

**Circuit Analysis for Analog Designers**  
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**Lecture - 40**  
**Input referred noise and the noise factor**

(Refer Slide Time: 00:18)

\*  $v_a, i_a$  are properties of the network  
\*  $v_a, i_a$  are correlated sources

So, in the last class we basically concluded that, if you have an amplifier or actually any linear system with internal noise sources you can represent the effect of all the noise sources inside by thinking of this as a noiseless and accounting for the effect of all the internal noise sources by using, I think we call them  $v_a$  and  $i_a$  right. The different notations to use it does not matter what. The key points to note are that  $v_a$  and  $i_a$  are properties of the network and the noise sources inside right. And therefore, you know.

If you want to kind of hand this black box over to somebody else who might be using the box you not only you know give them the 2 port parameters and so that they will be able to figure out what the input impedance is and what they must drive this network with you also give them  $v_a$  and  $i_a$ .

And in principle you know that should be good enough to be able to figure out what the output noise would be regardless of what source impedance you drive the; you drive the network that will make sense. And in general  $v_a$  and  $i_a$  are they independent both of them are you know noise sources in general are  $v_a$  and  $i_a$  independent they are not because  $v$

$v_a$  and  $i_a$  are both derived from the same independent noise sources inside right so  $v_a$  is some linear combination of  $v_1 v_2 v_3$  all the way through  $v_n$  and likewise  $i_a$  is some other linear combination of  $v_1$  through  $v_n$ . So, clearly you know they are related to each other so they are not independent so in general  $v_a$  and  $i_a$  are correlated sources ok.

(Refer Slide Time: 03:10)

And so, a couple of observations let us say for example, that we are driving the network with some  $R$  s and then here is our  $v_a$  and here is our  $i_a$  and this is the internals of the networks. So, there is some  $R$  in here that represents the input impedance of the network looking in alright. So, what comment can you make about the, what comment did we make about the Thevenin voltage looking in here? It is simply  $V_i + v_a + i_a R_s$  correct.

$$v_i + v_a + i_a R_s$$

So, if the  $r$  s if we were driving it with if we were driving it with an ideal voltage source then what comment can you make? The source is an ideal voltage source what comment can you make a, which of this input referred noise sources is inconsequential. Well,  $i_a$  is of no consequence alright. On the other hand, if the source is a; is an ideal current source what is of no consequence? I mean  $R_s$  therefore, tends to infinity right. And then what is of no consequence?

The voltage sources of no consequence right; so, in reality  $R_s$  is not neither in practice  $R_s$  will neither be 0 or infinity ok and. So, both at least in principle  $v_a$  and  $i_a$  will both be will be important right. And if you I mean this also basically points to the fact that there is a switch spot as far as choosing, I mean.

Remember that this Thevenin voltage here is this right and you know once you have that Thevenin voltage that  $v_{th}$  times you know  $R_n$  by  $R_n$  plus  $R_s$  is going to get amplified by the or processed by the rest of the network correct. So, if you basically want to minimize within quotes the noise right.

The noise evidently depends on I mean this is the signal and this part corresponds to the noise ok. So, both the signal and  $v_a$  plus  $i_a$  times  $R_s$  are going to be processed by the same transfer function going forward right. So, maximizing the signal to noise ratio therefore, means that you need to you know you would like to make sure that the signal power to you know the noise power is as large as possible and.

So, and there is you know evidently a switch spot let us say at a certain frequency you know the strength of  $v_a$  is you know is very large compared to  $i_a$  times  $R_s$  ok. There are two quantities which cause noise one is  $v_a$  the other one is  $i_a$  times  $R_s$  ok. So, if  $i_a$  times  $R_s$  is very small let us say somehow it turns out that the input referred noise current source is very small in magnitude right then what comment can you make about  $R_s$ .

You know you can I mean as I said this basically means that, that  $R_s$  can be made you know orbitally large ok, alright. But on the other hand, I mean, but if  $i_a$  is not 0 but is finite, but small in the beginning for some value of  $R_s$  let us say  $i_a$  times  $R_s$  is very small compared to  $v_a$  and you say well, great let me go and crank up  $R_s$  right. As you go on increasing  $R_s$  what will happen eventually? I mean that basically if I increase  $R_s$  basically means I can use a poorer and poorer source that is what it means right. So, if I go on cranking up or yanking up  $R_s$  what happens eventually, yes?

What happens? Well, that  $i_a$  times  $R_s$  starts to you know starts to become very large right. So, I mean there is evidently you know a switch spot for you know. So, given the strengths of  $v_a$  and  $i_a$  and in fact you know that they are also correlated given the properties of  $v_a$  and  $i_a$  you will find that there is a switch spot for the source in this in our example source resistance in general source.

I mean all these are basically you know we just assume that there is no memory and then you know did everything finally; everything is going to be a function of frequency. So, all that it means is that this is that  $z_s$  of  $j\omega$  at a certain frequency there is some you know there is some switch spot for the source impedance which minimizes the total amount of noise right.

