Circuit Analysis for Analog Designers
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Lecture - 50
Example scattering matrix calculations

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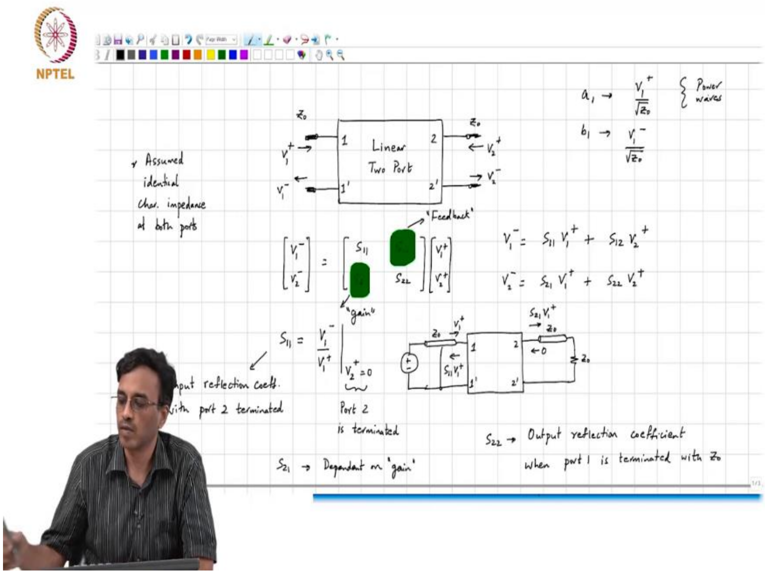

Well, we say ok. What is S 1 1? S 1 1 is V 1 plus by sorry V 1 minus by V 1 plus when V 2 plus equals 0 and how do you ensure that V 2 plus is 0?

$$S_{11} = \frac{V_1^-}{V_1^+}|_{V_2^+=0}$$

Right, I mean unlike in the admittance parameters where you terminate the port 2 in a short right here you want to make sure that you know V 2 minus is what this linear two port is pushing onto the output port you want to make sure that V 2 plus is 0 and that is only possible when you terminate the output port with the you know resistance which is equal to the characteristic impedance of the line right.

So, the test setup therefore, is you have our two port ok. So, we have some voltage source here and then we have this transmission line and then we have another transmission line here. The length of these lines can be 0 this is Z naught and then this is Z naught and this is Z naught and. So, this is V 1 plus right what comes back is what comes back? Right.

So, this is basically this V 2 plus is 0 because you terminated the output port right. So, this is S 1 1 times V 1 plus alright and so, alright. So, what common can you make about the about V 2 minus now is it 0 is it nonzero? It is nonzero because well the 2 port is basically you know whacking this the second port, I mean the output port. So, this is basically S 2 1 times V 1 plus right.

And this is analogous to what we do with the y parameters and so on. What would we do? We would you know in the first experiment we will short the output port and excite the input port of the voltage the input admittance gives you y 1 1 and the current in the output port measured will give you the ratio of that to the applied voltage will give y 2 1.

Similarly, here you know you terminate the output port you know you excite the input port, what is reflected or in other words S 1 1 is the input reflection coefficient. I hope you remember that you know V 1 minus by V 1 plus is you know is the reflection coefficient. So, input reflection coefficient with port 2 terminated alright. And S 2 1 I mean is I mean can we make any comment about S 21 do you know what parameter of the 2 ports do you think you know it would be related to?

Yeah, it depends I mean S 2 1 is basically quantifies within quotes the gain or y 2 1 or whatever the forward transfer parameter of the 2. Now ok so ok, now likewise what about S 2 2? It is output reflection coefficient, when?

When port 1 is terminated with Z yeah, clear people? Alright and what comment can you make about S 1 2. What does that quantify?

I mean just like y 1 2 what does y 1 2 quantify? It quantifies the effect of an excitation at port 2 on the response at port 1 right. And likewise, S 1 2 basically is telling you if I throw something at port 2 how much of it you know comes into comes out of port 1 right. So, this is again quantifying the amount of feedback within the device ok. So, you know a caveat we have assumed identical characteristic impedance at both ports ok in which case it is you know it turns out that you know it does not matter.

