

**Circuit Analysis for Analog Designers**  
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**Lecture - 52**  
**Measuring the S-parameters of a one-port**

(Refer Slide Time: 00:16)

Reciprocal N-ports  $\Rightarrow$  Y matrix is symmetric  
 $\Leftrightarrow Y = Y^T$   
 show that  $S = S^T$

In a reciprocal two-port  $S_{12} = S_{21}$

Measurement of a 1-port

Idea 1:

$V = U_T z_L$   
 $V = U - Z_S I$

Let us get to the practical aspect of things. So, it is all you know very fine to say well I want to for example, if I want to measure the  $S_{11}$  so of a for one port. So, this is basically a transmission line, which you connect to some  $z$ . In a lot of electronics, you will of test related topics you will find this DUT or DUT as people like to call it often and the DUT stands for Device Under Test right. And the idea is the you know you want to be able to measure the simplest case of measurement is you know you have one port and you want to measure it right.

And even with the one port, even though there is a this potentially no issue with stability, you know you cannot go and put the voltmeter and ammeter you know right at the two terminals of the impedance, you want to measure right.

I mean you presumably there is some distance between, I mean this is the measurement equipment is some kind of box and this  $Z$  DUT some tiny thing somewhere and you need you know some kind of cable or interface to go from the box to the DUT. And therefore, you have some kind of transmission line right.

So, what you want to do therefore is to measure, remember for a one port you want to basically be able to measure the ratio of. I mean if you measure  $V^-$  by  $V^+$  ok. So, if you measure  $V^-$  by  $V^+$ , you basically know the, well what all you the I mean ok. Now, actually let me ask you the question ok. Well, this is our setup right. How do you think we should go and measure? How we can go and measure  $Z_{DUT}$ ? What all do you think we can do either means this can be done many ways, what do you think you would do?

Ok, that only gives you. Ok, I mean you know yeah tell me exactly what you would want to do. Let us say we know  $Z_0$  and is that all that is needed do you need something else or what? t d ok fine. So, he says well I need to know  $Z_0$  and t d alright, and then what come on folks. Well, he says ok, let us let us see if this is  $V^+$  what is this?  $V^+$  e to the minus you know  $j\omega t d$  right and what is this?

I mean we will use the short form for this is this  $\gamma$  times  $V^+$  e to the minus  $j\omega t d$ . So, what is  $V^-$  here,  $\gamma V^+$  e to the minus  $2j\omega t d$ , correct. So, if we were somehow able to measure, let us assume that we are I mean we have a transmission line and we are able to measure the forward going and the backward going waves right. That is equivalent to measuring what? I mean how will we, I mean what do you think conceptually how will you measure  $V^+$  and  $V^-$ , you have transmission line.

Do you want to measure  $V^+$  and  $V^-$ , what do you think you should you know, you could do in principle? In practice it says, you know what how you do it is another matter, but in principle what would you do or do you need to know?

Yeah. So, basically remember that you know if you want to measure the voltage and current, I mean you want to measure the forward and backward going waves you know at some point you measure the voltage and the current. And recognize that you know  $V$  is nothing but  $V^+ + V^-$  and  $i$  is nothing but  $V^+ - V^-$  by  $Z_0$  right. And I mean even in spice this is a good way of measuring the forward going wave and the backward going wave. Basically, you know you have it using you know voltage controlled so basically  $V^+$  is nothing but voltage plus  $Z_0$  times current by 2, and similarly  $V^-$  equals  $v^- - Z_0 i$  times 1 half correct.

$$v = V^+ + V^-$$

$$i = \frac{V^+ - V^-}{Z_0}$$

$$V^+ = \frac{v + Z_0 i}{2}$$

$$V^- = \frac{v - Z_0 i}{2}$$

So, if you measure the voltage and current at a point you will be able to find out what the forward going and the backward going waves are. And measuring voltage is easy, I mean in practice you know if you I mean at least in principle you know measuring the voltage is very straightforward. What do you think you can do? Well, you know you just need to stick something in parallel with that node and you will be able to find the voltage. Measuring current on the other hand is, what do you have to do to measure current?

You put something in series, you have to cut the transmission line and put your ammeter there right. So, obviously this is you know only ok in the textbook it is not; it is not ok in practice right. So, if you want to find a current which is flowing like this right, you know how will you, what do you think you can do to measure that current without actually cutting the wire?

