Circuit Analysis for Analog Designers
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Lecture - 44
Transmission line circuit analysis: The short circuited and open circuited line

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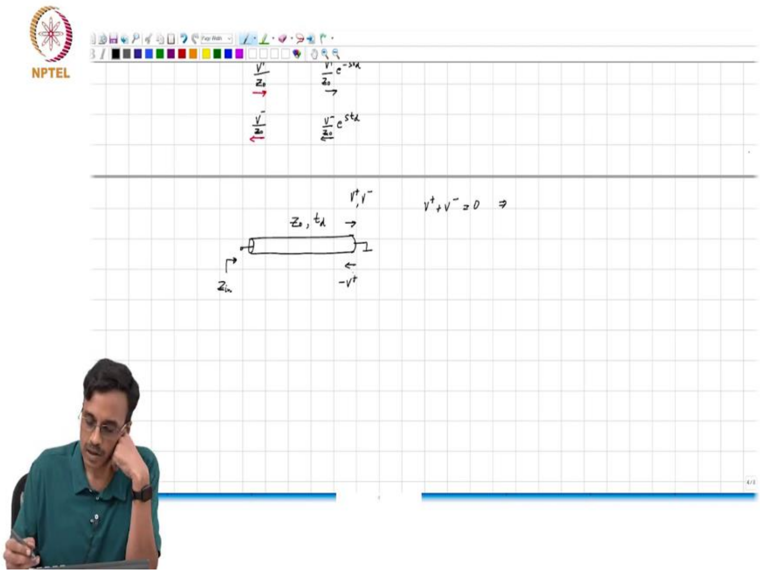

At that end, on the right end it is a short circuit, what is the meaning of short circuit? The voltage is 0. So, if we call this V plus what comment can we make about uh the total voltage is V plus, plus V minus that must be equal to?

What is that? If this the forward and backward going waves here are V plus and V minus right, what is the total voltage at that at that end? It is V plus, plus V minus and that must be equal to? 0, right?

$$V^{+} + V^{-} = 0$$

Which basically means that what is V minus?

Minus V plus, correct? So, alright. So, if this forward going wave is V plus what comment can you make about the forward going voltage wave there?

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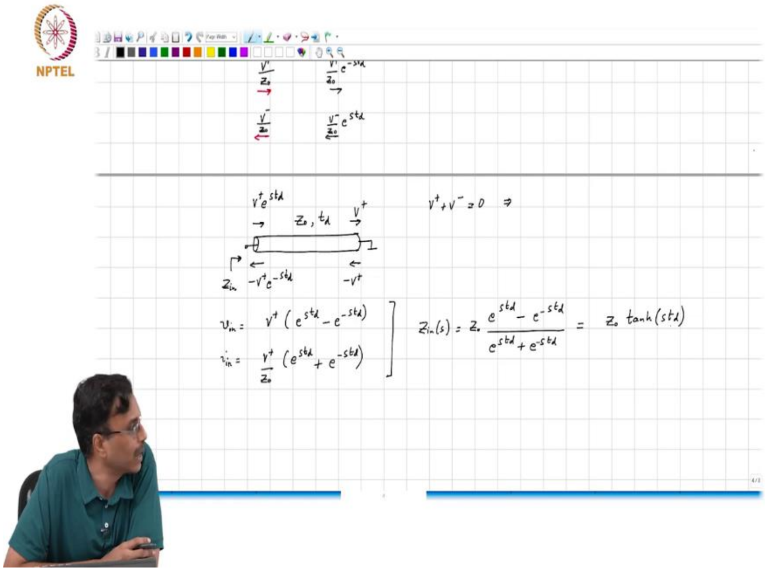

The voltage on the right is V plus it is going this way. So, what must have been what must be here at the left end must be an advanced version. So, that basically is V plus e to the s times t, $(V^+e^{st_d})$. And likewise, the voltage and the backward going wave must be minus V plus e to the minus s times t, $(-V^+e^{-st_d})$, correct. So, what is the total voltage of the left end? What is the total voltage in the left end?

