Circuit Analysis for Analog Designers

Prof. Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 39
Input referred noise in electrical networks - part 2

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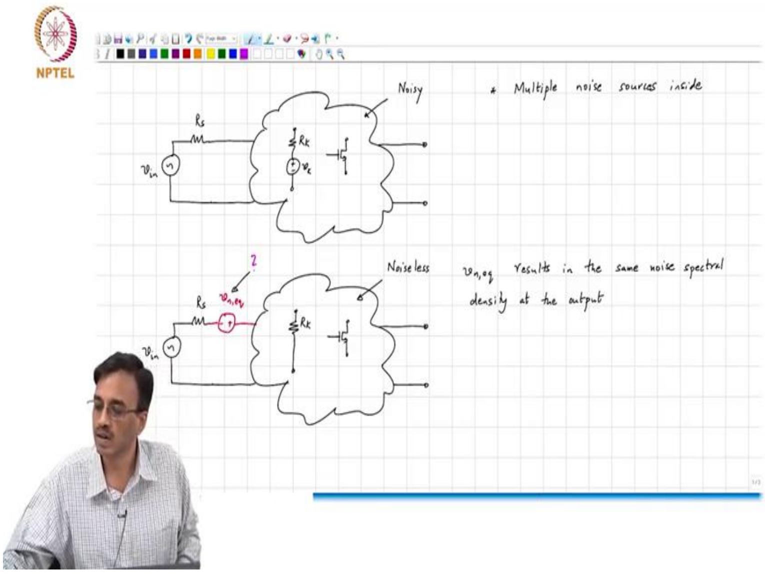

In the last class we were talking about how one would represent the noise of a large network right. So, as you have definitely aware we are always dealing with networks with whole bunch of components resistors, some transistors you know whatever and presumably the network has an input, which I will call this resistance R s and couples to network in some fashion like this.

This is the output and so, there are multiple noise sources inside and therefore, there is a associated with each noise source there is a transfer function from the noise source to the output right. And you know all these myriad noise sources go and influence the output in some way and we know how to calculate the spectral density at the output due to all these noise sources.

Now as we were discussing yesterday when you are giving this block away to somebody else for use, I mean the other person is probably not really interested in the gory details of what is going on inside the box right. So, he is more he or she is more interested in figuring out you know how does this you know this amplifier or filter or whatever affect the signal to noise ratio of my signal right.

My signal is there it is being processed by this animal right and the output is of course, consists of two parts one is the signal that is processed by the transfer function that it is supposed to that is the within quotes desired output, on top of it there is noise that is added from within the box and you know an obvious question to ask would be you know how badly is my circuit block degrading the signal to noise ratio of my input signal right.

So, for instance you know if you had an amplifier, the job of the amplifier is to take a small signal and make it large right in some sense. Now and the reason why you need a large signal is because this the circuitry following this amplifier basically is able to discern what is going on, but in the process the amplifier also adds its own noise.

So, the signal increases at the output port of the amplifier, but likewise there is also a lot more noise at the output than there was at the input and why is there a lot more noise? There are two reasons for this; one is that the noise that was inherent in the signal itself before it hit the amplifier will get amplified because amplifier does not know what is signal and what is noise. On top of this amplified noise from the signal source itself there is internal noise that is generated by the amplifier ok.

And therefore, the total noise at the output will in the best case be simply that which is amplified that of the signal source which is amplified by the amplifier right. And if your amplifier is really bad then well it does a great job of amplifying my input signal, but also adds so much noise in the process right that in effect even though the magnitude of the output signal or the power of the output signal at the is very large right. The fidelity of the signal is degraded so much because in addition to this large signal you have you know more than proportional noise amplification correct.

So, therefore, when you are characterizing amplifiers and so on or filters for that matter or any circuit for that matter you would like to be able to estimate or figure out how within quotes you know how bad is my signal to noise ratio getting, I mean how badly is my signal to noise ratio getting affected because of you know whatever signal processing I am doing inside this box right. And since we are not really interested in the gory details of what is happening in the box reasonable question to ask would be ok.

Well, what if I need to compare the you know how the I mean compare the signal to noise ratio of my signal source which is that of this guy here to what the signal to noise ratio would be here an equivalent way of doing this would be the following right. So, let us we ask the question, well if I found an equivalent noise source here where I went and disabled all noise sources inside the network. So, what do I do? So, this is a noisy network and, in the picture, below I say the network let us assume its noiseless right.

