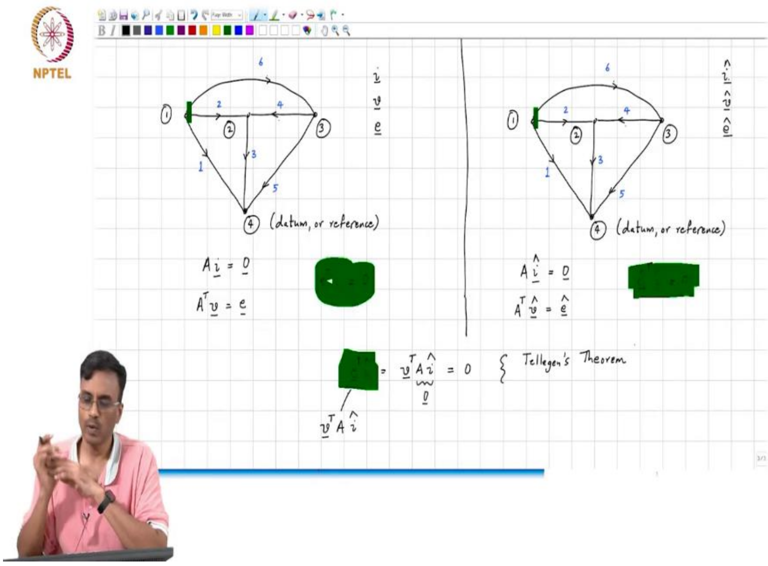
Circuit Analysis for Analog Designers
Prof. Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras

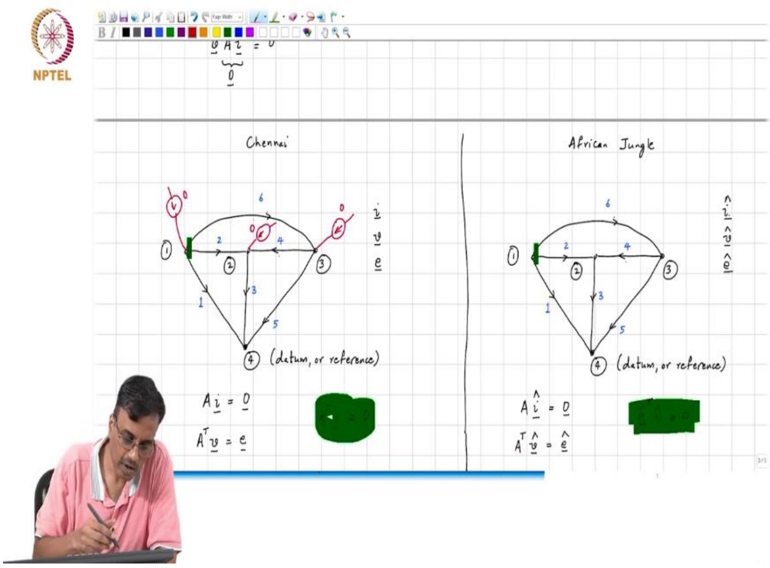
Lecture - 04
Intuition behind Tellegen's Theorem

(Refer Slide Time: 00:16)



Now let us see why this it makes intuitive sense right.

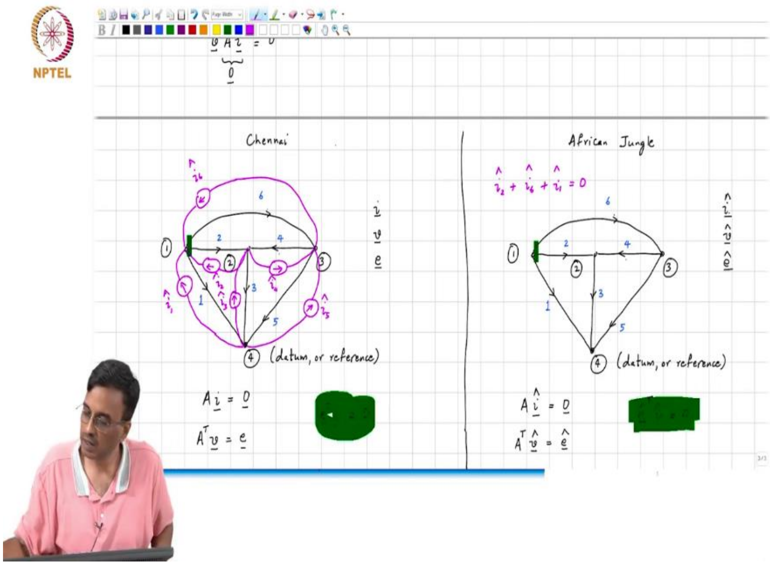
(Refer Slide Time: 00:20)



Now, let us say I add here is a network which has some node voltages and node currents I mean branch currents. Now let us say I add a current source into this node of value 0 right likewise, I am going to add another current into that node with a current 0 and into this node with a current 0. What comment can we make about the node voltages in the branch voltages when we do this.