It is V plus times e to the s times t d minus e to the minus s times t d.

$$v_{in} = V^+(e^{st_d} - e^{-st_d})$$

And what is the total current? The forward wave is associated with the current flowing in the I mean going into the line which is V plus e to the s t d by Z naught and the backward wave is associated with the current which is flowing in this direction. So, the total current is simply the forward current minus the backward current. So, that is basically V plus time by Z naught times e to the s times t d plus e to the minus s times, correct.

$$i_{in} = \frac{V^+}{Z_0}\,(e^{st_d} + e^{-st_d})$$

So, therefore, what is the input impedance? It is e to the minus e to the minus s times t d by and this is nothing but Sorry, time Z naught, Z naught tan hyperbolic s times sanity check.

$$Z_{in}(s) = Z_0 \frac{e^{st_d} - e^{-st_d}}{e^{st_d} + e^{-st_d}} = Z_0 \tanh(st_d)$$

Pardon. At t d equal to oh well yeah sure ok fine, but t d is ok that is one thing. If t d is not equal to 0, what must be the DC impedance looking in?

What must be the DC impedance looking in? It is a short circuit right ok. It is a short circuit that we have done it through cable, but we you know hopefully the cable basically does not mess up the DC shock, correct? And, so, as s tends to 0, clearly that tan hyperbolic s times t d goes to 0. So, fortunately we still have a short circuit even though we have an expensive cable. Does it make sense? Alright.

And, so, as frequency becomes uh you know higher as s starts to increase from 0 what do you see? How does it look like? At low frequency at DC of course, it is a short, at low frequency what does it look like? Or.

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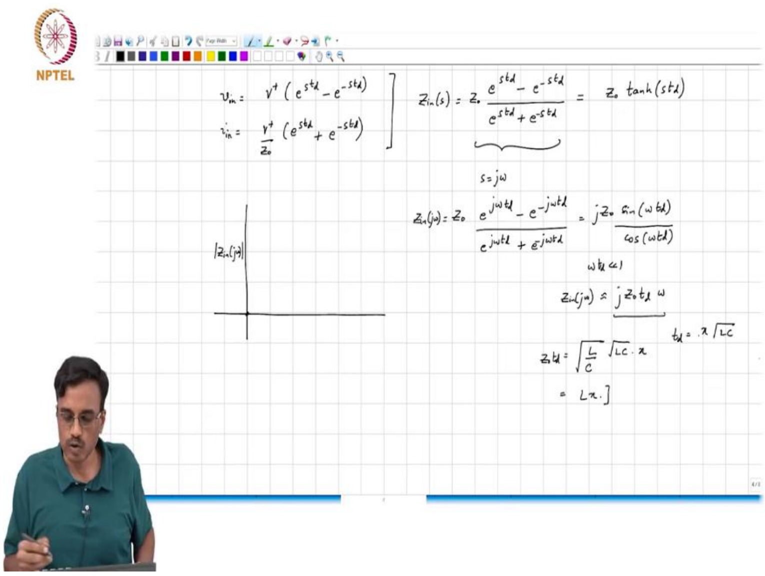

So, well I would say put s equal to j omega the Z naught. So, this is e to the j omega t d minus e to the minus j omega t d by which is Z naught, what is the numerator? j sin omega t d by cos omega t.

$$Z_{in}(j\omega) = Z_0 \frac{e^{j\omega t_d} - e^{-j\omega t_d}}{e^{j\omega t_d} + e^{-j\omega t_d}} = jZ_0 \frac{\sin(\omega t_d)}{\cos(\omega t_d)}$$

If you plot the magnitude of the impedance, you can see that at DC it is 0, right. So, Z in of j omega that is at DC it is 0, as omega increases what do you see? How is it go?

Well, it goes as? Yeah, I mean for small omega what do you see? Well, it is Z naught times much much smaller than 1, Z in of j omega is approximately j Z naught t d times, alright.

$$\omega t_d \ll 1$$

$$Z_{in}(j\omega) \approx j\, Z_0 t_d \omega$$

So, it looks and how does this, what does this remind you of? It is an Inductance and why does this make intuitive sense?

Ah? Yeah, inductance is a short, but I am asking you what is the value of that inductance? What is the value of that inductance?

Z naught times t d, correct? The question is why does this Z naught times t d does it make any sense or does it or did you say well you know what can I say it comes out of the math.

Very good. Z naught. Remember, what is Z naught? It is square root of L by C where L and C are per length unit length values. And what is t d?

t d is nothing, but length by? Length by velocity which is basically velocity is nothing but Square root? Correct.