Pardon. Yeah; so, mean yeah. So, I mean yeah basically use you use you couple an inductor somewhere and the you know so that you will get a couple I mean, you basically use mutual induction right ok. So, in reality it turns out, I mean I am not sure I will have time to discuss it in this course, but it turns out that it is possible and the principle is this. Eventually if you want to find the positive and the power and the backward going waves, you have to do voltage plus  $Z$  times current and voltage minus  $Z$  times current.

The only question is how you accomplish this right. It turns out that there are you know, if you use a couple transmission lines. Couple transmission lines means what? Well, you have a conductor like this and you have you know you put another. So, for example, let us say I, you have a conductor running over the ground over a piece of shield of metal ok, and you have another conductor like this ok.

Now, these two guys are also talking to each other, there is capacitive coupling because of well you know that you know two conductors with separate in space there is some

capacitance. There is also inductive coupling because of, there is going to be some current induced in the second one due to the first one right. So, in principle it is possible to measure both current and voltage flowing on a line right, without actually physically going and cutting it to measure the current ok.

And so, that is I mean and eventually they use the coupled voltage and the coupled current to go and compute V plus and V minus right. And that contraption which is used to do it is what is called a directional coupler alright.

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The slide contains the following handwritten text and diagrams:

- Top Left:** NPTEL logo.
- Top Center:** "In a reciprocal two-port  $S_{12} = S_{21}$ "
- Top Right:** A diagram of a directional coupler with incident wave  $V^+$  and reflected wave  $V^-$ . Below it, equations:  $V = V^+ + V^-$  and  $I = I^+ - I^-$ .
- Middle Left:** "Measurement of a 1-port". Below it,  $\Gamma_{V^+} = \frac{V^-}{V^+}$  and  $Z_{in} = Z_0 \frac{1 + \Gamma_{V^+}}{1 - \Gamma_{V^+}}$ .
- Middle Center:** A diagram of a 1-port network with input impedance  $Z_{in}$  and characteristic impedance  $Z_0$ . It shows incident wave  $V^+$  and reflected wave  $V^-$ . Below it, "Idea 1: We know  $\Gamma_{V^+} = \frac{V^-}{V^+}$  is known  $\Rightarrow$  Find  $\Gamma = \frac{V^-}{V^+}$ ".
- Middle Right:** A box containing the equations:  $V = V^+ + V^-$  and  $I = I^+ - I^-$ . Below it, "Directional coupler".
- Bottom Left:** A small inset image of a man speaking.
- Bottom Center:** "Problems:  $Z_0$  &  $t_d$  need to be known exactly". Below it, a diagram of a directional coupler with a "Calibration" label.

So, a directional coupler is basically a contraption that basically that you can use to go and figure out what are the forward going wave is and what the backward going waves alright. And the principal is you know at what do you call you know a 10000-meter view from the sky is what it is essentially doing is computing  $v$  plus  $Z$  naught times  $i$  by 2 and  $v$  minus  $Z$  naught times by 2, except that you know there is no calculator it is punching right, it uses Maxwell's equations to do this computation.

So, the idea is basically like oh the idea is therefore, if we are able to separate out the forward and the backward going waves right which we will do using this magic device called the directional coupler. Then how will we find the impedance? What will we do?

Oh well, it is I mean we know  $\Gamma_{V^+} = \frac{V^-}{V^+}$  and we know  $V$  plus. So, taking the ratio you get  $\Gamma_{V^+} = \frac{V^-}{V^+}$ . Well  $t_d$  is

known right and omega is known because I mean you know the frequency at which you want to measure right. And therefore, this is known ok which basically means that you can find. So, find gamma and once you find gamma, how will you find Z DUT?

Oh well this is nothing but Z DUT minus Z naught by Z DUT plus Z naught alright and all this I mean.

$$\Gamma = \frac{Z_{DUT} - Z_0}{Z_{DUT} + Z_0}$$

So, once you know gamma you can go and you know Z naught already, so you can find Z. Does makes sense folks? But what are the problems with this approach?

Pardon. I cannot hear you. I mean, he is saying oh you must I mean you are sloppy and you do not make the measurement properly. Yeah of course, if you do not measure properly nothing will be correct right, but the assumption is that you know either you do it properly or you hire somebody to do it properly ok, alright. So, that is you know what you call it; where are all the problems where you can get, where all can you get errors?