I mean you can simply use the amplitude to the forward and backward voltage waves strictly speaking the definition of the S parameters is to define first an incident within quotes you know wave which is what which is actually normalized to the to the characteristic impedance in this way right. And similarly, the reflected wave is basically V 1 minus by square root Z naught ok. And you find ratios of b 1 by a 1 and b 2 by a 2.

If all the port impedances characteristic impedances are the same of all the transmission lines connecting the both ports 1 and 2 then the square root Z naught is you know is redundant and it simply you can simply talk about V 1 minus V 2 minus V 1 plus and V 2 plus ok. Sometimes these are often also called power waves and why? The square of this directly gives you power ok alright.

So, if all the impedances are the same there is no point in carrying that square root Z naught all the way there is something just to bear in mind right and not get all flustered when you see you know b 1 a 1 I mean or whatever b 1 b 2 equals S matrix times a 1 a 2 right it is in most cases it is 1 and the same alright.

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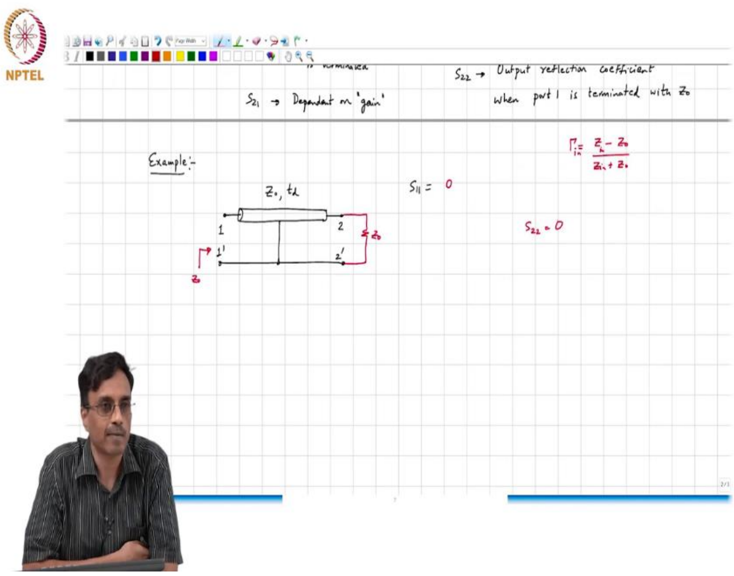

So, now that you know we know what these S parameters are we let us try and find the S parameters or some commonly used two ports right. And this is analogous to the first the first thing you do the first time you do y parameters you basically you know throw some example networks and find the Y parameters and the same thing you would like to do with the S parameters right.

So, this is let us say you have a transmission line of length Z naught and length of characteristics impedance Z naught and length t d and you would like to find the S parameters of this guy. What do you think we should do? What would be? Well to find S 1 1 what would you need to do?

Ok. You terminate port 2 with Z naught and find the reflection coefficient at port 1 right if you terminate port 2 with Z naught what is the impedance looking in here? It is Z naught very good. So, what common can we make about the reflection coefficient at port 1? What do we understand by the reflection coefficient? Remember, the reflection coefficient is nothing but gamma is Z minus gamma n is nothing but Z in minus Z naught by Z in plus Z naught.

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

So, if the looking in impedance is Z naught then what comment can we make about S 1 1, it is 0 alright ok. And this is a symmetric 2 port. So, if S 1 1 is 0 what comment can we make about S 2 2 is also 0 right. Now the next thing to do is find you know S 1 2.

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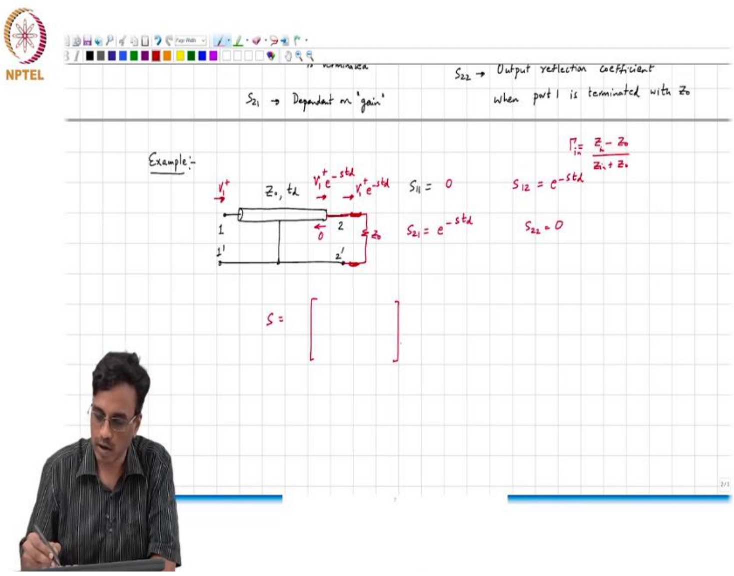

