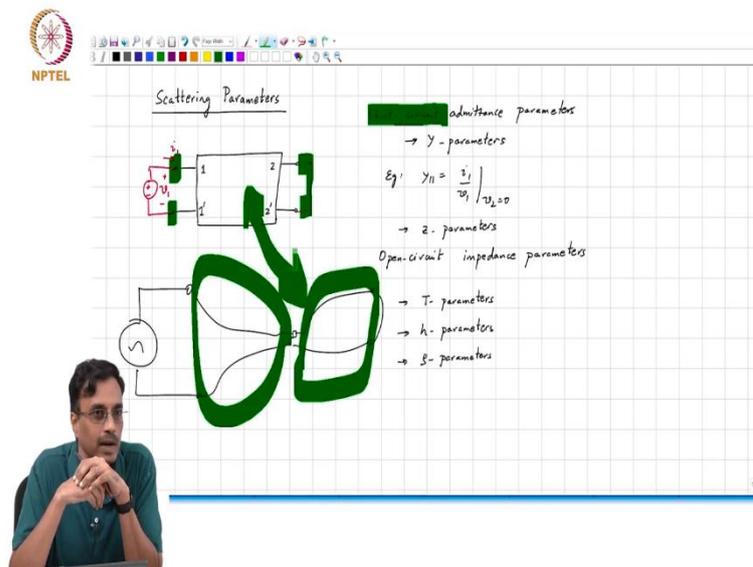


Circuit Analysis for Analog Designers
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Lecture - 42
Introduction to disturbed networks, the ideal transmission line

Today we will start talking about a new set of two port parameters that I am not sure you have seen before and these are called the scattering parameters.

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All of you have no doubt you know exposed to various parameter sets used to characterize networks, namely the short circuit admittance parameters. And what are they called; what parameter set? It is the y parameter, right.

And why are they called short circuit admittance parameters? The admittance, because the units of the are admittance and short circuit, because the way you measure for example, y_{11} is nothing but. Yes, what is y_{11} ? i_1 by v_1 when v_2 is 0, right.

$$y_{11} = \frac{i_1}{v_1} \Big|_{v_2=0}$$

So, basically your short circuit the second port and you apply a voltage v_1 here and this is i_1 . So, the short circuit comes about, because other port is short circuited and are terminated in the short circuit and i_1 by v_1 is the dimensions of admittance, fine. And

likewise, you see in the z parameter set and what are these? These are open circuit impedance parameters and you see in other parameter sets; what others are you aware of, ok? Great I mean. So, the T parameters, the T parameters they are called the chain parameters sometimes and then you also see in the h parameters.

How many of you have heard of the h parameters set, ok? You have seen this and then you have seen g parameters right, ok. And you can have come up with a whole bunch of I know interesting, but mostly useless problems are trying to convert from one set to another right, ok. Given h, you find g; if I you know given h, you find y I mean all these things you have done earlier blue in the face right, ok.

Now, the question now is you know they are already enough, they seem to be at least, you know every alphabet seems to have one parameter set. There is you know there is a, b, c, d parameters; we have missed e, f and there is g, there is h, there is y, z, ok. So, now the question, the obvious question is you know why do we need one more set of parameters that describe the same old two port all over again; I mean clearly you know that you know if you know one set, you know all the you know the rest.

So, the question is you know why on earth do we need a new parameter set. And so, what is the you know I mean is it simply, is it simply masochism or is it you know is a, is there a practical use case that necessitates a new parameter set, right. It turns out that there is a it is actually grounded in the scattering parameters are grounded in practice and they satisfy basically a requirement.

And to see what that requirement is, I will have to kind of go back a little bit and talk about transmission lines, right. And the reason I need to talk about transmission lines is the following. So, you know when we basically say we are going to, when let us say we are trying to measure y parameters in a lab, right. It is one thing to draw you know a two-port box like this and draw a perfect short circuit at the at port 2, right.

And quite another thing to basically to do the same thing in the lab; because many times you know you not only do the theory, you also want to make measurements and prove that you know whatever your device or your amplifier or your system, you know has the two port parameters that you think it should have, correct. So, if you wanted to measure the y parameters; what you would have to do in principle is to basically short circuit port 2. And remember that you know and apply a voltage at port 1, ok.

And as far as this diagram is concerned, the voltage has to be applied right at the; right at the terminals of port 1, alright. It is not I will apply you know v_1 a mile away and then you know hope that the same voltage will appear at the terminals of port 1. Likewise, the short circuit at port 2 means that the two terminals 2 and 2 prime are you know that short basically is you know right at the port, right.

It is not you know for instance you know it is not 2 centimeters away, it is not a foot away, it is not a mile away, alright. And that obviously, brings up you know a whole new set of practical problems, right. So, let us say you are trying to measure the y parameters of a transistor correct ok; the transistor is this microscopic small thing which is like sitting somewhere, ok.

And you know your voltage source is this big dabba right and your whatever box that you have to measure current or whatever is another big dabba, right. And so, basically now the question is ok, I mean how do I get the two terminals of this box measurement box or the source box to this microscopic animal which I can see under you know only under magnification, correct ok.

And so, then the answer is you know why are we worried, you know have you not heard of a cable, right. So, you take you know you basically say here is my big box, the voltage source which is some box like this, right. And here is my transistor with you know three terminals right, let us assume magically it is biased.

