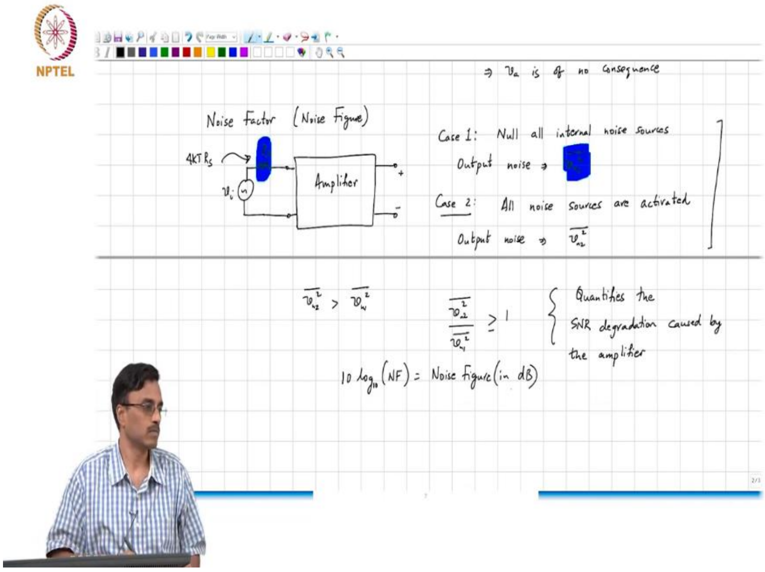
Circuit Analysis for Analog Designers
Prof. Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras

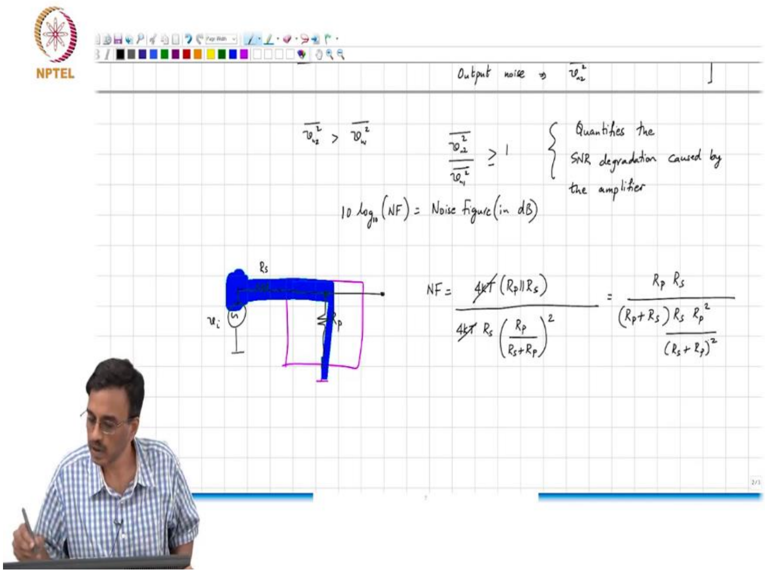
Lecture - 41
Noise Factor Examples

(Refer Slide Time: 00:16)



So, alright so, let us do you know some quick examples to help you figure out what the noise factor is.

(Refer Slide Time: 00:33)



So, let us say these are source, this is R s and this is within quotes are black box and I am going to say well, my black box simply is got a resistor R p alright. Now, what comment can we make about the noise factor, what do we do? We need to find the output noise correct and we need to find the output noise assuming the box itself is noiseless. So, what is the output noise spectral density? What is the output voltage noise spectral density?

It is 4kT times, Pardon. Why R p by R p plus R s is a whole square? It is simply. What is the output noise voltage spectral density? 4kT R p parallel R s. Any confusion about this ok?

$$NF = \frac{4kT\ (R_P||R_S)}{}$$

Now, you have to divide this by the output noise spectral density that would have resulted provided which was noiseless? R p was noiseless, now what would be the output noise spectral density? It is 4kT times It is 4kT times R s that is the noise spectral density of this resistor right and how what is the transfer function from this noise source to the output?