And I am trying to replace the effect of all these multiple noise sources inside the box with a single noise source right, which for obvious reasons I will call the equivalent noise source and in what way should the two be equivalent? I mean in what sense when we say equivalent what do we expect? Well v n equivalent has the same or rather results in the same noise spectral density at the output.

Does it make sense right? So, all that we are saying is this is supposed to be an equivalent representation of this myriad noise sources inside the box and evidently there is I mean if we are if we are able to club the effect of all these noise sources into one source at the input right that is what we call the input referred noise source. Now, let us discuss how we are going to find this input referred noises? How I mean can we think of how we will be able to do this? What do you suggest?

Exactly yeah.

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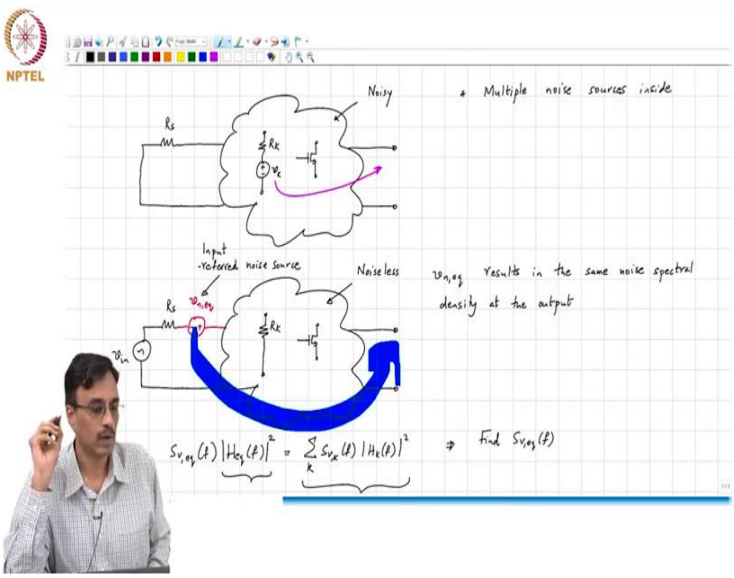

So, basically the idea is very simple right we from v n equivalent to the output here there is going to be a transfer function correct and therefore, S v equivalent of f will be whatever transfer function there is from v n equivalent to the output right. So, that is basically call that H equivalent times sorry this times H equivalent of f the whole square right must be equal to well there are multiple noise sources inside.

So, each one of them will yield, well S v k of f times mod H k of f the whole square and you do all this over all the noise sources ok and which basically means that you can go and just do the math and find the spectral density of the equivalent noise voltage source that would result in the same noise spectral density at the output.

$$S_{V_{1eq}}(f)\left|H_{eq}(f)\right|^2 = \sum_k S_{V_{1k}}(f)|H_k(f)|^2$$

Does it make sense? And you know as he in English all that this means is that this is nothing but what does this represent? The output noise spectral density right and this represents well recognize that the gain from v in and from v n equivalent is the same this H equivalent of f simply represents the gain from the input voltage source to the output and therefore, it stands to reason that you know you find the total spectral density divided by gain square and this is the spectral density of the equivalent input referred noise.

So, this is an equivalent noise source and this is referred to the input referred noise source right. So, is the motivation and the jargon clear now alright. So, now, let us get started couple of things that I would like to draw your attention to one is in our job therefore, seems to be a to find the transfer functions from each of those input sources to the output right and also from v n equivalent to the output right.

And well you know how given a general network we can only come up with some broad guidelines, it is without knowing the network it is evidently not possible to come up with the transfer function correct? But a couple of observations that I would like to make are the following and to do that I as usual I will what do you call use our you know KCL KVL type analysis let us say I am interested in finding the transfer function from v k to v output ok.

