Circuit Analysis for Analog Designers
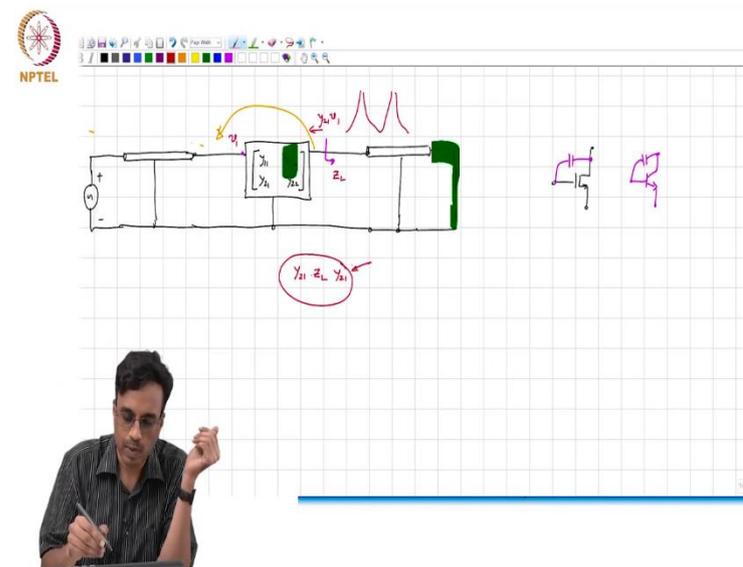**Prof. Shanthi Pavan**
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Lecture - 49
Scattering Parameters: Introduction

A quick recap of what we were doing yesterday.

(Refer Slide Time: 00:24)

So, we were talking about the problems with trying to measure the Y parameters of for instance an active network right. Here again I just take a simple example of some amplifier do not worry about the biasing details. It has got you know some Y matrix say Y 1 1, Y 1 2, Y 2 1 and Y 2 2.

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

And you know as we have been talking well it is not physically possible to have a perfect short right at the terminals of the box.

So, there is some cable of some lengths and then you know you are kind of terminating the other end of the cable and likewise here and this is the source and is basically yeah you hoped that these cables are small enough. So, that the voltage applied here appears across port 1 and likewise the short circuit that you placed here is effectively a short here.

But as we have already seen with our analysis you know the that is only true when the frequency of operation times you know the delay of the transmission line or equivalently the length of the cable is very small compared to 1. And so, in most practical cases you know it is that omega times t d being much smaller than 1 is not satisfied.

At any rate the even for a short line and the perfect short is only true at DC and it is not true at any other frequency. But there is a more serious problem and that has to do with feedback within the device right. So, for example, this term Y 1 2 quantifies the effect of you know when you excite the output port with voltage what current flows in the input port. Ideally remember that; that Y 1 2 must be I mean a transistor or you know any amplifying device you would like that Y 1 2 to be 0, but you know as you have seen you know in your both your device classes as well as your circuits classes.

The equivalent circuit for this for the MOSFET is that there is a parasitic capacitance between the gate and the drain. Likewise, if you have a bipolar transistor there is again parasitic capacitance between the collector and the base.

And it follows that you know if you make you know if you use multiple devices. And then you know you make a bigger amplifier through such effects since each one of these active devices is not truly unilateral. When you put them all together you will find that the resulting device is also the resulting amplifier is also not unilateral.

So, any amplifying kind of two port will have some undesirable you know within quotes feedback right. Now, the impedance looking here unfortunately right because of the transmission line terminated in the short circuit or an open circuit if you want to measure Z parameters.

Basically, you know can be thought of as a bunch of tuned impedances right each resonating at what do you call you know 1 by, I do not remember the I think it is 1 by what was that 1 by 4 t d and you know 3 by 4 t d and so on it looks like open circuits right.

And so, if you inject, I mean remember the amplifying device is reacting to voltage that you apply here by pulling a current which is Y 2 1 times let us call this v 1 is Y 2 1 times v 1 right. And that is this current is going through potentially an impedance whose you know magnitude looks like this right.

So, you can see therefore, that there is you know this this current is subject to a to a load impedance with a high order load impedance with whose impedance can become very large right. So, if there was no feedback if Y_{12} was 0, there would be no problem at least as far as stability is concerned right.