R p by; R p by R s plus R p so, what should I do?

$$NF = \frac{4kT\ (R_P||R_S)}{4kT\ R_S \dfrac{R_P}{R_S + R_P}}$$

This the noise spectral density is going through within quotes again so, we have to multiply by again square correct

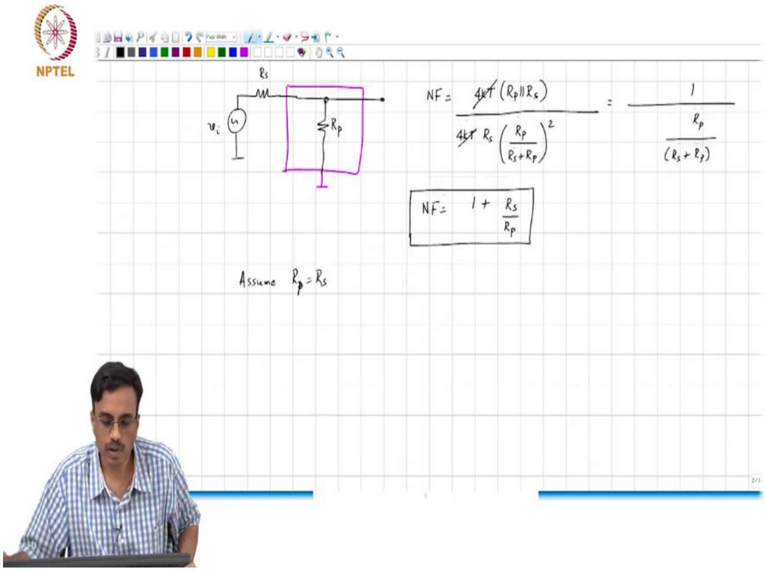
$$NF = \frac{4kT\ (R_P||R_S)}{4kT\ R_S \left(\dfrac{R_P}{R_S + R_P}\right)^2}$$

And therefore, this is nothing but 4kT goes away so, this is R p R s divided by R p plus R s times R s times R p square by R s plus R p d whole square alright

$$NF = \frac{4kT\ (R_P||R_S)}{4kT\ R_S \left(\dfrac{R_P}{R_S + R_P}\right)^2} = \frac{R_P R_S}{(R_P + R_S)\dfrac{R_S R_P^2}{(R_S + R_P)^2}}$$

and therefore, this goes away, this goes away, this goes away, this goes away, this goes away this goes away.

(Refer Slide Time: 03:42)



So, this is nothing but, the noise factor is nothing but 1 plus R s over R right,

$$NF = 1 + \frac{R_S}{R_P}$$

Sanity check well, R p is infinity, what should we expect? Well, you are not doing anything so, presumably you are not losing SNR so, I mean sure enough you get noise factor of 1 or 0 right and I mean well, within I mean we said this is our amplifier so to speak, but you know what you are actually doing is attenuating correct ok. So, if you simply attenuate with the resistive attenuator as you can see this is telling you that you are always going to be. I mean you are going to be degrading the SNR right ok and you know why does that make sense?

Student: (Refer Time: 05:00) added additional (Refer Time: 05:08).

Ok well, yeah that is I mean I guess that is right. So, what comment can you make about the absolute noise spectral density at the output has that let us assume; let us assume for argument's sake that R s is equal to R p is made equal to R s right. So, what comment can we make about the noise spectral density at the output when compared to R p is equal to infinity?

Student: (Refer Time: 05:40) R p (Refer Time: 05:43).

It is what comment can we make about the output noise spectral density if R p is equal to R s?

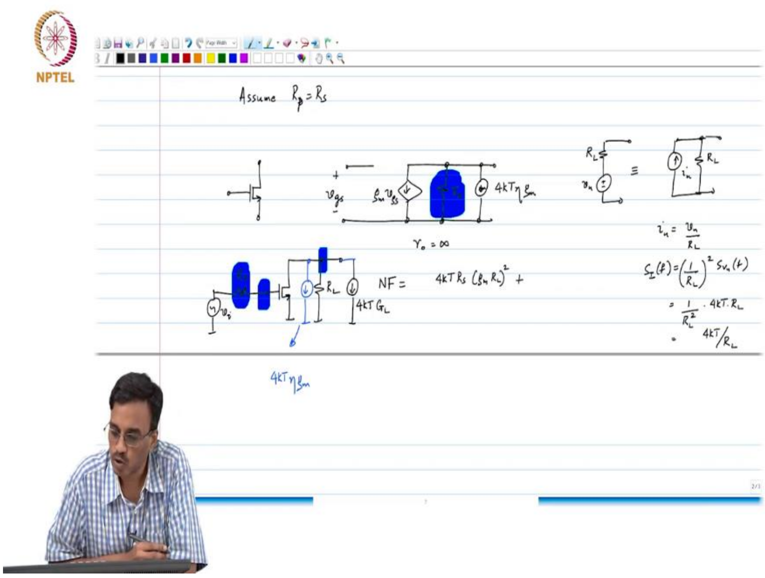
It is you know 4kT times R you know R s by 2. So, the noise spectral density at the output is actually gone down, that does not mean that the signal is becoming any I mean the signal to noise ratio is improving why?