Well, we are adding 0 currents into every node. So, nothing will happen to the node potentials and therefore, nothing will happen to the branch voltages and to the branch currents correct. Now what I am going to do is derive this 0 in a rather special way right.

(Refer Slide Time: 01:42)



What I am going to do is recognize that 0 can be written as i 2 hat, plus i 6 hat plus i 1 hat is equal to 0 correct.

$$\hat{\imath}_2 + \hat{\imath}_6 + \hat{\imath}_1 = 0$$

So, what I am going to do is basically add a current here which is $\hat{\imath}_6$ a current here which is $\hat{\imath}_1$ and here going to add a current which is $\hat{\imath}_2$ ok.

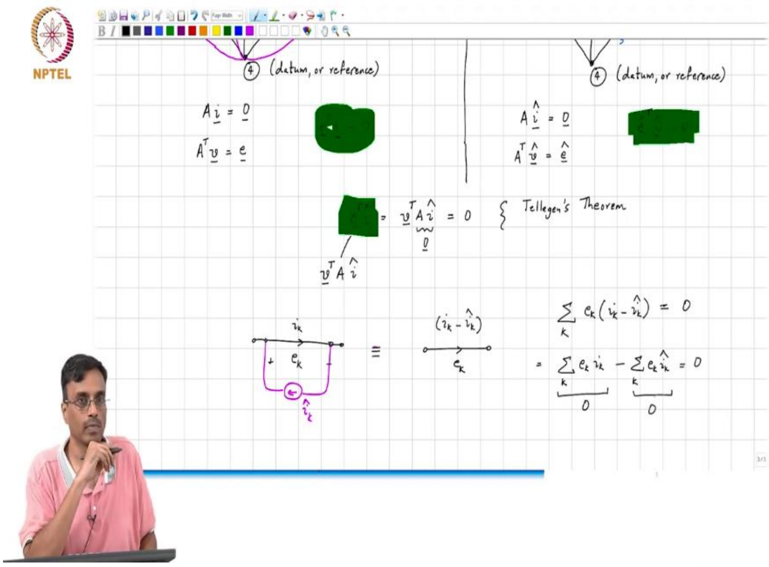
Likewise, in other words I am going to replace put in parallel with every branch in this network a current in the opposite direction and whose value is where do I mean where am I getting these currents from. These hatted currents basically are pertaining to the second network alright.

And please note that I am putting the current across every branch right. But these currents are not chosen in an arbitrary way they chosen in a they chosen in a special way what is. So, special about the way I have chosen these currents in pink that I have added here what is so, special about the way I have chosen those currents?

Yeah. So, basically all those currents you know basically satisfy Kirchhoff's current law correct. So, even though I have added these new currents in pink that is not what I have in effect done is added a current, what is the net current I have added at each node?

Right; the current I have added at each node is 0 alright. So, nothing is happened to the branch voltages or the branch currents ok. In other words, I have replaced in the original network if there was a branch.

(Refer Slide Time: 04:20)



Let us call this some e_k and i_k is the current, e_k is the voltage alright. What we have done is in effect in parallel with every branch we have put in $\hat{i}_k$ alright.

Now this network is as legitimate as any other network correct. So, energy conservation must be valid for you know I am sorry power conservation must be valid for this network too correct. And therefore, what is what comment can you think make about this branch this is equivalent to if you think of this is a branch what is the voltage across this branch? e k. What is the current through this branch? i k minus i k hat correct.

$$(i_k - \hat{i}_k)$$

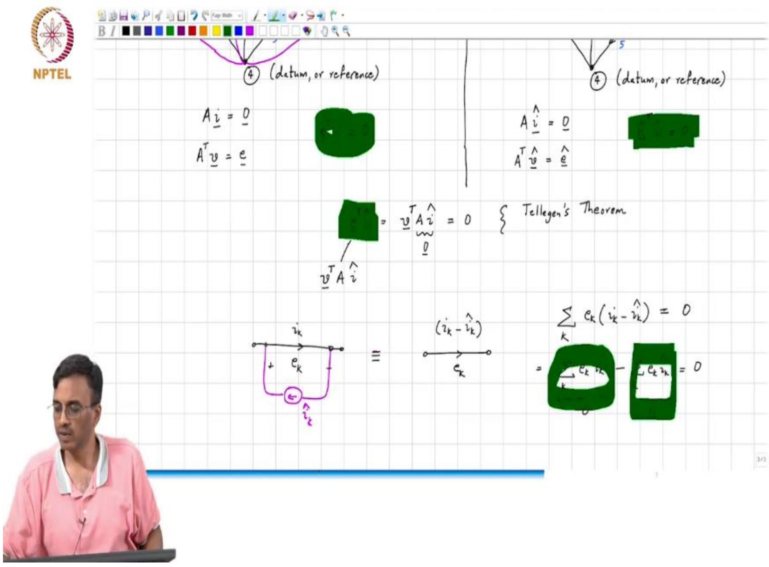
So, what comment can we make about the power dissipation instantaneous power in the entire network is simply k times sum over k e k times i k minus i k hat alright.