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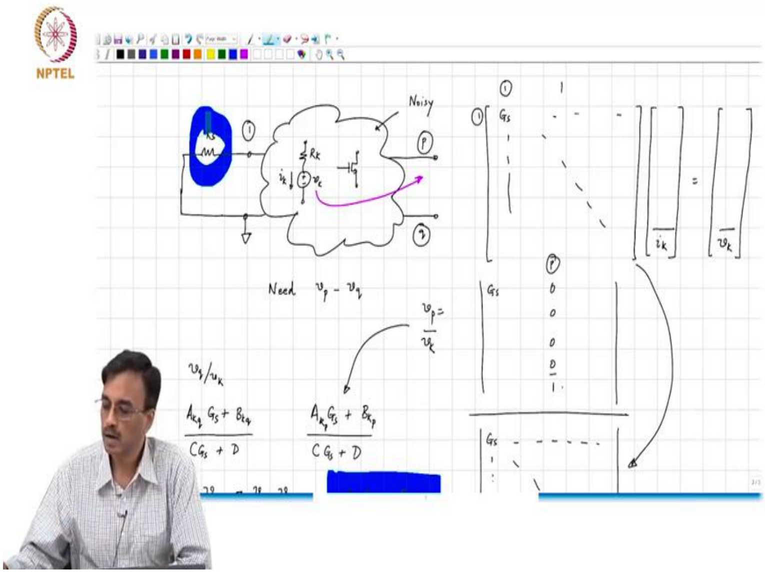

So, let us say I am interested in finding the transfer function from v k to the output and as usual I choose you know this node as my reference node that is ground 0 this is 1 this is I do not know node p this is node q and we write the nodal equations you know as we are used to and there is only one source here right.

And so, if we write the nodal equations what do we get? We get the m n a matrix we know how to do this right and let us and the unknowns will be you know all the node voltages alright. And well there is only one voltage source. So, the unknown basically is say some i some k right and what do we get on the right-hand side?

Yeah, and which are the independent sources here. Its only v k that appears here, correct and what else do we know? Well, we know that there is a resistance R s from node one to one to ground. So, if you look at this MNA matrix this is node 1 alright and this is the 1st row. So, this will be G s alright and then you have you know the rest of the matrix alright. So, we are interested in finding we are interested in what are we interested in finding? This is the set of equations and what are we interested in finding?

$$\begin{bmatrix} G_s & \cdots & \cdots \\ \vdots & \ddots & \vdots \\ \cdots & \cdots & \vdots \end{bmatrix} \begin{bmatrix} \\ \\ i_k \end{bmatrix} \begin{bmatrix} \\ \\ v_k \end{bmatrix}$$

We are interested in finding v p minus v q correct and how do we do that? Well good old Cramer's rule right. So, what do you do? So, v p for instance is nothing but yes what do we do people? I cannot hear you at all man.

Very good right. So, basically you replace you find the determinant of this matrix right in the in the numerator what do you do? You have of course, you have G s here and in the pth column what do you do?

You replace the stuff with you basically you will get v k right all these will be 0 does it make sense people and what we have in the denominator? Is simply the determinant of that matrix? So, this will be G s and what we had, correct? So, of course, this basically means that you can take v k out as common and therefore, you have v p over v k which is the transfer function we are after as the ratio of determinants which is like which is like this right.

$$\frac{v_p}{v_k} = \frac{\begin{bmatrix} G_s & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 1 \end{bmatrix}}{\begin{bmatrix} G_s & \cdots & \vdots \\ \vdots & \ddots & \vdots \\ \vdots & \cdots & \vdots \end{bmatrix}}$$

So, if you expand the determinant what comment can you make about the numerator? It will be of the form what comment can we make about can we make any comment about the form of the numerator ok. There will be a whole bunch of terms when you expand the determinant out to calculate it ok. A term can either consist of G s or not consist of G s? Correct further if it consists of G s, G s will appear only in the first power does it make sense right?

So, in general therefore, the determinant corresponding to this matrix can be written in the form let us say some A sub k right because we are doing this with respect to the kth source right. A sub k times G of s or G sub s plus this A k times G s basically that A k term simply clubs all those terms with G s in it correct and therefore, and that is possible only because G s appears in the first power and then you will have terms without G s ok

And what comment can you make about the denominator? Well, it will also be of the same form; obviously, you know the you will not have the same A k and the same B k. So, you call this C times G s plus D ok

$$\frac{A_k G_s + B_k}{C G_s + D} = \frac{v_p}{v_k}$$

And as you discussing yesterday you know such a form is called a bilinear form. So, if v p by v k is of this form what comment can we make about v q by v k? Yes, people you will get a? Similar form except that it will be some you know whatever right I am going to call this a A k moreover just to make running out of subscripts here. So, this is A k q times G s plus. Yes, Prashanth B k q what comment can you make about the denominator?