So, what do we do? We have some V 1 plus here correct ok. And if this is remembered that the reflection coefficient is 0 ok. So, there is basically you have some incident wave here. This is V 1 plus what comment can we make about the forward wave their pardon.

It will be simply V 1 plus e to the Minus S times t d or if you are doing j omega it is minus j omega t d and. So, therefore, what comment can we make about the forward going wave here. I mean imagine that you this is a infinitesimally small transmission line. What is continuous across boundaries?

Voltage not impedance folks come on. Current and voltage are always continuous across boundaries. When you go from 1 transmission line you have a junction and you go elsewhere KCL and KVL must be valid at every point correct ok. Here well we know that the reflected wave what is the reflected wave here?

We know that it is 0 because you know everything is terminated right. So, the voltage here what is the total voltage at this point?

It is simply the sum of the forward and the backward waves which is simply V 1 plus e to the minus S times t d right and likewise on this transmission line we know that there is no reflected wave.

So, the voltage at this point is the same as the voltage at this point and that must be simply equal to the amplitude of the I mean whatever the forward going wave there correct. So, what comment can we make about the about S 2 1 it is simply e to the power e to the minus S t d or on the j omega axis it is e to the minus j omega.

And of course, now this being a symmetric two port now what do they expect for S 1 2 it is the same thing e to the minus S times. So, the S matrix therefore, the scattering matrix of this two port.

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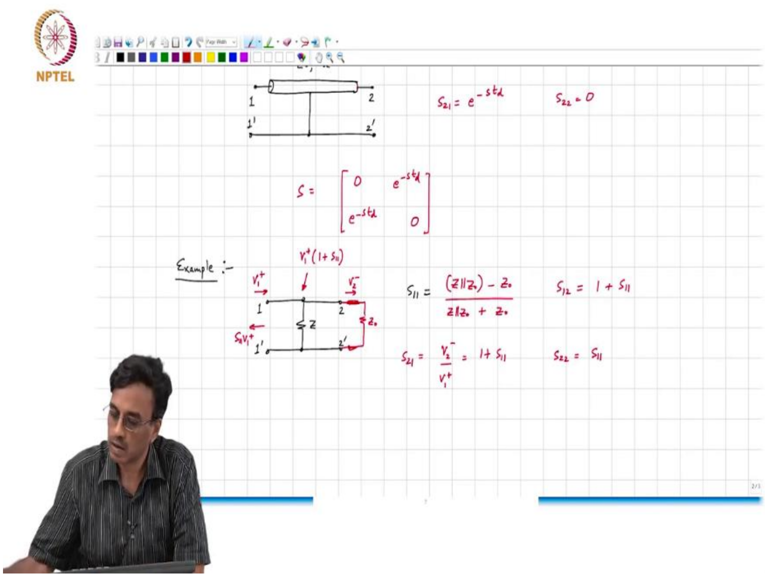

Now, I can remove all these extras is 0, 0, e to the minus S times t d and e to the minus S times td alright ok excellent. Let us do another example let us say we have an impedance Z here which is all you know not to be confused with Z naught which is the characteristic impedance of the transmission lines. So, what is S 1 1? Please calculate it come on people what is the definition of S 1 1? What do we need to do?

We terminate port 2 with Z naught and we would like to find the input reflection coefficient right. I mean I hope all of you are aware of why S 1 1 is the input reflection coefficient is the ratio of V 1 minus 2 V 1 plus you know when you terminate port 2 and V 1 minus by V 1 plus is nothing but the reflection coefficient right. So, what is this, what is it now.

Z parallel Z naught minus Z naught by alright ok.

$$S_{11} = \frac{(Z||Z_0) - Z_0}{Z||Z_0 + Z_0}$$

So, if this is V 1 plus what gets reflected back is S 1 1 times V 1 plus because we have terminated port 2 with Z naught correct. So, what is the voltage at this node? Pardon.