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$t_d = x\sqrt{LC}$$

So, Z naught times t d therefore, is nothing, but L times C times square root of L C times x which is therefore, L times x the C goes away.

$$Z_0 t_d = \sqrt{\frac{L}{C}}\,\sqrt{LC}\,x$$

$$Z_0 t_d = Lx$$

So, this is nothing, but I mean does it make intuitive sense or no? Right. So, you have this L C, L C; where were we? Yeah.

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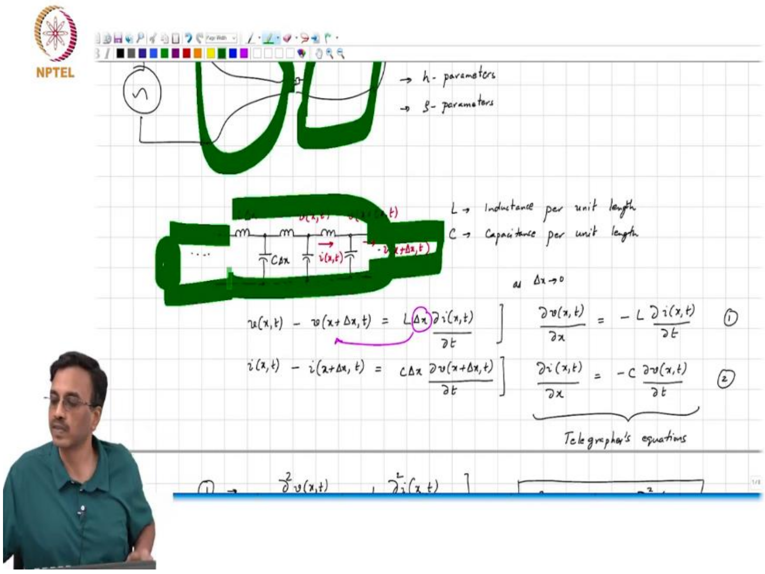

Network, right? Each one of these inductances is this some L delta x, you have a transmission line, right. You are shorting the output and you are looking at the impedance at this at this end, at low frequency. At low frequency what are the current flowing through all those infinite similar capacitors? 0.

So, what you essentially look at is you are looking at the total inductance, right. So, basically, so this is a, it is the total inductance of the lag, alright.

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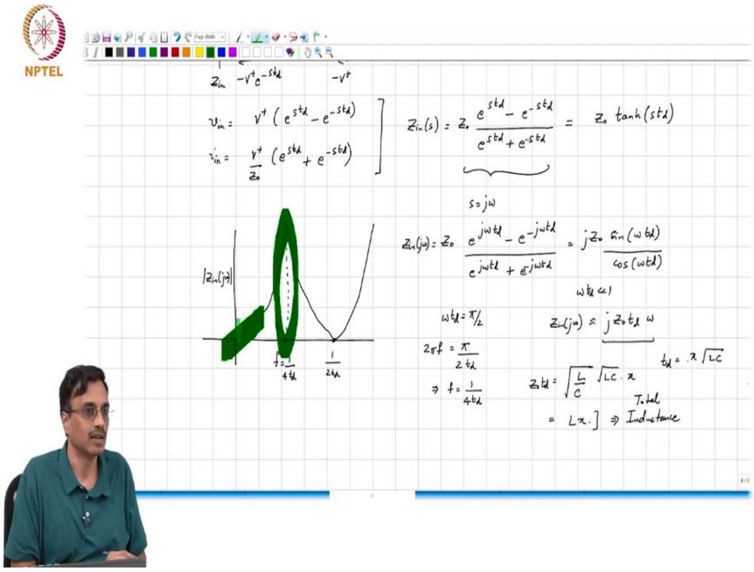

And, so, that is one thing to bear in mind. The next thing is what? If I plot this Z magnitude of the impedance, well at low frequency does this; as the frequency starts getting higher and higher what happens? When omega times t d becomes pi by 2.

$$\omega t_d = \frac{\pi}{2}$$

So, that basically is omega equals pi by 2 d or this is nothing, but 2 pi f equals pi over 2 d, f is nothing, but 1 over 4 times t d, right.

$$\omega = 2\pi f = \frac{\pi}{2t_d}$$

$$\Rightarrow f = \frac{1}{4t_d}$$

What happens? Well, it goes the magnitude of the impedance goes to infinity ok and at 1 over 2 times t d it comes back to 0 and then again goes up, alright. So, what is the moral of the story? Well, we were expecting a short circuit, correct and we naively said oh we will take cable and then short the other ends of the cable and I mean you know at DC of course, it is a shot, right.