C G s plus D and why is the denominator the same?

$$\frac{A_{kq} G_s + B_{kq}}{C G_s + D} = \frac{v_q}{v_k}$$

Well, the matrix below is the same I mean the determinant below is the same. So, the key take away is that.

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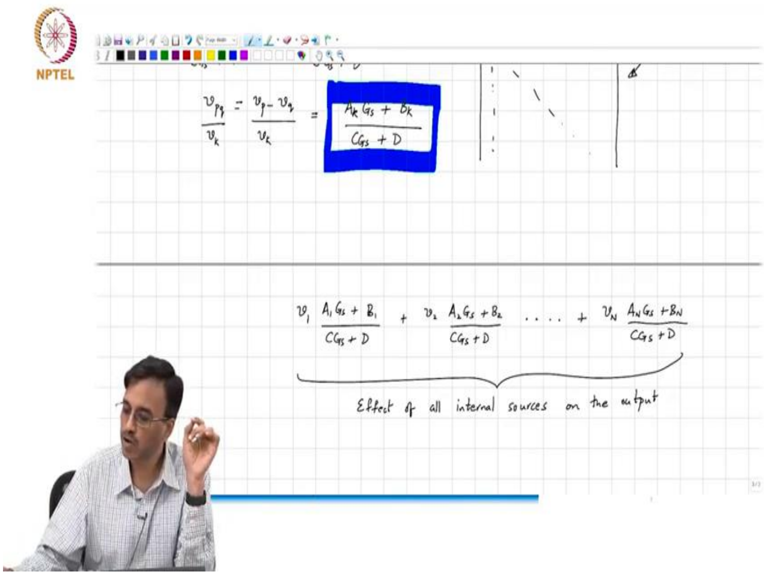

Therefore, if you do v p q which is v p minus v q by v k what do you get? What do you think this will the form of this will be? What will it be? It will also it will be again of this form it is A k times G s plus B k divided by C G s plus D ok

$$\frac{v_{pq}}{v_k} = \frac{v_p - v_q}{v_k} = \frac{A_k G_s + B_k}{C G_s + D}$$

I hope you guys are convinced about this right ok. So, therefore, the transfer function from any internal noise source to the output will be of this general form where all I have done is basically brought out the explicit dependence on the explicit dependence on the source resistance ok and.

So, therefore, if we want, I mean let us now, at this point not worry about those sources being noise sources if there were deterministic sources, the total output because of these multiple sources would be v 1 A 1 G s plus B 1 divided by CG s plus D correct this is the first noise I mean first source inside the box plus v 2 times A 2 G s plus B 2 over C G s plus D blah blah blah until you are blue in the face. So, this is you know whatever v N A N G s plus B N divided by C G s plus alright.

$$v_1 \frac{A_1 G_s + B_1}{C G_s + D} + v_2 \frac{A_2 G_s + B_2}{C G_s + D} + \cdots + v_N \frac{A_N G_s + B_N}{C G_s + D}$$

So, this therefore, is the effect of all internal sources on the output. Does it make sense people? The key point to note is that the denominators of all these transfer functions remains the same ok and this must also gel with your intuition I mean with your you know prior background namely that you know this is why the poles of any transfer function that you compute in the same network.

You know is independent of where you put the input and where you take the output right is a inherent property of the network itself and that makes sense because that is coming from the denominator the determinant of that MNA matrix which is a characteristic of its got no sources inside right the MNA matrix is just you know got to do with the network and its topology excellent.

So, now we want to find a single voltage source v equivalent whose effect at the output of the amplifier is exactly the same as the effect of all these internal noise sources on the output. This is the effect of all the internal noise sources all the internal sources on the output correct ok. And we would like to find what that equivalent input source is which will have the same output correct. So, what comment can we make about. So, with therefore, we need to find the transfer function from v n equivalent or v equivalent to the output.