Now, you have a high order with Y_{12} unfortunately there is if there is a large voltage here. Then through Y_{12} there is some there is some feedback through the two port and you know that also sees some very large frequency selective impedance.

So, if you want to think about the loop gain remember the loop gain is basically of the form you know the exact expression is messy. But you can see that it depends it should depend on Y_{21} times whatever load you have there right ok that is the voltage and then part of it is fed back right. So, that times Y_{21} times you know whatever this impedance is and so on.

So, this basically you know appears in the loop gain. And now Z_L is the order of that Z_L is very high and Y_{21} times Z_L is you know the peak value of Y_{21} times Z_L can become very large, because Z_l is virtually an open circuit. So, this therefore, the loop gain function if you want to think about it that way basically is a high order transfer function with a lot with a lot of gain right.

And you know that if you have a high order transfer function with a lot of gain then it is very easy to you know for the closed loop system to have poles in the right half s plane. And therefore, the closer I mean the moment it turned out that when people were trying to measure the admittance parameters of transistor amplifiers.

They found that all of a sudden you know the whole set up if they were unlucky could just go in flames and then (Refer Time: 07:17) or which simply burn the transistor because well this whole system would start to oscillate right because of the feedback within the device and these you know high order effectively high order impedances have the load and the source right.

So, in one of your assignments you will actually you know put a transmission line and see how simple it is actually to make something you do not even have to try ok just it will simply oscillate right. And so, if the device was small, I mean and you know you if there was a lot of oscillation, I mean the oscillation amplitude was very large. Then the power

dissipated inside the device would be much larger than it is rated power and the transistor would just burn right.

And if you had a power device for example, you were trying to make a power amplifier and were trying to characterize a transistor for instance. Then you know because of the large parts involved you could even basically kind of get into a small fire accident ok.

So, clearly you know the whole notion of trying to measure voltages and I mean the admittance or the impedance parameters by where the two port is terminated with open or short circuits is simply does not work in practice ok.

And the reason is that you know if you have a long transmission line as we have seen you know terminating it in a short basically causes these weird impedances at the other end of the line. And therefore, we need to find a way around this problem. In other words what we need to do is figure out a way in which we can measure you know whatever I mean you know the parameters or the of these two ports right where the length of the transmission line has.

So, what is the root cause of this problem is that?

The length of the transmission line depending on the length of the transmission line you basically start to see different impedances at this point right or this looking back even ok.

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And the longer the transmission line you know the more frequently you know you start to see resonances. And what you want to do therefore, is to make sure that the length of the transmission line at least to first order has no influence on the impedance that you see the transfer the two port sees.

So, what do you think now that you know all about transmission lines what do you think you want to do? If you want the impedance, you know you, I mean physical reality, is you cannot get away from the cable right ok. So, and you want the impedance looking in here to be independent of the length of the transmission line right. So, what do you think we can do right?

Yeah. So, basically you say oh well you know I know the characteristic impedance of my transmission line and you know it is some it is a long cable with some length t sub d.

And I should basically whatever I do right I should make sure that I terminate my transmission line with Z and likewise my source also should have an impedance of 0. So, this is Z naught say t d 1, Z naught times t d 2 ok. And fortunately, if the moment I do this what happens? What is the impedance looking in here?

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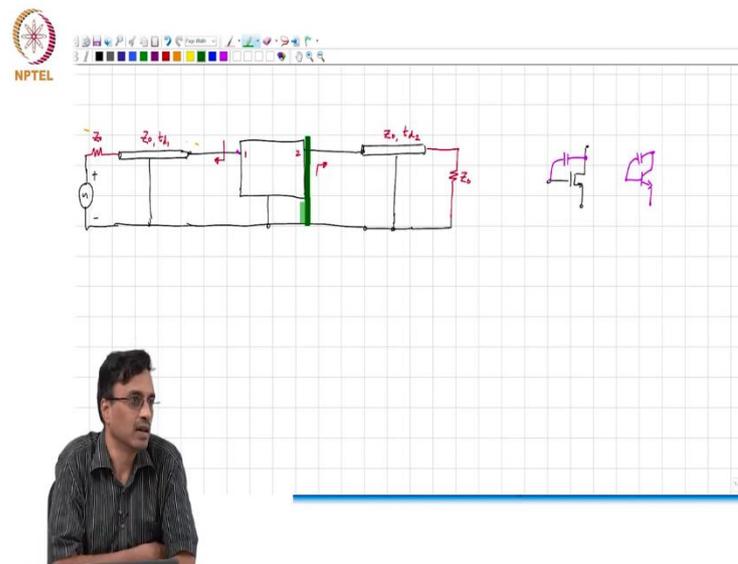
What is the impedance looking in at the I mean at port 2?