So, if you choose for example, you know two small source impedance right  $v_a$  will dominate correct if  $R_s$  is 0 its only  $v_a$  that is of consequential. If you use two large  $z_s$  is the  $i_a$  will dominate right, but perhaps it is possible to choose something somewhere in the middle where this  $v_a$  plus  $i_a$  times  $z_s$  is small right. It depends on the relative values of  $v_a$  and  $i_a$  as well as the correlation between them ok. So, and those of you who have done you know an RF class basically you know have seen this in that context, but I think doing that now basically is not particularly useful.

(Refer Slide Time: 10:51)

The slide contains the following content:

- Diagram:** A block diagram of an amplifier. The input is connected to a source with impedance  $z_s$  and noise voltage  $v_s$ . The amplifier has an input noise current  $i_s$ . The output is a noise voltage  $v_o$ .
- Handwritten Notes:**
  - Top right:  $\Rightarrow v_a$  is of no consequence
  - Left: Noise Factor (Noise Figure)
  - Case 1: Null all internal noise sources. Output noise  $\Rightarrow v_o$
  - Case 2: All noise sources are activated. Output noise  $\Rightarrow v_o'$
  - Equation:  $\overline{v_o'^2} > \overline{v_o^2}$
  - Equation:  $\frac{\overline{v_o'^2}}{\overline{v_o^2}} \geq 1$
  - Text: Quantifies the SNR degradation caused by the amplifier
  - Equation:  $10 \log_{10}(NF) = \text{Noise Figure}$

The next thing I would like to talk about is this concept of again this is largely jargon right it is a noise factor or also called noise figure and the idea is the following. So, we have our let us say we have our source and we have our amplifier or filter or whatever you have there is a whole bunch of internal noise sources.

And you look at the output and there let me ok. So, let us again assume that the input perhaps is coming from some kind of antenna, the antenna has got some source resistance

and this source resistance adds noise right. So, this noise is basically, noise spectral density is  $4 kT R_s$ .

So, in the first case what I am going to do is simply look at the noise spectrum let us call the or assuming there is no memory  $v_{n1}^2$  square right that is the mean square noise of the output of the amplifier ok. Next thing I am going to do is you know all noise sources are active and let us say the output and I again measure the output noise alright, which will be which will be larger it is a no brainer right.

So, basically  $v_{n2}^2$  square on a mean square value of that will be much greater than  $v_{n1}^2$  square correct. So, this ratio is a measure of I mean what physical meaning can we attach to this quantity the mean square value again as I said this is a we were assuming that there is no memory inside the amplifier. So, at this point it is just simply the mean square noise integrated over some bandwidth.

So, but what physical significance can we attach to this quantity? What does the number quantify? Well, this basically is telling you how badly the, I mean  $v_{n1}^2$  square is simply the input noise that is processed by the amplifier correct. And that input I mean the input signal is also processed by the same transfer function because you know the noise source corresponding to  $R_s$  and the input voltage  $v_i$  have the same transfer function correct ok.

So, this noise so this noise basically in case one corresponds to the noise that would have been there at the output even if the amplifier was perfectly noiseless right; the second case is you know what you actually see which is simply you know consists of two parts one part is the input noise processed by the amplifier on top of it there is extra noise that the amplifier adds you know on top you know of its own alright.

And clearly the best you can do is if  $v_{n2}^2$  square is the same as  $v_{n1}^2$  square right. And so, this number must always be greater than or equal to 1 right and this is a you know this quantifies the SNR degradation caused by the amplifier alright. Remember, if the amplifier is perfectly noiseless what comment can we make about the input SNR and the output SNR?

$$\overline{v_{n2}^2} > \overline{v_{n1}^2}$$

$$\frac{\overline{v_{n2}^2}}{v_{n1}^2} \geq 1$$

Well, it is the same thing right, you taken a noisy waveform and you amplified it up you amplified the signal and the noise by the same factor and therefore, the SNR does not change ok. Now, what are you doing now you are amplifying the signal and the input noise by the same factor on top of it you are adding your own noise right.

And therefore, your you have effectively degraded the signal to noise ratio and that is equivalent to simply saying you know you might yeah, I mean the degradation in SNR is simply found by just taking the ratio of the noise that you would see that you see at the output divided by the noise you would have seen provided the amplifier was noiseless right. So, and this is therefore, this is what is called the noise factor or the noise figure.

So, it is common to I mean noise I mean there is also I mean there is some confusing notation here. This absolute number is often called the noise factor people also like to do I mean the something else that you will see is  $10 \log$  to the base 10 of noise factor is noise figure and some people use it interchangeably and so on, but you know if they say the noise factor is 3 db it basically means that the noise factor is 3 db. What does it mean, the output SNR is?

$$10 \log_{10}(NF) = \text{Noise Figure}$$

I mean or  $v_{n2}^2$  square by  $v_{n1}^2$  square is a factor of two, which basically means that the amplifier is adding as much noise as the input noise source after going through the amplifier ok.