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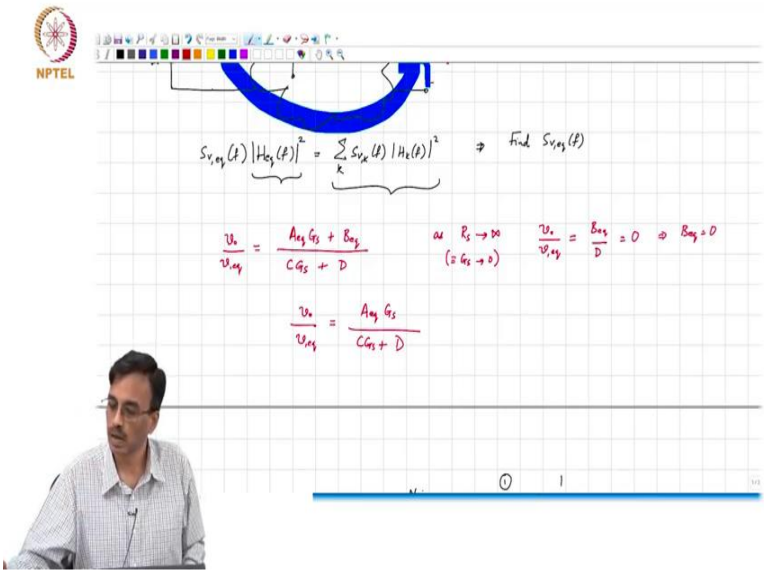

So, what comment can you make about the transfer function from v over v equivalent? Pardon yes people. Well, you I mean you would say well it is the same form what should be the denominator? C G s plus D very good and the numerator is some A equivalent times G s plus B equivalent,

$$\frac{v_o}{v_{1eq}} = \frac{A_{eq}G_s + B_{eq}}{CG_s + D}$$

but there is a there is a twist right as R s tends to infinity what comment can you make about v o by v equivalent? As R s tends to infinity in other words G s tends to 0 what comment can we make about the transfer function from that equivalent source to the output? Which is the same as asking the question what is the transfer function from the input to the output right when that R s becomes R s becomes infinite right. So, what do you what comment can you make?

Yeah, it is B equivalent by D that is fine right so; obviously, as G becomes 0 G s tends to 0 this is B equivalent by D, but simply inspecting the network tells you that if I remove the resistor there is no output to talk of and therefore, this must be equal to 0 right and therefore, what does this mean? What does this mean?

B equivalent must be must be 0 is this clear?

$$as\ R_s \to \infty\ (\equiv G_s \to 0)$$

$$\Rightarrow \frac{v_o}{v_{1eq}} = \frac{B_{eq}}{D} = 0$$

$$\Rightarrow B_{eq} = 0$$

So, therefore, while v o by v equivalent is of the same form what we need to understand is that this must be that B equivalent must be 0. So, this must be of the form A equivalent G s divided by C G s plus alright.

$$\frac{v_o}{v_{1eq}} = \frac{A_{eq}G_s}{CG_s + D}$$

So, with that in mind what is the single equivalent voltage source that has the same effect as all these multiple sources inside? What should we put on the left-hand side?

Yes.

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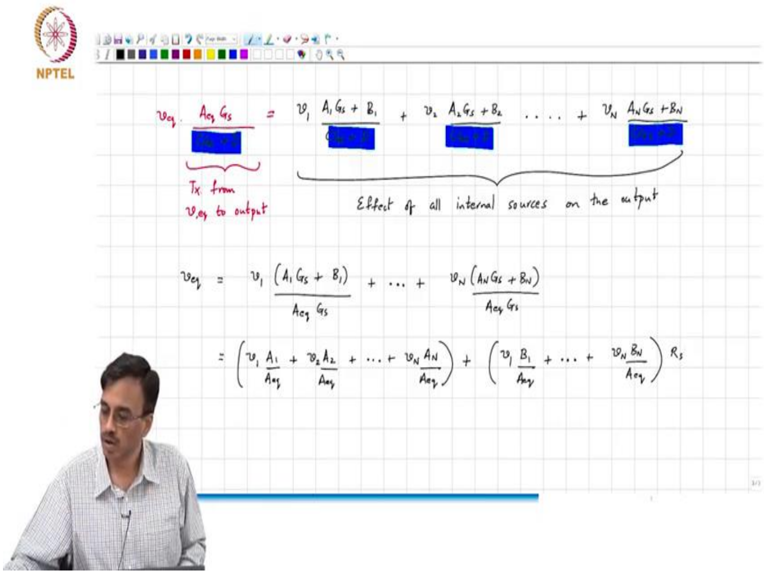

Well, v equivalent times A equivalent times G s divided by C G s plus right. So, this is the transfer from v equivalent to the output ok.