But, it does also the wild things right ok. At, if the input frequency is you know some magic value which is you know as you can see an odd multiple of 1 over 4 times t d right, but t d is the length of the cable, then what was supposedly a short circuit can actually look like an look like an open circuit alright, ok and between frequencies of 0 and 1 over 4 t d it can look like anything between a short end or open end ok.

So, like all things in life you know right things are more complicated than they seem, alright uh. So, you know this whole the notion of getting a short circuit, a short circuit is supposed to be zero impedance at all frequencies, right. So, like most things in a textbook it does not exist, right because you know the moment you put a you know two conductors with some finite length right the best you can do is as you keep making the length shorter and shorter what will happen?

Well, the frequency at which this happens becomes? Higher and higher, but you cannot escape from that slope. The best thing you can do is you can get a small inductance, correct, alright.

Now, I am going to quickly take a few minutes and then finish off the open circuit stuff too. So, the bottom line therefore, is that you know the perfect short circuit does not exist, that is what that is your take home message, alright. And, if a perfect short circuit does not exist what comment can you make about the open circuit?

Well, a perfect open circuit does not exist probably does not exist either, right. A perfect I mean in the best case you can make I mean a short circuit to first order looks like an inductance, right. So, an open circuit to first order will look like a?

Will look like? A capacitor, right.

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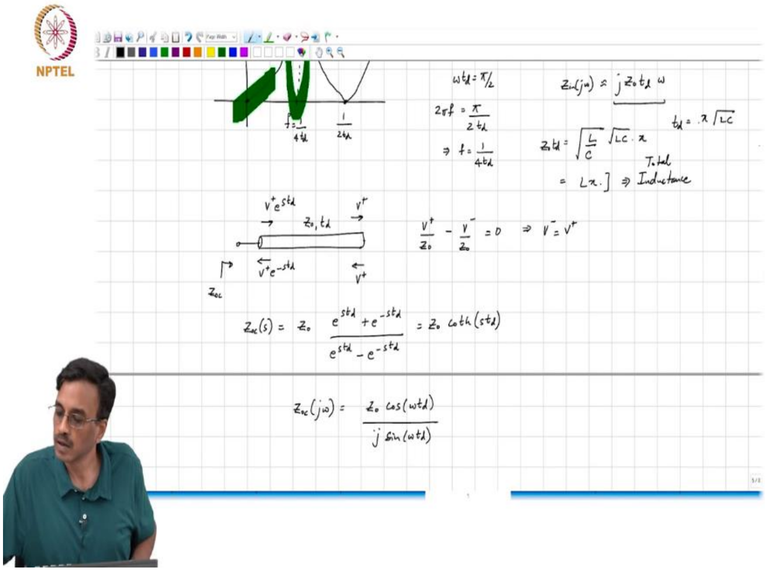

So, let us quickly uh. So, this is an open circuit. So, what is the definition of an open circuit? The current is 0. So, if this is V plus, the current flowing in corresponding to the forward wave is? Is what? V plus by Z naught and the current in the backward wave this is V minus. So, V plus by Z naught minus V minus by Z naught that is the total current flowing out of the transmission line, that must be? 0, which basically means V minus equals? Equals V plus, right. So, this is basically V plus.

$$\frac{V^+}{Z_0} - \frac{V^-}{Z_0} = 0 \ \Rightarrow V^- = V^+$$

Now, what must this forward wave be? This must be V plus e to the? Plus - s t d right, $(V^+e^{st_d})$ and what must this be? V plus e to the minus s t d, $(V^+e^{-st_d})$. So, what is the I am going to skip all the algebra. You should now be able to tell me what the impedance is? What is the impedance? Is Z naught times what is the voltage? e to the s t d plus e to the minus s t d divided by s t d minus minus s t d, which is Z naught cot hyperbolic s times t d right.

$$Z_{oc}(s) = Z_0 \frac{e^{st_d} + e^{-st_d}}{e^{st_d} - e^{-st_d}} = Z_0 \coth(st_d)$$

which is basically therefore, Z naught open circuit of j omega therefore, is Z naught by j Z naught cos omega t d divided by j sin omega, correct.

$$Z_{oc}(j\omega) = \frac{Z_0 \cos(\omega t_d)}{j \sin(\omega t_d)}$$

So, at low frequency I mean sanity check add DC. Well, it is an open circuit and you know fortunately it does indeed turn out to be infinity, correct. So, at the frequencies which are small compared to uh.