Yeah well, we need; remember that the reflection coefficient you measure what dimensions does it have. We got dimensions, it does not have any dimensions. The Z DUT what dimensions does it have.

Well, it is got ohms right. So, whatever you do to transform you know gamma into Z Z DUT right will have something inside which has got dimensions of, you have box you put in gamma you are getting Z DUT right. You are putting in something dimensionless you are getting ohms. So, inside the box there must be something where you are multiplying by ohms and that happens to be the Z naught here right. So, the Z naught is not accurate or not exactly known ok.

Then you will have trouble ok. And likewise, I mean we said that the length of the line is you know is t d and well, if the t d is not accurately known ok. Well, if you have a cable in the lab well you know that it is kind of you know kind of what that t d value is, but you do not know it exactly. So, if Z naught and t d are not known properly clearly our estimation of first our estimation of this guy will be wrong right.

Once you that basically means the gamma that you get will be in error, if the gamma is in error and  $Z_{naught}$  is an error then the  $Z_{DUT}$  made will be an error, correct. Well, we have a problem right. So, what do we do to fix this problem, what do you think we can do? So, basically what he says is very good. So, basically the thing his idea is, oh well we do not know  $Z_{naught}$  and we do not know  $t_d$  correct ok. We do not know them exactly at any rate, so let us try and figure them out ok.

So, every time you are making a measurement right before you actually make your measurement, you let us try and figure out  $Z_{naught}$  and  $t_d$  and use that  $Z_{naught}$  and  $t_d$  obtained you know just before you put plug  $Z_{DUT}$  in, right. In these formulas and then you should be able to; be able to get  $Z_{DUT}$  exactly correct does this make sense. I mean it is obviously, more complicated in simply plugging and playing here right, but I mean hey you know if you do not know what  $Z_{naught}$  is and if you do not know what  $t_d$  is you know be prepared to do some work a little harder correct ok.

So, in general therefore, let us look at this you know what we actually have. So, we have an unknown two port between, correct. So, we will be able to measure the reflection coefficient here correct, which is equivalent to saying we know the impedance looking in here ok. When we are trying to find the impedance there, but we do not know this, you know the box that stuck between in the unknown 2 port and you know what you are actually measure correct, alright.

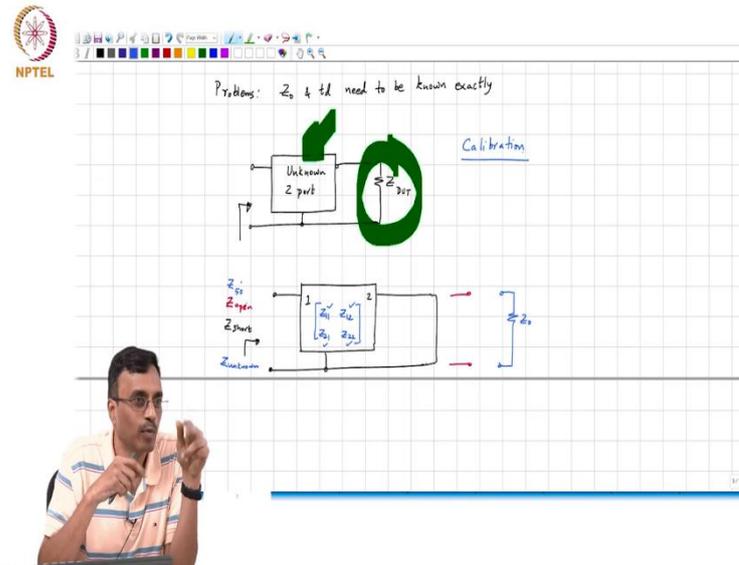
So, what do you think we should be doing therefore? Well, we do not know I mean; obviously, with an unknown 2 port you cannot make any progress. So, you have to convert that unknown 2 port into a known 2 port. Now, you know so how many parameters do you need to characterize that unknown 2 port?

I mean in general for a 2 port you need? Four parameters, but what is you know so special about this unknown, two port between your measurement equipment and the actual  $Z_{DUT}$ .

It is you know is basically a its passive right, and time invariant and therefore, it is I mean it is reciprocal. So, how many parameters do you actually need? You do not really need, in general you need 4, right. Reciprocity you know ensures that ensures what?