$$v_{eq}\frac{A_{eq}G_s}{CG_s + D} = v_1\frac{A_1G_s + B_1}{CG_s + D} + v_2\frac{A_2G_s + B_2}{CG_s + D} + \cdots + v_N\frac{A_NG_s + B_N}{CG_s + D}$$

So, what do you see as happening here? Well, one thing you notice is that all these guys cancel out and therefore, we are left with v equivalent times v equivalent is simply v 1 times A 1 G s plus B 1 divided by A equivalent times G s plus blah blah blah alright.

$$v_{eq} = v_1 \frac{(A_1 G_s + B_1)}{A_{eq} + G_s} + \cdots + v_N \frac{(A_N G_s + B_N)}{A_{eq} G_s}$$

And this can be written as v 1 times well A 1 over A e q plus v 2 times A 2 over A e q all the way up to the Nth plus?

Pardon. By the way is this dimensionally consistent well a A and A basically have the same dimensions. So, this is indeed a voltage correct plus v 1 times B 1 by A equivalent plus all the way up to v N times B n over A equivalent divided multiplied by divided by G s which is equivalent to saying it is alright.

$$v_{eq} = \left(v_1 \frac{A_1}{A_{eq}} + v_2 \frac{A_2}{A_{eq}} + \cdots + v_N \frac{A_N}{A_{eq}} \right) + \left(v_1 \frac{B_1}{A_{eq}} + \cdots + v_N \frac{B_N}{A_{eq}} \right) R_s$$

So, what are the dimensions of this? Some voltage and; obviously, the strength of that voltage source depends on it depends on the strengths of what are v 1 through v N?

They are the internal sources right. So, clearly it makes sense that the voltage source depends on the internal noise sources and A 1 by A e q A 2 by a A e q etcetera depends on the transfer functions from the internal sources to the output divided by these those A 1 by A eq basically quantify the relative strengths of the transfer functions from the internal sources to the output ok.

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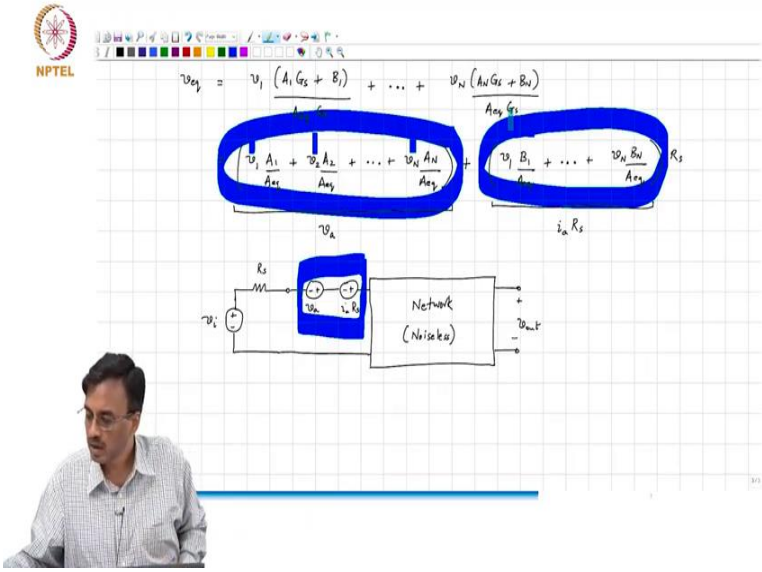

Now, what comment can you make so, this is let us call this v_a alright and what comment can we make about the dimensions of this quantity here this is current and this is $i_a \times R_s$ alright. So, therefore, to come back to our equivalent noise source; so, we have v i this is R s ok and our equivalent noise source consists of two components one is v a and the other one is i a times R s and well, this is my internal network whatever it might be, but here I have gone and made sure that all my noise sources are null in other words the network is noiseless and this is the output ok.

Now, does it make intuitive sense that this noise source this equivalent noise that you have here? It will make intuitive sense that that depends on R s I mean one thing you can say that what can I say you know it just comes out of the math right, but is there intuition domain does it make intuitive sense that we should have a term which is proportional to R s. Any thoughts?

Yeah ok, fair enough right I mean and the easiest way of understanding this is the following as R s becomes larger and larger right what comment can you make about the influence of v i on the output?

Well as R s becomes tends to starts becoming larger and larger you know the connection between v i and the network becomes weaker and weaker and therefore, you should expect the transfer function from v i to v o to be to kind of become smaller and smaller right ok, which basically so and, but the noise due to the internal sources that is what it is ok.