V 1 plus times 1 plus S 1 1 correct. So, that is you know evidently the voltage also at node at that is the voltage at port 2 correct and therefore, what comment can you make about the forward going wave here? Pardon, why am I interested in this forward going wave here? That is. Yeah, that is V 2 plus or minus. V 2 minus very good ok. So, what comment can we make about V 2 minus?

It will be equal to the voltage at port 2 which happens to be V 1 plus times 1 plus S 1 1 correct. So, what comment can you make about S 1 2 therefore? 1 plus S 1.

So, that says 2 1 sorry is V 2 minus by V 1 plus which happens to be 1 plus S 1 and S 1 2 is the same as S 2 1 is 1 plus S 1 1 alright. And S 2 2 is same as S 1 does makes sense ok alright.

$$S_{12} = 1 + S_{11}$$

$$S_{21} = \frac{V_2^-}{V_1^+} = 1 + S_{11}$$

$$S_{22} = S_{11}$$

So, now, if Z is a capacitance, you know you can go and substitute arbitrary things for Z and you know get a whole lot of problems like this.

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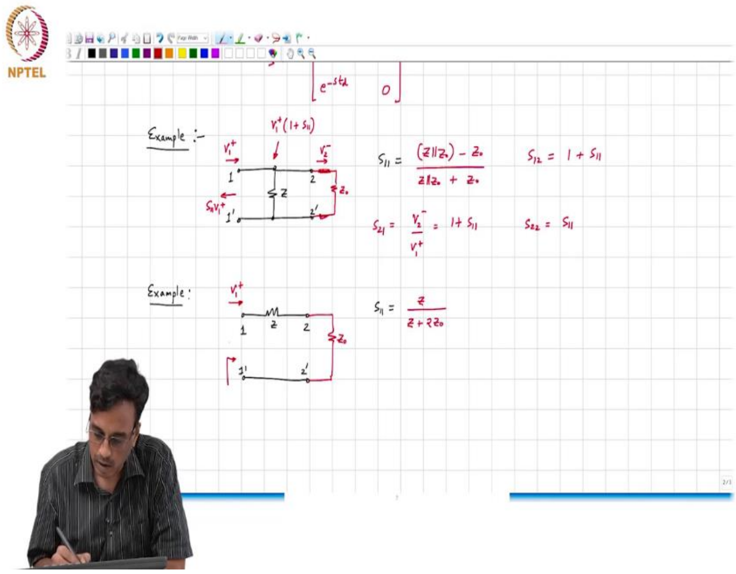

Then let us do another example ok. If shunt is here series cannot be far behind. So, this is 1, Z alright now what is S 1 1? Again, same volt, same volt we terminate port 2 with Z naught ok. So, basically, we terminate and the input the looking impedance is; obviously, Z plus Z naught. So, the reflection coefficient is Z by Z plus 2 Z alright.

$$S_{11} = \frac{Z}{Z + 2Z_0}$$

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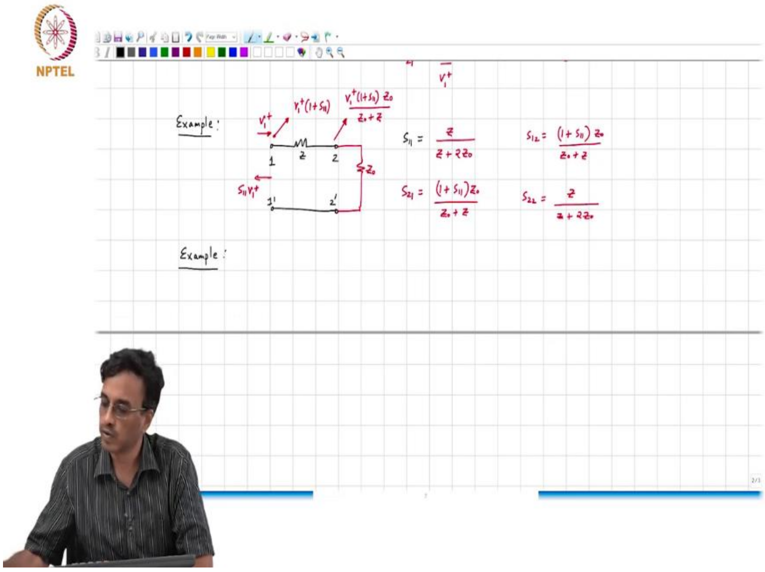

So, this is so, this is V 1 plus what is reflected back is S 1 1 times V 1 plus, $(S_{11}V_1^+)$. So, the voltage here is V 1 plus times 1 plus S 1 1 correct, $\left(V_1^+(1+S_{11})\right)$. So, what is the voltage here guys? We know the voltage at port 1 what is the voltage at port 2?