Well, you know the  $S_{11}$ , the  $S_{21}$  and  $S_{12}$  of these two ports are of the same or the  $Z_{12}$  and  $Z_{21}$  or  $Y_{12}$  and  $Y_{21}$  are the same. So, you need? We need three parameters, if we need three parameters how many equations do you need? We need? 3 equations. So, therefore, before you put Z DUT right, we need to find the 3 you know the 3 unknowns of the 2 port right.

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So, therefore, before measurement what do you do? You terminate these two ports with, I mean instead of Z DUT you put in how many impedances must you put in there? We need 3, we need to find 3 unknown's guys we need 3 equations. So, you basically put in 3 test impedances there right ok. So, in other words we first find and what comment can you make about 3 and what is your choice of 3 test impedances? Well, you can say I will put 50 ohms, 51 ohms and 51.1 ohms. Is that a good smart way of doing things pardon?

Why cannot oh rather why cannot I put 50 ohms, 50.1 ohms and 50.2 ohms. Pardon. I cannot hear you man. Will become, hey you have a computer 3 equations, 3 unknowns I mean how difficult can it be? Pardon. Measurement precision is a problem right. I mean you will I mean you know I when you try to invert those matrices, if you put 50 ohms, 50.1 ohms, 50.2 ohms right. Then the they will become; the equations will become in conditioned because you know you pretty much have the same equation right. You know just copying the equation 3 times does not make it; copy and pasting the equation saying I

have 3 equations now, does not make, does not help right because the rank of that matrix is basically still close to 1 ok.

So, you want to choose impedances here which are within quotes as different from each other as possible right. So, in other words the reflection coefficient you get must be as different as possible. So, what do you think the best choices are?

Well, when you were you terminate it with a short, right where the reflection coefficient is of the impedance is, if you have a short circuit what is the reflection coefficient? Minus 1, if you have an open circuit, it is? Plus 1 and if you have you know nominally something which is we know that that unknown 2 port is kind of like a transmission line. So, you if you have  $Z_{\text{naught}}$ , then you basically have 0 reflection coefficient right.

So, the first experiment is to measure  $Z_{\text{short}}$  right, where you short port 2. Then the second experiment is you find  $Z_{\text{open}}$  when you open port 2 alright.

And the third experiment is you find  $Z_{50}$  when you terminate it with  $Z_{\text{naught}}$  which is I mean the standard impedance that is used in microwave work is rf and microwave work to connect up blocks together is 50 ohms. So, in the  $Z_{\text{naught}}$  is 50 ohms. So, basically from  $Z_{\text{short}}$   $Z_{\text{open}}$  and  $Z_{50}$  and if you know those you will in principle be able to find the? Right, you will be able to find the  $Z$  parameters for instance of this unknown 2 port.

I mean if you do not like the  $Z$  parameters you can find the  $S$  parameters, it does not matter right. But bottom line you will be able to you know what that unknown 2 port is right. Then what will you do? Yeah, then you put finally, you put the  $Z_{\text{DUT}}$  and then right and the resulting impedance you measure is  $Z_{\text{unknown}}$  and from; since you know  $Z_{50}$  is  $Z_{\text{open}}$  and  $Z_{\text{short}}$ , you will be able to find  $Z_{11}$ ,  $Z_{12}$ ,  $Z_{21}$  which is the same as  $Z_{12}$  and  $Z_{22}$  right. And once you know this matrix, I mean this matrix and you know  $Z_{\text{unknown}}$  you will be able to figure out what?  $Z_{\text{DUT}}$ s alright.

And this is this process of finding no this unknown or characterizing this unknown animal between the measurement port and the and  $Z_{\text{DUT}}$  this is called calibration. I mean you might be wondering hey you know why what is this guy talking about right, you know in my electronics lab, you know I all the time I was measuring you know resistance by taking a multimeter and then you know. In fact, the multimeter was on my neighbor's desk and I

ran these 4 feet long cables and then you know measured the resistance and the resistance turned out to be ok alright ok.

So, how come we need all this complicated thing now? I mean why did this all the suddenly become important?

I mean, are we smoking low power Z DUT is it like you know; what seems like a trivial thing is you know connecting a multimeter across two terminals I mean, should it be this complicated?

Uh. I understand we need to do it, but I mean in your electronics lab when you were measuring like you know resistance, did you do all this?

So, how do we reconcile that? At low frequency which is equivalent to saying  $\omega$  equal to, when you are doing this at low frequency. I mean low frequency basically means that I mean  $\omega$  is close to 0, then what happens? What happens to the transmission line?

