sinusoid whose average value is 0 yeah I mean which basically means that the input spends equal amount of time on the right as well as on the left.

Right. Clearly you can see that when the input is on the right the output is always. It is basically, Its output is always larger. Right and likewise when the input is negative the output is always larger than the linear output.

Right. So, since whether the input is positive or negative you are always getting a value higher than what you put in basically means that there is Dc offset right. Then of course, this is the linear term there is no surprises there right and this is the second harmonic distortion right.

(Refer Slide Time: 10:42)



So, in other words we put in an input frequency of f, but our output frequencies are 0, f and 2 f ok, and anyway well you might argue that even in linear periodically time varying system you get an output frequency which is different from the input frequency. But there the output frequencies are f, f plus you know f s, f plus 2 f s and so on and not f and 2 f and 3 f and so on right. So, the next thing is you know how do you quantify you know how linear or non-linear the amplifier is. So, one way to do that is to calculate what is called the second order harmonic distortion, (HD_2) right, which is the ratio of this the of the strength of the second harmonic to the ratio of the fundamental at the output ok

$$HD_2 = \frac{\frac{a_2 A^2}{2}}{a_1 A}$$

And this is as you can see this is a 2 A divided by 2 A 1 right

$$= \frac{a_2 A}{2a_1}$$

And in dB this is simply 20 log to the base 10 in dB alright.

$$HD_2 = 20\log_{10}\left(\frac{a_2 A}{2a_1}\right)\ in\ dB$$

Another thing I like to draw your attention to is the strength of the second harmonic. See that the notice that the strength of the second harmonic is proportional to A square right. So, in other words if you double the input amplitude, let us say it goes up by a factor of 2 which is 6 dB. What comment can you make about the amplitude of second harmonic?

Yeah, that becomes is A square. So, it goes up by a factor of 4 and that is goes up by a factor of 4, which is 12 dB right. So, in other words in general for every 1 dB increase in input leads to 2 dB increase in second harmonic amplitude, ok. And why does it make intuitive sense that the second harmonic strength is proportional to A square?

Yeah, so basically.Yeah, so the distortion terms are basically coming in because of v i square which is v i multiplied with itself. So, if you increase the input amplitude right and you obviously, multiply it with itself then you basically get a squared effect ok and so, ok, so, this is also equivalent to saying that if you increase the input amplitude by 1 dB, what comment can you make about HD 2.

H D 2 will also increase by 1 dB.