Simply V 1 plus times 1 plus S 1 1 which is the voltage at port 1 times potential divider very good. So, that is basically Z naught divided by Z naught plus Z correct, $\left(\frac{V_1^+(1+S_{11})Z_0}{Z_0+Z}\right)$. So, now, what is S 2 1? What is it? 1 plus S 1 1 times Z naught by Z naught plus.

$$S_{21} = \frac{(1+S_{11})Z_0}{Z_0+Z}$$

What about S 1 2? Well, the same as S 1 1, S 2 1 which is basically 1 plus S 1 1 times Z naught by Z naught plus Z.

$$S_{12} = \frac{(1+S_{11})Z_0}{Z_0+Z}$$

And S 2 2 is same as S 1 1 which is Z by Z plus 2 Z that makes sense people ok.

$$S_{22} = \frac{Z}{Z+2Z_0}$$

So, I mean you can keep now doing this until you are blue in the face ok. You can generate as many examples as you want and keep calculating right. And the next thing that next example I would like to do is something.

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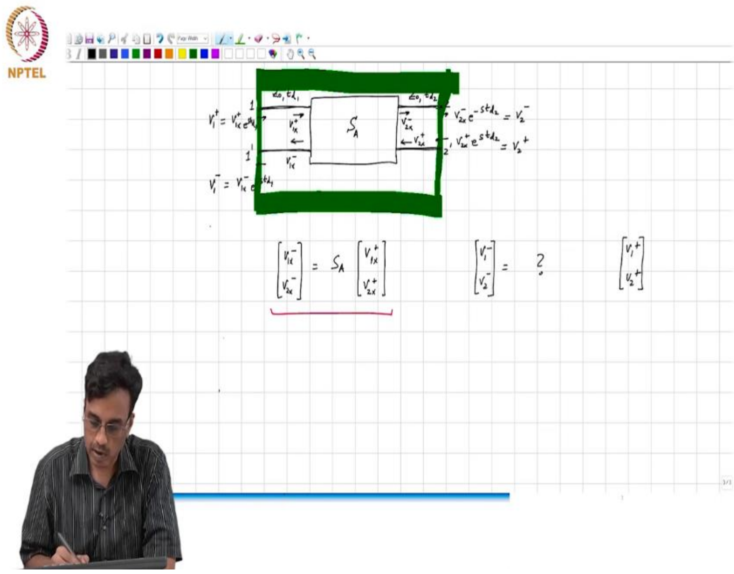

So, let us say you have a two port already right with a known S matrix ok. Let us call this S A. And now I connect transmission line. So, this is Z naught t d 1, $\left(Z_{0_1} t_{d_1}\right)$. This is Z naught t d 2, $\left(Z_{0_1} t_{d_2}\right)$, alright. So, what are the S parameters of this new animal? How do we figure it out? Well, what is let us call this forward wave let us call this V 1 x plus, (V_{1x}^{+}), correct and what comes back therefore, is V 1 x minus, (V_{1x}^{-}), and what must this therefore be?

V 1 x plus times e to the power S times t d, $(V_{1x}^{+} e^{st_{d_1}})$, and what comes back? The reflected wave here is V 1 x minus times e to the minus S times t d, $(V_{1x}^{-} e^{-st_{d_1}})$ ok.