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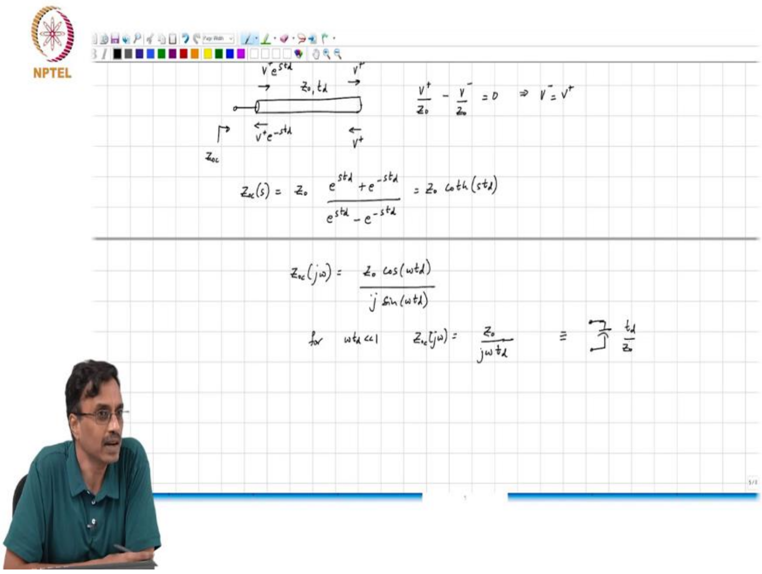

So, for omega times t d much smaller than 1, the impedance is nothing, but Z naught by j times omega t d, correct.

$$for \;\; \omega t_d \ll 1, \;\; Z_{oc}(j\omega) = \frac{Z_0}{j\omega t_d}$$

So, this looks like a, this looks like a capacitance. So, what is the capacitance?

The capacitance. t d over Z naught and a straightforward math will tell you that this is simply nothing, but the total capacitance in the transmission line and that makes intuitive sense because at low frequency the voltage drop across the inductors is, it is an open circuit.

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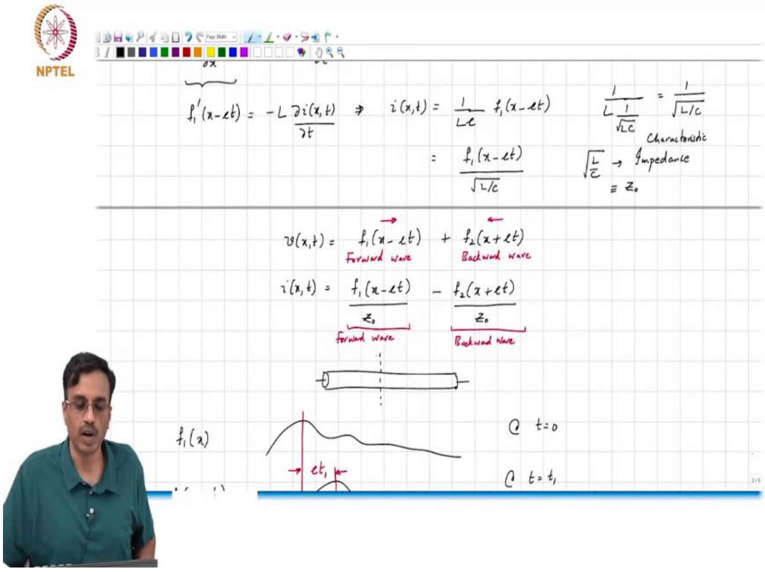

So, the voltage across the drop across the inductors is very small. So, the applied voltage applies is appears across all the capacitors. So, the current that flows in is simply nothing, but uh 1 over j omega C, where C corresponds to the total capacitance of the lag, right.