Oh well, the signal is also attenuated by the same factor which basically means the signal power has gone down by 1 by 4.

Right ok noise has gone down by half, but the signal power has gone down by a factor of 1 by a factor of 4 so, the SNR has gone down by a factor of. The noise power has gone down by factor of 2, signal power has gone down by a factor of 4, what comment can we make about the SNR?

The signal they say as the SNR has actually gone down by a factor of 2 even though the noise flow has actually reduced by a factor of 2 right and that is also, that is what the equation is telling us if R s is equal R p then the noise factor is 2 which means that the output SNR is lower than the input SNR by a factor of 2. The noise factor does not say anything about the absolute noise flow, it is only quantifying how badly you done with respect to 2 SNR right. Let us do another example.

(Refer Slide Time: 07:24)



Now, let us take a transistor and at low frequency, what comment can we make about what is the small signal equivalent of the transistor? It is nothing but this is the incremental v gs so, this is g m times v gs, this is r o and as I mentioned the transistors accompanied by a noise source whose spectral density is for a long channel MOS transistor can be shown to be 8kT over 3 times g m, but then you know long channel MOS transistors you know only exist in the textbooks.

So, what a better formula is to use 4kT eta times g m right.

$$4kT\eta g_m$$

Well, that eta is some number that fits measurements right eta or gamma or whatever they are they have some that is the usual non-ideality factor, ideally eta should be two-thirds, in reality it is more like three-half's ok so, whatever right I mean this the designer has no control over what you get right, it is what you get.

And so, another thing I would like to point out is that even though you have a r o which a models the finite or non-zero lambda of the device, it is not a physical resistance right and therefore, there is no noise as that current source is not the noise associated with it is not noise associated with that r o ok and so, this is a simple model for noise in fact, it turns out that in reality, there is also another noise source in the gate, but that only becomes shows up at really high frequency, but for most calculations at least at low frequencies, this is good enough.

Now, let us do a simple common source amplifier whose incremental picture is shown here, and this is v i, this is R s and that is R L. So, what comment can we make, how do we what is the noise factor, what do we do? First thing is to find the total output noise. So, what will be the total output noise, noise spectral density? Well, what is the gain from R s to the voltage source and series with R s to the output?

Yeah, well, let us assume again you know let us make life simple and assume that r o is equal the infinity ok. So, what is the gain from the noise source to the output?

Minus g m R L right. So, what will be the output noise spectral density due to R s? It is 4kT R s times g m R L minus g m R L the whole square, which is the same as plus g m R L the whole square right plus what else? Is that all?

$$NF = 4kTR_S(g_m R_L)^2 +$$

How many noise sources are there?

Student: (Refer Time: 11:02) R L (Refer Time: 11:05).

I mean the main thing here is the transistor man, how many noise sources do we have? 3 noise sources. Now, you know I mean with you know even I mean of course, the three you know noise sources we can say we can all do this you know in our minds, but you can imagine what will happen if you have you know 50 transistors like this and you know there is tons of noise sources and there is one output and you need to now calculate you know.

And I made r life simple by assuming r o is infinity and there is no parasitic capacitances etcetera, in reality all that stuff is going to go come in and then, it is going to be a mess so, that is where the adjoint is the inter reciprocal network is so useful right, because you write down the MNA equations of this big network, you solve it once ok and you get all the transfer functions that you are looking.

So, back to our example so, there are three noise sources fortunately, it seems like we can use some simple trick to convert that into two so, this noise source is that of the transistor and it has a 4kT eta g m, the noise source resistor is a voltage source, but we know how to convert from not an equivalent to a Thevenin equivalent so, v n in series with R L is equivalent to i n in parallel with R L.