$$\sum_k e_k(i_k - \hat{\imath}_k) = 0$$

And so this therefore, now what must this be? What must this be? Well energy conservation says that this must be 0 well this is nothing but must be 0 what we know about this is 0; because that is energy conservation with respect to the original network. And therefore, it must follow that this chap is also equal to 0, right.

$$= \sum_k e_k i_k - \sum_k e_k\, \hat{\imath}_k = 0$$

(Refer Slide Time: 06:41)



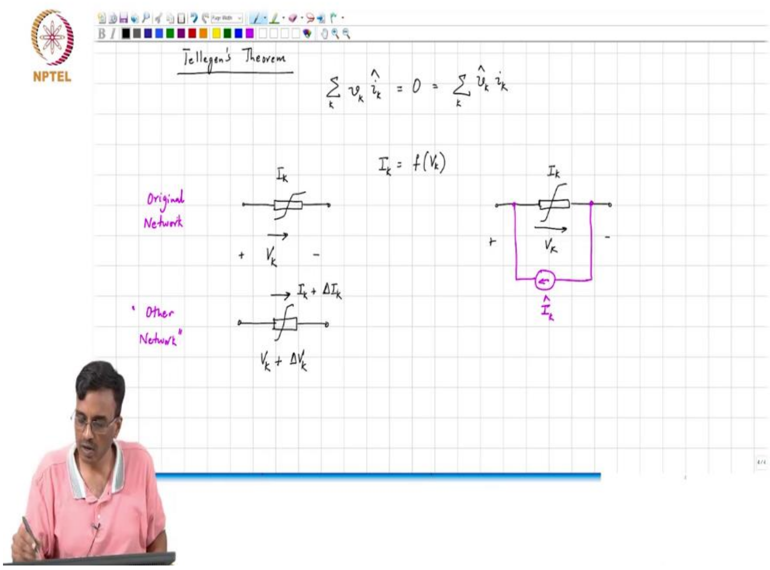
Of course, if I just showed you just this result, it would be quite puzzling as to how you multiply the voltages in one network and currents in some other network and how the sum magically turns out to be turns out to be 0. I mean if you go think about it this way you see that it is not that surprising after all right.

Because energy conservation must hold for this network alright and in and we already know that this must be 0. So, the other term must also be 0. So, Tellegen's theorem is basically a statement of instantaneous power conservation right.

And this physically appealing because you know that you know at any instant of time if the total power consumed in some branches is not exactly equal to the power generated in other branches.

Then you know if that was true then we not all be sitting here we will be selling power right this is a way of making more energy than you dissipate and therefore, ok. So, that is the alright now it turns out that Tellegen's theorem has you know whole bunch of interesting applications.

(Refer Slide Time: 08:13)



And let me talk about a couple of them. Tellegen's theorem can be applied to two networks of having the same graph which basically means that there are whole bunch of obvious things implied here it could be applied the second network could be the same as the first network the second network could be the same as the first network, but at a different time.

The second network could be completely different from the first network as long as it has the same skeleton or the same graph right ok. All that this is saying is that you know v k some more all branches of v k times i k hat is equal to 0 which is the same as the sum overall branches of v k hat times i k alright.

$$\sum_k v_k \, \hat{\iota}_k = 0 = \sum_k \hat{v}_k \, i_k$$

And one example all of you have seen taken an analog circuits class and where all the network elements are non-linear correct. And what you call we know that analyzing non-linear networks is difficult. So, we resort to what do we do? We linearize the network and work with small signal equivalence right.

And the underlying basis for small signal equivalents are the following I mean you have some you have some sources and you change the sources by a small amount. And then you know you only write KCL and KVL for the incremental quantities correct and the basis for that is again you know you can think of it as a principle that we used while deriving Tellegen's theorem.