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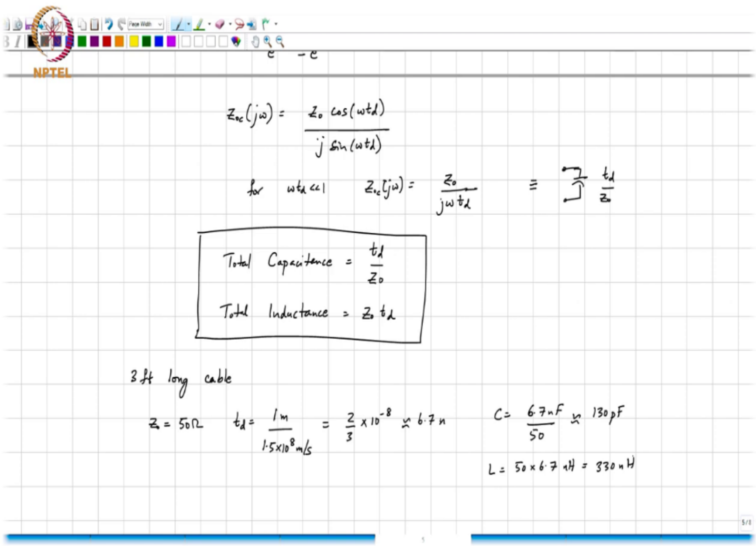

So, therefore, so, the capacitance is t d over Z naught, total inductance is Z naught times d, right. So, these are good relationships to know, alright.

$$Total\ Capacitance = \frac{t_d}{Z_0}$$

$$Total\ Inductance = Z_0 t_d$$

So, ok. So, now, this is the theory uh. So, let us say you walk into a lab, right and you see a cable which is you know 2 feet long or let me make it uh 3 feet long, right.

Now, you want to figure out whether a CMOS inverter will be able to drive this load, right ok. Now, the question is how much capacitance does this cable have? It is a great interview question, right? Here is a lab you know you walk in right your host and you company takes you into a lab, right shows you a cable they say how much capacitance is there in this cable uh.

And particularly because you put on you know four internships in your you know on your CV which right at various impressive companies right doing things that only you can do in two months, alright. How will you figure it out?

Pardon. So, well the cultural part of the question is you know what is Z naught. You have done enough courses to know this. What is the standard impedance used? 50.

$$Z_0 = 50\ \Omega$$

So, well you got that out of the way, right. We need to find t d ok. Well, you know 3 feet is approximately a meter, right. So, that is 1 meter divided by Velocity of light, right and even if you I did not know the velocity of light in the medium right you know that it is going to be faster or slower than 3 in to 10 to the power 8.

Slower than 3 into 10 power 8. And do you have a can you hazard a guess of what the dielectric constant is? Well, what is the dielectric constant of you know I do not know maybe glass or silicon dioxide?

Student: (Refer Time: 22:31).

Well, this looks like glass in the middle I mean obviously, this inner conductor cannot be floating inside the cable, there must be some mechanical medium which is must be holding it in place, right. It turns out that you know 4 is a good number to use right, ok. So, what is the speed of light in the medium?

The nice round number is 1.5 into 10 power 8 meters per second.

$$t_d = \frac{1m}{1.5 \times 10^8 \; m/s}$$

So, this is how much? It is two-thirds in to 10 power minus 8 which is about? 6.7 nanofarad, nanofarad, nano seconds, right (Refer Time: 23:22) yeah, alright.

$$t_d = \frac{2}{3} \times 10^8 \approx 6.7nF$$

So, the capacitance therefore, is 6.7 nanofarad divided by 50 which is 130.

$$C = \frac{6.7nF}{50} \approx 130pF$$

Does it make sense? Right ok which is why a 1 farad capacitance is a huge capacitance right. So, this is a like 1 and half foot long, 3 foot long cable right and it is got only capacitance only 100 picofarad or so, right to get therefore, you know what do you call microfarad or you know nanofarad I mean or farads of capacitance you better pack things now very very very.

Now, what is the inductance? What are the estimate of the inductance? 50 times 6.7 nanohenrys which is 330 nanohenry, right.

$$L = 50 \times 6.7nH \approx 330nH$$

So, we short the ends of the cable and you know look at the other two terminals and then you know at low frequencies it look like an inductance of 330 nanohenrys. If you open circuit the cable it will look like capacitance of value, 130. Does it make sense people? Ok, right.