I mean  $t d$  equal to 0 right, that basically means what? Well, just behaves like a short circuit and then you know you know what do you measure you know even 4 feet away at DC is the same as you know what I mean, it does not matter how sloppy you are correct ok. Of course, the transmission line is not, I mean if your cable has got some finite resistance, then there will be an error right. And to kind of fix that error I mean you know if you want to be really careful you terminate the other end of the cable with the short circuit.

And see what do you measure on your multimeter. And then you connect your unknown resistance and then you know the difference will give you the actual resistance that you want to measure, correct. But if the cable is lossless at DC right, then you know all this does not matter because if  $t d$  is equal to 0, then well this is  $V$  plus, this is  $\gamma$  times  $V$  plus. So, the voltage here is the same as the voltage here right and similarly the current here and the current here are the same.

So, it does not matter that you have a long cable right. Only when that  $\omega$  times  $t d$  product becomes large is when you start to have errors in measurement. If you simply stick a cable and say you know oh, I will you know I will continue what I did to do what I did in my you know basic electronics lab, where everything was running at you know half a kilohertz right. If I try to do this when you know at say, a gigahertz right ok. Then you

know if you have a foot long cable right, a foot long cable has got a  $\tau$  of how many seconds?

Ok a foot is how many meters guys? One-third of a meter right and the speed of light is?  $3 \times 10^8$  is only when you know, when you have free space men. Typically, 1.5 is a reasonable estimate right ok. So, you know you do the math. So, one-third into whatever, 1.5 into  $10^8$  and you know 8 meters per second. So, that is what 1.5 into 3 is about 5, right. So, it is something like 20 nanoseconds is it ok. I hope that is right ok. So, 20 nanoseconds times gigahertz is times  $2\pi$  is yeah. So, that  $e^{-j2\omega\tau}$  is a significant phase shift. It is not, it is not it is not 1 alright. So, as frequencies I mean this, I hope.

So, if say you know if, I mean as you can see that  $\omega\tau$  for at a gigahertz is you know several multiples of  $2\pi$ . So, if you want that to be a short fraction or small fraction of  $2\pi$  which corresponds to the phase of 1 wave length, then that the length of the cable better be very small in relation to the wave length, correct. Now, if you are now working not at 1 gigahertz, but if you are working at 60 gigahertz now, right.

What qualified as a small distance at 1 gigahertz now qualifies as can be you say you know I say effectively looks electrically 60 times larger, if you are working at 60 gigahertz right. So, at high frequencies you know this the if the dimensions of the physical circuit that you are working, I mean the physical dimensions of the circuit you are working with are comparable to a you know a small fraction of the wavelength is when all these distributed effects become important right.

We should understand this very clearly right, because I do not know about you, but when you know I was studying all this in my EM class, you know 30 years ago right yeah. I always used to wonder you know how come you know when you move from basic electronics to the transmission line course, you know even things as simple as measuring an impedance become all of a sudden becomes so complicated right, I mean were we doing things wrong in the basic electronics class.

Or you know; or you know is this guy is simply making a big fuss out of nothing right ok. So, as you can see there is, I mean there is method to this madness right. I mean there is a reason why these things are important right. So, at high frequencies and what is the meaning of high frequency therefore? This  $\omega\tau$  is, I would say you know a

significant fraction of I would say like maybe one-tenth let us say. At arbitrarily, I am just saying 0.1 is bad enough right ok.

And what is  $t d$  in terms of length and omega? In  $1 t d$ , I mean remember that frequency is nothing but omega by  $2\pi$  correct and basically in  $1 t d$ ; I mean in  $2\pi$  over omega seconds you will basically, the wave will move by how much? 1 wave, by 1 wavelength right. So, basically this  $t d$  is related to the I mean which is related to the physical length of the cable is eventually related to the wavelength of light at that frequency correct.

So, this omega times  $t d$  product being you know not sufficiently close to 0 basically means the physical dimensions of you of your transmission lines of the circuit that you are working with become comparable to the wavelength of light at that frequency. The moment that becomes the, it becomes comparable then all distributed effects I mean this then this notion of the voltage here.

I mean hey I have shorted these two nodes together right; it does not make any sense because the voltage here and the voltage here are different. And therefore, all these lumped element approximations all become right be become unknown right ok.