And then you say well you know I will connect you know you know a cable like this; the two conductors of the cable perhaps are like this and then you know I will have to short circuit these two. So, I will connect another big cable like this right; that looks like a short circuit, because well these two cables are short, right. But as frequencies become higher and higher, the meaning of what a short circuit is and what an open circuit is become more and more fuzzy as we will discuss, right.

And you know while we are at this point, we are now we are no experts as to you know to figure out what these two long cables will do to the measured y parameters. One thing we can be absolutely sure is that, this is definitely not the same as, these two situations are definitely not the same; they would be the same if the length of the cables tended to tended to 0, but if they do not, then you know the situation is not the same, right.

And where we have to figure out you know what these cables are actually doing to our measured y parameters, right. And as it will turn out it turns out, I mean it so happens that these cables basically can potentially send the whole system into oscillation right; if the transistors got sufficiently high gain and so on right, when you drive it to the voltage source and when you terminate it with a short circuit as shown here through those long cables, right. So, our first order of business is to understand you know what the cable does, right.

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And I am sure you seen most of this in e m; but I am just going to quickly go over this stuff right, you know cable or I mean a coaxial cable or a parallel plate the you know what do you call system with two conductors separated by some kind of dielectric, it turns out can be modeled by a series of L C networks right.

The inductance of each one of them is an infinitesimal inductance, which is basically L times Δx , ($L\Delta x$) right; L by the way is the inductance per unit length. And likewise, every capacitor, I mean C is the capacitance per unit length, right. And well this is at this point you know an infinitely long cable; we are trying to figure out what happens. And remember that the as we go along like this, you know this is x and therefore, the voltage at any point on this network is basically a function of both space and time, right.

If this is x comma t , ($v(x, t)$), this is going to be v of x plus Δx comma t , ($v(x + \Delta x, t)$), alright. And if this is current is also a function of space and time. So, this is i of x comma t , ($i(x, t)$), this is i of x plus Δx comma, ($i(x + \Delta x, t)$), alright. Now, what

comment can we make about and what are we supposed to do? We are trying to figure out what equations govern the voltage and current as a function of space and time.

So, well we only know one, I mean it turns out by the way I am sure all of you done an EM class and the equivalent circuit for this comes out from Maxwell's equations, right. So, to solve this what do we do? We simply write Kirchhoff's laws.

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The slide shows a circuit diagram with an inductor and a capacitor in series over a length Δx . The voltage across the inductor is $v(x,t) - v(x+\Delta x,t)$ and the current through the capacitor is $i(x,t) - i(x+\Delta x,t)$. The equations are:

$$v(x,t) - v(x+\Delta x,t) = L\Delta x \frac{\partial i(x,t)}{\partial t}$$

$$i(x,t) - i(x+\Delta x,t) = C\Delta x \frac{\partial v(x+\Delta x,t)}{\partial t}$$

In the limit $\Delta x \rightarrow 0$, these become:

$$\frac{\partial v(x,t)}{\partial x} = -L \frac{\partial i(x,t)}{\partial t}$$

$$\frac{\partial i(x,t)}{\partial x} = -C \frac{\partial v(x,t)}{\partial t}$$

These are the telegrapher's equations.

And recognize that v of x comma t minus v of x plus Δx comma t is simply the drop across that infinitesimal inductor is nothing but, is nothing but L partial derivative of i with respect to, with respect to time, pardon.

Student: $L \Delta x$ (Refer Time: 13:41). Oh, sorry L , that is correct, thank you; $L \Delta x$, right.

$$v(x,t) - v(x + \Delta x, t) = L\Delta x \frac{\partial i(x,t)}{\partial t}$$

And similarly, i of x comma t minus i of x plus Δx comma t must be equal to $C \Delta x$ times partial derivative of, by, that makes sense people?

$$i(x,t) - i(x + \Delta x, t) = C\Delta x \frac{\partial v(x + \Delta x, t)}{\partial t}$$

Are you just saying that you know the current on the right is current in the left minus what current is lost in the capacitor, right. Now, as Δx tends to 0, what do you see? The left-hand side basically you can write this as; well, this goes this way goes into the denominator there. And what do you see?

The partial derivative of voltage with respect to time, with respect to space sorry is the negative of L times, correct.

as $\Delta x \rightarrow 0$

$$\frac{\partial v(x, t)}{\partial x} = -L \frac{\partial i(x, t)}{\partial t}$$

And likewise, the partial derivative of current with respect to space is the negative of the voltage with respect to, that make sense people, alright.

$$\frac{\partial i(x, t)}{\partial x} = -C \frac{\partial v(x, t)}{\partial t}$$

So, these are two first order linear coupled partial differential equations. And you know whenever you have two equations and two variables you know what do you do to find the solution? Well, you eliminate one of them, right. So, what do you do here? And by the way what are these called, you have seen this before I suspect, transport equation you know.

Yeah yes. Telegraph what? Telegraph. Telegraphers equations actually because there are two of them and as you can imagine.

I mean all these kinds of analysis all became important in the early days of telegraphy, where you were you know you sending Morse code and you know you want to analyze how far it will go and what happens and so on, right. And so, you have two ways, I mean you need to find v and i and therefore, you two couple first order differential equations, partial differential equations; you would to find one or the other, you basically eliminate the other variable.