I mean if you remove if you remove the input source and R s altogether there will be the internal sources will cause some noise at the output correct ok, but as R s becomes larger and larger the noise caused by the internal sources remains largely the same at the output right; however, the transfer function from v i which is also the transfer function from that equivalent noise voltage source keeps dropping correct.

So, if you want to achieve the same noise that you see right as R s becomes infinity or R s as R s becomes larger and larger, the only way to do that is if the strength of the of that equivalent input noise voltage becomes larger and larger and must be proportional to R s right because asymptotically the transfer function from v i to v o will fall off as R s ok I mean unless you consider you know pathological cases like input impedance being infinity which is which you know does not exist in practice right ok.

Input impedance of a network being infinity only exists in the textbook ok. So, in reality there will be some capacitance between the input and ground and so, basically as R s becomes larger and larger you will find that the transfer function must basically become the transfer function from v i to v o keeps falling down.

And if you want the equivalent noise source to have the same effect as all these myriad internal noise sources the strength of the noise source, better go up in proportion because otherwise you know this equivalent noise source has to get multiplied by the transfer function to give the same output noise.

So, if the transfer function is going down the strength of the noise source better get keep going up that is the intuition because you see i a times R s alright. Now, we are in a bit of a bind right because remember what was the aim of our whole project when we started off what were we trying to do I mean when we started off this whole discussion on you know equivalent noise source?

What we were try I mean the big picture is that you know here is a network that I am going to give to you the consumer right and just like how I give you the two port parameters of my amplifier I would also give like to give you something which has the same noise behavior right ok and in other words that is the reason why we started off this input referred noise discussion. So, that you the user can take this information and put that in your I mean you are presumably building a bigger system you want to see how the noise of my block impacts your system.

So, what you would like to do is take the model of my two-port including noise and you know plop it inside a inside you know a bigger model right ok. Unfortunately, the way we have it right now seems like and we can say well here is the input referred noise ok or the strength of the input referred noise source, but there is a problem what is the problem you think?

Student: (Refer Time: 37:14).

Well, you know we would ideally like that input referred noise source to be a property of the network right, but it you know the way it appears here is that it looks like it; obviously, depends on R s and we also saw you know why that makes sense right, but we are now stuck with an input referred noise spectral density or you know whatever voltage which depends not only on the network through v a and i a. Remember that v a and i a are only dependent on all these quantities only depend on the network they do not depend on R s ok. Is that clear by the way? Because that those a b c and d terms are all independent of R s right.

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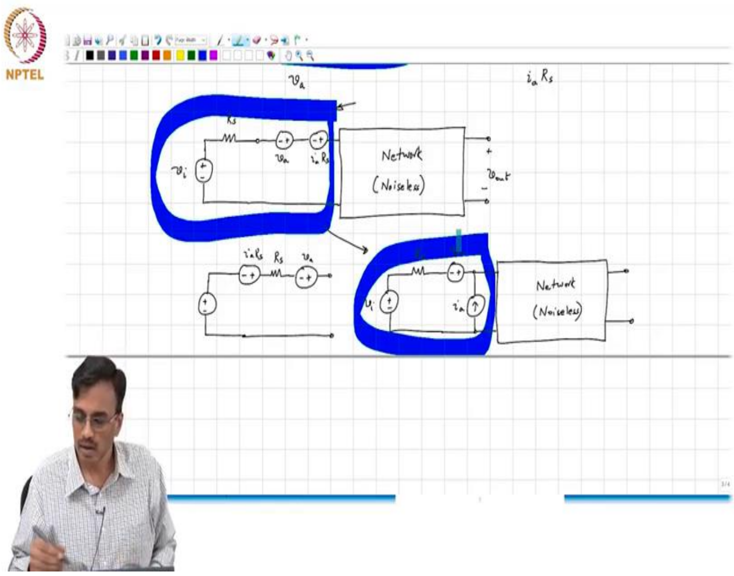

So, these are only network dependent though the one circled in blue and so, we are in a bit of a bind fortunately it turns out that you can if you stare at this, right, you can it turns out you know that I can move voltage sources around. So, if I make this i a times R s, R s and then v a right and then you recognize that you can do a Norton to Thevenin transformation there.