It is Z naught and the important thing is that it is independent of it is independent of the length of the transmission line t d2, so t d 2. So, in principle you can have a long cable and

long lossless cable and the impedance will still be Z naught right no matter how long t d 2 is. Is that clear people?

Alright, likewise with the transmission line in on the input side right ok. So, regardless of however, large t d 1 is what comment can we make about the Thevenin equivalent of I mean Thevenin impedance looking in from port 1? Oh well for that you short you know de energize the source. So, source becomes a short circuit and the effective impedance seen by port 1 is also Z naught right.

So, this solves a big practical problem right. Because now the load is no longer some you know high order frequency dependent network that we had earlier, it is simply a resistor of value Z naught right.

So, this so, as a result it is equivalent therefore, even though you have terminated the transmission line you know right. I mean we could have port 2 here and you know you could run a cable at least in principle you know all the way to your homes right. And if we terminated the cable at Z naught it is as if you are terminating it right at the port 2 ok.

So, the stability problems that are associated with the measurement of Y and Z parameters is now avoid alright. And so, this you know it turns out that this was the motivation to come up with the with an alternative set of alternative two port parameter set ok and again you know the practical motivation is this.

Now, you have to fill in all the theory and that is oh well on a transmission line there is no notion of I mean voltage and current keeps changing at every point. The only thing you can be sure is that you know if you know what the forward going wave and the reverse going wave is at a certain point you can figure out what it is at some other point on the line by appropriately delaying or advancing the forward and reverse going waves alright.

So, that is the so, with this background. So, let us say you have a two port and you think of this the two ports as being excited by you connected to the rest of the world through transmission lines right with and for the purposes of this course, we will assume that the impedance the characteristic impedance, the transmission lines on both ports is the same that need not necessarily be true.

But, most of the time in practice you know this is a good assumption to make. And if you do that then basically you know a on this transmission line well this is port 1 and this is port 2 alright. And you have basically the incident wave here and that is on port 1 and what do you call the how do you denote the incident wave.

Yeah. So, V 1 is on this is incident on port 1. So, that is V 1 plus and what gets thrown back is what do you call that folks come on V 1 minus ok alright, and similarly what is incident on port 2 is V 2 plus and what gets thrown back is V 2 minus alright and this is this is a linear two port correct.

And therefore, and if you think of V 1 plus and V 2 plus as the cause right, they and they and V 1 minus and V 2 minus are the effect. Then simply by super position we should expect, what should we expect?

Well, what is reflected back at port 1 must not only depend on what is the incident on port 1, but also what is incident on Port 2 alright.

And the assumption is that these transmission lines you know are terminated at their far ends with a characteristic impedance of Z naught. And therefore, you know V 1 minus you know what is reflected from port 1 right is reflected and it is gone right it is not going coming back again ok. So, V 1 minus V 2 minus is basically a linear combination of V 1 plus and V 2 plus. And basically, these are the two port scattering parameters S 1 1, S 1 2, S 2 1 and S 2 2 right.

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

All that in English all that this is saying is that the reflected wave at port 1 is S 1 1 times V 1 plus which is the S 1 1 times the incident wave at a port 1 plus S 1 2 times V 2 plus. And similarly, V 2 minus which is the reflected wave at port 2 is S 2 1 times V 1 plus plus S 2 2 times V 2 plus correct.

$$V_1^- = S_{11}V_1^+ + S_{12}V_2^+$$

$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+$$

And like in yeah you know with our usual two port parameters Y parameters, Z parameters. We kind of interpret Y 1 1 as you know whatever right I 1 by V 1 when V 2 is 0 and so on. Likewise, we would like to interpret I mean you always want to kind of you know place something new in a framework where you have already done something similar in the past.