And what is i n? v n by R L

$$i_n = \frac{v_n}{R_L}$$

So, the noise spectral density of the current source is nothing but well, you are multiplying that current by 1 by R L times v n so, you multiply this by a constant so, you have to I mean you multiply that by 1 by R L square times S v n of f which is nothing but 1 over R L square times 4 kT R L so, this is nothing but 4 kT over R L that makes sense guys.

$$S_z(f) = \left(\frac{1}{R_L}\right)^2 S_{V_n}(f)$$

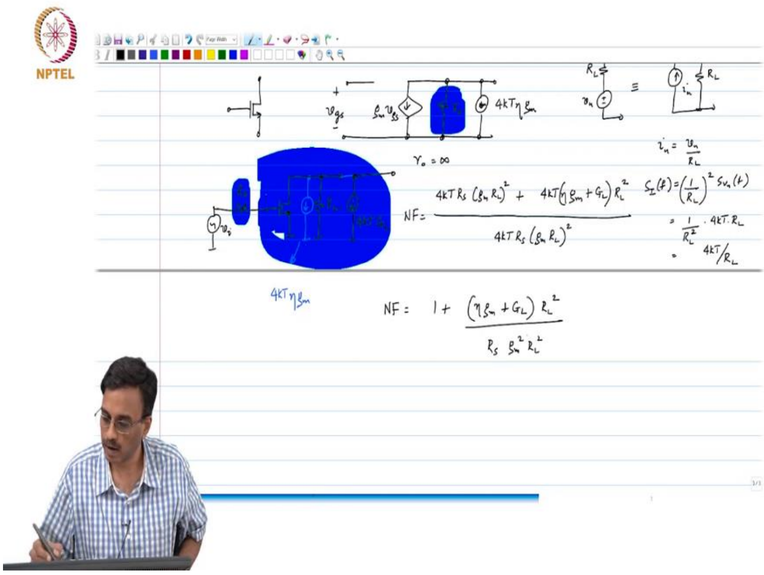
$$= \frac{1}{R_L^2}\ 4kT\ R_L$$

$$= \frac{4kT}{R_L}$$

So, we have two noise sources here, one which corresponds to the transistor, one that corresponds to we are going to call that 4kT G L, ($4kTG_L$) ok. So, what is the total noise the at the output therefore? This is the input source plus what is the total noise voltage at the output now?

Yes Danish.

(Refer Slide Time: 14:04)



Student: 4 kT eta g m plus G L that is the current flows into R L so, what should I; what should I do here? Very good R L square, this is the total noise at the output of the amplifier right

$$\underline{4kTR_S(g_mR_L)^2 + 4kT(\eta g_m + G_L)R_L^2}$$

And you have divided this by the noise that you would have got if the amplifier itself was noiseless right. So, in other words everything inside this blue box if it was noiseless, what is the noise you would have got? What would you have got?

That is easy to do; it is simply 4kT R s times g m R L whole square.

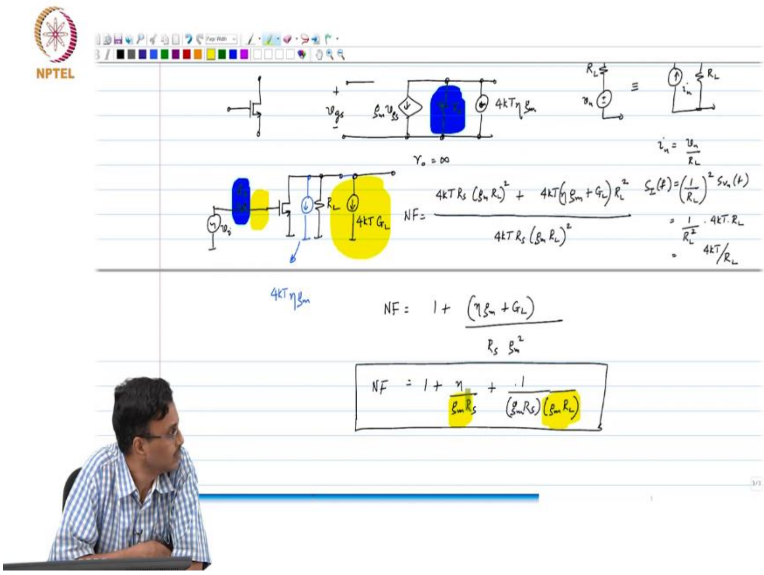
$$NF = \frac{4kTR_S(g_mR_L)^2 + 4kT(\eta g_m + G_L)R_L^2}{4kTR_S(g_mR_L)^2}$$

So, this is the noise factor and therefore, the noise factor is 1 plus eta g m plus G L into R L square divided by R s times g m square R L square ok

$$NF = 1 + \frac{(\eta g_m + G_L)R_L^2}{R_S g_m^2 R_L^2}$$

And so, this goes away, this goes away.