Likewise, this is V 2 x minus, (V_{2x}^{-}) and this is V 2 x plus, (V_{2x}^{+}) and this will be V 2 x minus e to the minus S t d 2, $(V_{2x}^{-} e^{-st_{d_2}})$, t d 1 sorry here. And this will be V 2 x plus e to the S t d, $(V_{2x}^{+} e^{st_{d_2}})$ alright. So, we are; however, interested in finding. So, V 1 plus equals this V 1 minus equals this is V 2 minus and this is V 2 plus because this is our 2 port correct alright.

$$V_1^{+} = V_{1x}^{+} e^{st_{d_1}}$$

$$V_1^- = V_{1x}^- e^{-st_{d_1}}$$

$$V_2^- = V_{2x}^- e^{-st_{d_2}}$$

$$V_2^+ = V_{2x}^+ e^{st_{d_2}}$$

So, how do you relate and we know, what do we know? We know that V 1 x minus V 2 x minus equals S A times V 1 x plus and V 2 x plus alright.

$$\begin{bmatrix} V_{1x}^- \\ V_{2x}^- \end{bmatrix} = S_A \begin{bmatrix} V_{1x}^+ \\ V_{2x}^+ \end{bmatrix}$$

And we are interested in finding V 1 minus V 2 minus equals what into V 1 plus V 2 plus correct. So, now, can you help me out?

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = ? \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

Ok, very good. So, what is your suggestion? So, you say which are you talking about this equation ok. So, what do, what did you want to do?

So, what should I do I will take.

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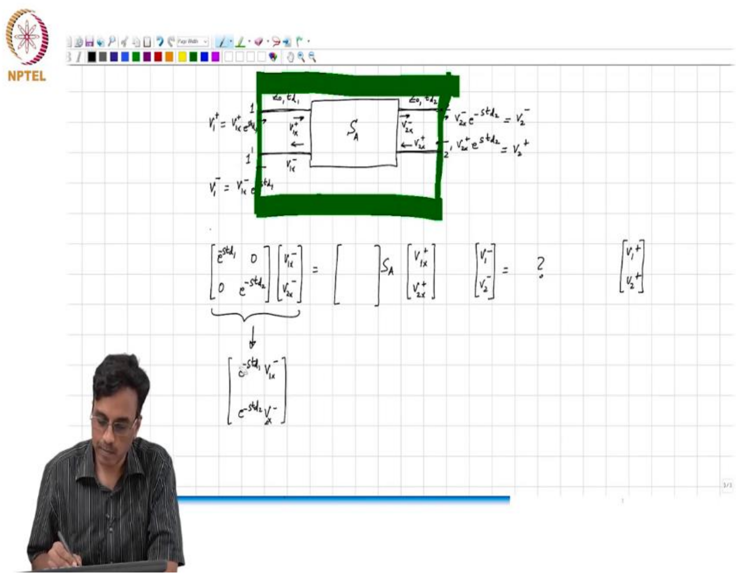

So, you want me to pre multiply this with suggestion is the following this is you multiply this with a diagonal matrix which is e to the power S t d 1, 0, 0, e to the minus S t d 2.

$$\begin{bmatrix} e^{-st_{d_1}} & 0 \\ 0 & e^{-st_{d_2}} \end{bmatrix} \begin{bmatrix} V_{1x}^- \\ V_{2x}^- \end{bmatrix} = \begin{bmatrix} e^{-st_{d_1}} & 0 \\ 0 & e^{-st_{d_2}} \end{bmatrix} S_A \begin{bmatrix} V_{1x}^+ \\ V_{2x}^+ \end{bmatrix}$$

So, that will give me what will this give me? This will give me e to the S t d 1 times V 1 x minus right ok. So, what now or did you mean minus S t d 1 and minus S t d 2?

Ok, alright. So, that should give me S t d 2 V x e 2 x minus.

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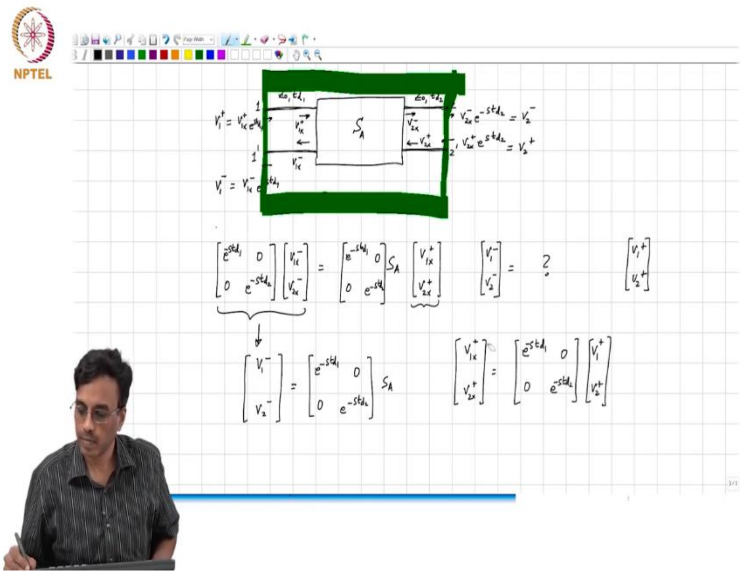