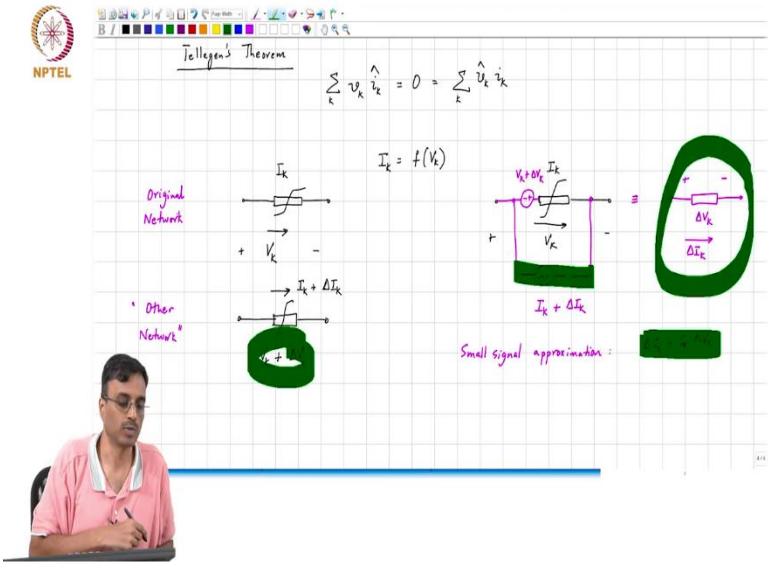
The original network for instance let us say you had some non-linear element where the branch current and the branch voltage are related in some really non-linear fashion ok. And remember that Kirchhoff's laws are not going to be or the branch voltages in the branch currents are not going to change if you replay if you place in parallel with every branch.

An arbitrary current $\hat{I}_k$; where this $\hat{I}_k$ is derived from another network. That another network is where this is original network, the other network is where a particular source has been changed by some amount correct.

So, if you go and change say a source voltage by some amount then all the branch voltages and the branch currents are going to be are going to get changed and therefore, this current is going to be $V_k + \Delta V_k$ and the current is going to be. You take a network and change one source; what happens? All the branch currents and the branch voltages Change;

So, therefore, this current is going to be $I_k + \Delta I_k$. Remember that this delta V ks and the delta I ks need not necessarily be. Small at this point right they change that change is that capital delta correct. So, what I am going to do is going to replace though you have taken an analog circuits class you have seen this before what I am going to do therefore, is in the original network I am going to add a branch current in parallel right.

(Refer Slide Time: 13:02)



Where I am going to call this $I_k + \Delta I_k$ correct. Does make sense people? Ok. Now, if I do this for every branch what comment can you make about the branch voltages and the branch voltages and branch currents in the original network.

What comment can we make? By making this construction with the original network, what is the net current I have added at every node? With in parallel with every branch I have added. $I_k + \Delta I_k$. So, what is the net current I have added at every node in the network? It is 0; does it make sense? Because all these $I_k + \Delta I_k$s must satisfy Kirchhoff's current law.

Correct ok. And likewise what I am going to do is also replace I am going to put in series with every node every branch I am going to put a voltage which is in the opposite direction. I am going to put $V_k + \Delta V_k$ what comment can I make about the what will happen if I put a voltage source of $V_k + \Delta V_k$ remember $V_k + \Delta V_k$ corresponds to the branch voltage in.

In that branch where I have changed a source right. So, what if I do this for every branch what comment can we make about the net voltage that we have added in every loop? Well, we know that the branch voltages you know are not some arbitrary branch voltages they all satisfy this $V_k + \Delta V_k$ therefore, satisfy KVL.

And therefore, the effect of adding all these is not is that it does not change the loop currents right. So, therefore, if the original network satisfies KCL and KVL its apparent that this must also if I replace every branch by this composite branch, it must also satisfy.

Student: (Refer Time: 15:55).

KCL and KVL alright. So, now, what is the net voltage across this branch? The voltage across this branch is ΔV_k and the current through the branch is ΔI_k.

Alright. I have flipped the picture on the right. So, that ΔI_k flows in that direction, but otherwise everything is the same does it make sense people right. So, what is the moral of the story? This is telling us that you know if you have a if you have a non-linear network and you change something alright well all the branch voltages and the all the branch currents will change.

And this is telling you that the change in the branch voltage and the change in the branch current right ok those changes also if you form a network where you have network elements whose branch voltages and branch currents are ΔV_k and ΔI_k then it will also follow KCL and KVL correct ok.

Now the next assumption; so, this is basically it is true for arbitrary changes in the sources alright. So, I like to retreat that this ΔV_k and ΔI_k need not be small the small signal approximation comes in when you assume that the changes are. So, small that you can relate the ΔI_k to the ΔV_k in a linear function alright.

So, therefore, you can see that the constructivity used to derive Tellegen's theorem in an alternative way also kind of throw light on why this small signal approximation is grounded in firm principles it is right the only approximation is this part right this is always true right.

So, the changes in the voltages and currents basically still obey KCL KVL. The moment you relate the change in voltage to the change in current through a linear equation that is when you have made an approximation alright.