(Refer Slide Time: 15:47)



And you have 1 plus eta over g m R s alight plus 1 over g m R s times g m R L alright. Now, why does this make intuitive sense or is there a sanity check?

$$NF = 1 + \frac{\eta}{g_mR_S} + \frac{1}{(g_mR_S)(g_mR_L)}$$

Well, if R L you know was noiseless, the noise factor would be 1 plus eta over g m R s ok. Now, the what comment can we make about the noise spectral density added by the resistor in relation to that added by the transistor? Actually, I missed the no, I think we are ok yeah. So, what comment can we add?

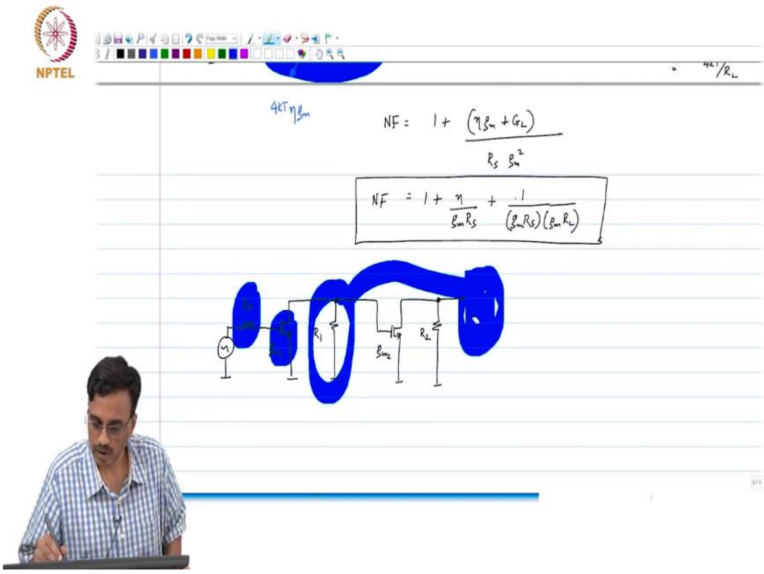
Student: (Refer Time: 17:05).

Yeah, so, basically you know when you refer, I mean adding a noise current here ok is equivalent I mean what disturbance at this node would cause a disturbance of 4kT I mean in other words, what disturbance must I add here at the gate to cause a current I in the drain?

Whatever disturbance there is in the drain current that has to be divided down by g m right in order to right and so, if you are looking at the output voltage, you know this current will cause a you know will have a gain of only R L whereas, a voltage here will have an effect of g m times R L ok.

So, you can see that the noise added by the resistor is I mean R L is lot less important than the noise added by the transistor. Because if you choose a large gain g m R L is a large number basically saying that the noise added by the resistor, noise current added by the resistor load resistor is much smaller than the noise current added by the transistor itself.

(Refer Slide Time: 18:47)



The next thing I am going to do another example, but I am not going to go through the algebra, I leave you to do the algebra, but I am going to what I would like to discuss is intuitively what we should expect so, the two-stage amplifier, this is g m1, R 1, g m2, R 2 alright and so, which I mean based on your experience so far, which, what do you think who I mean which of the noise sources do you think is the most contributes the most to the noise figure? g m? g m1 right why? Initially why does it make sense?

Student: (Refer Time: 19:51).

Well, yeah that is right I mean any the noise source, any noise added by the transistor, the noise current is now know converted into voltage that is further amplified so, a lot of the noise here will be due to g m1 and R s.

And as you keep going further down the chain let us say you have a long cascade right, the noise figure is going to be if you have; if you have a cascade of amplifiers each one which is amplifying the signal right and of course, also amplifying noise from preceding stages, which of the stages will be the most; which of the stages would you expect is the most critical as far as the noise is concerned?

The first stage right ok. And which do you think you know as you keep travelling down the chain, the signal is getting bigger and bigger so, I you know which would you worry about uh you know distorting the signal a lot?

Well yeah, the last stages are the ones which you should be primarily worried about as far as distortion is concerned and the first stages are the ones that you would be worried about primarily about noise right and that is how it goes.

So, that is all I had to say regarding noise factor and noise figure and you should be in a I mean and the you know again I would like to remind you that so far, we have just discussed you know circuits where there is no memory so that the algebra becomes easier on the blackboard right.

In reality, all these transfer functions will have will there be frequency dependence if all these gains and stuff like that so, basically wherever you see a gain, you replace it with a transfer function and therefore, in general, noise factor is a function of; it is a function of frequency because gain is a function of frequency alright. So, that basically is all I had to say about the noise factor.