So, that is basically we know this is nothing but V 1 minus sorry yeah this is V 1 minus and this is V 2 minus ok very good.

$$\begin{bmatrix} e^{-st_{d_1}} & 0 \\ 0 & e^{-st_{d_2}} \end{bmatrix} \begin{bmatrix} V_{1x}^- \\ V_{2x}^- \end{bmatrix} \rightarrow \begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix}$$

So, we are done with one side this is equal to e to the minus S t d 1, 0, 0, e to the minus S t d ok. Now what, how do we get I mean likewise how do you get from here, how do we get? Yeah. So, what is V 1 x plus and V 2 x plus in terms of V 1 and V 2, V 1 plus and V 2 plus.

Pardon. e to the S t d 1, 0, 0, e to the s correct is that right. V 1 x plus oh yeah sorry V 2 x plus yes are we good right. So, therefore, what should I, what should I do here?

$$\begin{bmatrix} V_{1x}^+ \\ V_{2x}^+ \end{bmatrix} = \begin{bmatrix} e^{-st_{d_1}} & 0 \\ 0 & e^{-st_{d_2}} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

So, V 1 minus is V 2 minus is simply e to the minus S t d 1, 0, 0, e to the minus S t d 2 times S A times.

No y inverse

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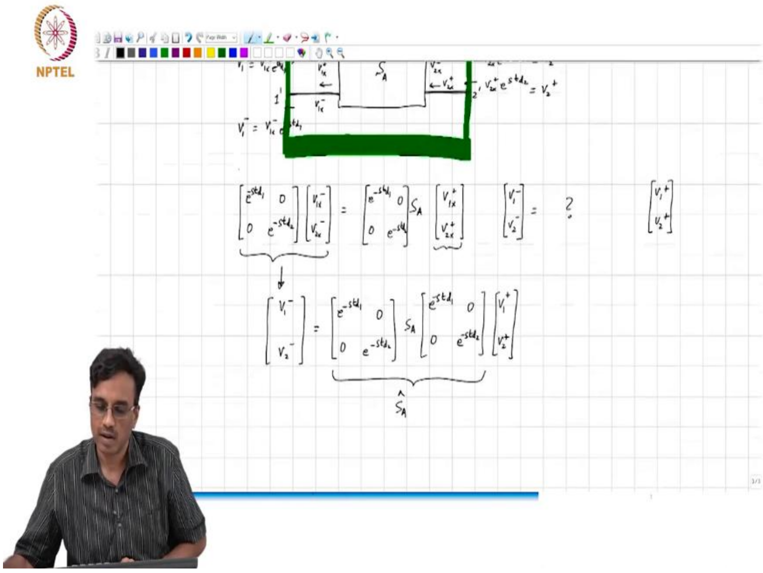

Right, ok. And. So, what is this now?

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} e^{-st_{d_1}} & 0 \\ 0 & e^{-st_{d_2}} \end{bmatrix} S_A \begin{bmatrix} e^{-st_{d_1}} & 0 \\ 0 & e^{-st_{d_2}} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

This is the modified one. Does make sense people? Ok. I hope we not made any algebraic goof it seems ok alright. So, this is the augmented you know S A hat.

$$\begin{bmatrix} e^{-st_{d_1}} & 0 \\ 0 & e^{-st_{d_2}} \end{bmatrix} S_A \begin{bmatrix} e^{-st_{d_1}} & 0 \\ 0 & e^{-st_{d_2}} \end{bmatrix} = \widehat{S_A}$$

So, anyway I mean so, to find I mean to find the S parameters basically as you can see I mean it is a little bit of algebra just like finding y parameters or Z parameters you know it takes some work right, but you should get used to it by simply by practice ok.