And you see that well you can get the same Thevenin equivalent for this box if you did this. So, you have v i R s then you have v a and you have i a and here is the network which is noiseless alright and you know how do we know that this is correct? Well, if you look there what is the Thevenin equivalent what is the Thevenin resistance? Rs what comment can we make about the Thevenin resistance here? R s ok and what comment can you make about the Thevenin voltage on top? v i plus v a plus i a times R s and what comment can you make about the Thevenin resistance I mean Thevenin voltage of the lower on the in the lower circle?

It is the same v i plus v a plus i a times R s. So, these two are evidently equivalent. So, now, what can you say? You said at this picture and then therefore, what do you give the user?

Student: (Refer Time: 41:04).

Well, you say that this is the equivalent noise representation of my network right ok.

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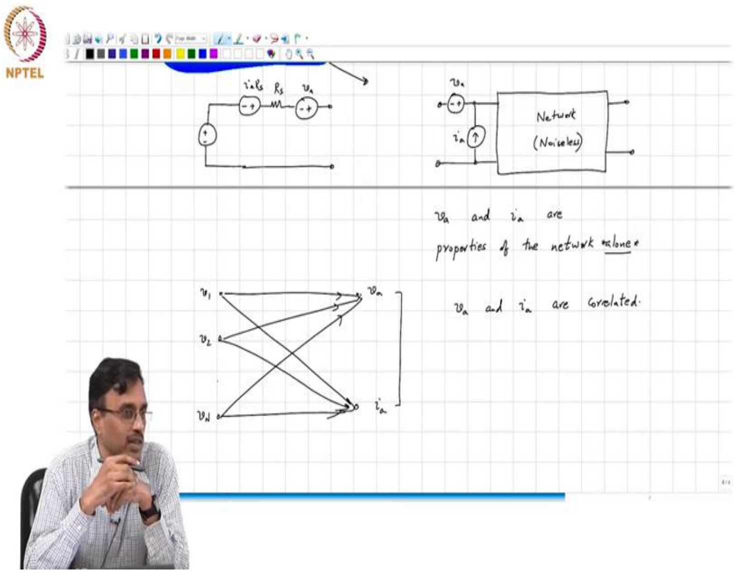

And so, now, this representation is independent of the source resistance and both v a and i a are properties of the network alone alright. And the reason is that the v a and the i a depend only on v 1 through v N which are noise sources that are internal to the network they depend on A 1 by A e q etcetera which all depend only on the a s and the b s remember are terms which do not are terms in the determinant of the MNA matrix or with you know appropriate columns zeroed out ok which do not depend on G s right. So, in other words they are only properties of the network.

Another thing that I would like to point out is that remember see finally, v a and you know i a are within quotes you know noise quantities because v 1 through v n are all noise sources. So, the spectral density of v a can be easily found by well you know s v 1 of f times mod A 1 by A equivalent whole square and so on and so forth right can we comment on whether v a and i a are independent or dependent?

Pardon? Why are they dependent? Well, remember that v 1 v 2 blah blah blah v N are they independent or dependent? They are generally independent because they are all noise from different resistors or different transistors they do not know what is going on in the other transistor right, but v a is some linear combination of v 1 through v N; i a is some other linear combination of v 1 through v N. So, what comment can you make about v a and i a? I mean is there some relationship between the two or they are completely independent.

Student: (Refer Time: 44:14).

Both are basically taking forming the linear combination of some bunch of independent sources correct ok and therefore, these two will be.

These two will be in general dependent. So, v a and i a are dependent in other words the noise sources v a and i a are correlated ok. So, this is like you know Indian marriage you know arrange marriage you basically you know each party is making inquiries about the bride or the groom as the case may be right now what do you do?

Well, you know you asked the neighbor you know is this fellow ok or like you know does this guy come home drunk at night at 3 in the morning and make a ruckus ok. Well, if you ask the neighbor, you know the neighbor on the left side and the neighbor on the right side ok.

The information you get is most likely going to be I mean this fellow is a random phenomenon, but you are simply right you are looking at you know alpha times random and then beta times the same random. So, you ask both neighbors you basically are not going to evidently get you know phenomenally new insight ok.

If you ask his boss in the office and you know his neighbor then you is you know each one is looking at an independent aspect of the persons performance. So, it is a you know you basically there also there will be correlation because you know it is still looking at the same person, but hopefully there will be lesser correlation ok.