

Circuit Analysis for Analog Designers
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Lecture - 36
Total integrated noise in RLC Networks

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Alright. So, let us try and see you know how we can do this. So, again, this is $R_{sub K}$, this is $V_{sub K}$, alright. And we have L , we have C . And we are interested in finding; what is the noise spectral density at the output? It is $4kT R_{sub K} |H_{sub K}(f)|^2$ the whole square is the output noise spectral density corresponding to the K th resistor.

$$4kT R_k |H_k(f)|^2$$

And what are we going to do? What do we need to do? Of course, we need to sum over all of them right and or you integrate this over. If you want if you are interested in finding the total integrated noise, what are we supposed to do?

You integrate this from 0 to infinity, and sum over, this is the mean square errors. Is this clear? Ok.

$$4kT \sum_k \int_0^{\infty} R_k |H_k(f)|^2 df = \overline{v_o^2}$$

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The slide displays a circuit diagram with a resistor R_k , inductor L , and capacitor C in parallel, connected to a current source $i_k(t)$. The output voltage is $v_k(t)$. To the right, the equation is written as $4kT \sum_k \int_0^{\infty} R_k |H_k(f)|^2 df = \overline{v_k^2}$. Above the circuit, there are labels C_1 and C_2 with arrows pointing to the circuit components. The NPTEL logo is visible in the top left corner.

So, in other words, the key aspect of the whole problem is, you know, how do we calculate this integral in a simpler way. I mean one is to say, so go and find all the transfer functions and do this. But you can see that it is a lost cause. So, again, we have not really exploited the fact that you have only R L C elements inside the network, ok, and, as we did with Nyquist theorem, we can always exploit reciprocity.

But now, I will do this in the time domain, right. So, let us say I put an impulse current here, ok. By the way, before I do that.

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The slide displays the same circuit diagram as the previous slide. The equation is written as $4kT \sum_k \int_0^{\infty} R_k |H_k(f)|^2 df = \overline{v_k^2}$. The NPTEL logo is visible in the top left corner.

So, this is the frequency domain and this can be written as $4kT \sum_k R_k \int_0^\infty |H_k(f)|^2 df = \overline{v_o^2}$, I can pull the R_k out of the integral, right, ok. And you do, $|H_k(f)|^2$ the whole square df , that is mean square. Does make sense? Now, let me rewrite this in a form that you are hopefully will ring a bell.

$$4kT \sum_k R_k \int_0^\infty |H_k(f)|^2 df = \overline{v_o^2}$$

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I write this as $2kT \sum_k R_k \int_{-\infty}^{\infty} |H_k(f)|^2 df = \overline{v_o^2}$, is there something?

$$2kT \sum_k R_k \int_{-\infty}^{\infty} |H_k(f)|^2 df = \overline{v_o^2}$$

You know have you, is there some other way of writing this? Pardon? Ok.

Right. Parseval's theorem, very good. What is that is, what is that saying? The mean square integral in the frequency domain is the same as the mean square this thing in the time domain. So, this is simply equivalent to saying well this is integral 0 to infinity. We know that the impulse response, corresponding to $h_k(f)$ if it is going to be causal. So, this is Parseval. Correct? Ok.

$$2kT \sum_k R_k \int_{-\infty}^{\infty} h_k^2(t) df = \overline{v_o^2}$$

So, now, we reducing to we have reduce a problem to finding this integral. Correct? And, we still have not used reciprocity.

So, since here it seems like in the time domain, it seems reasonable that use the time domain form of reciprocity. So, let us say I put an impulse current here. Correct? Ok. As soon as I put an impulse into the network, ok, what will happen? Where is that? If any of this impulse current goes into the network, where is it going to go?

It is only gone into the capacitors, right. It cannot go into the inductors, because inductors are going to be an open circuit at 0 plus, right. And the resistors will have an impedance, which is much larger than the capacitors. So, all the current goes into the capacitors, right. So, at t equal to 0 plus, let us call, I mean since there is some voltage across some capacitors inside the network, at t equal to 0 plus, the network has got some energy. Correct?

Because, these capacitors by you know by hook or by crook, have been charged by this this, impulse current source to some voltages and as a result the initial at t equal to 0 plus the energy in the network is non-zero and let us call that E_0 , correct.

Now, at t equal to infinity, what comment can you make you know about the capacitors?

Well, it is not necessary they be discharged, but there will be some I will show you examples where it is not necessary if that the capacitors be discharged, right. But there will be some redistribution of the, well, the impulses come, you know charged up some capacitors to some voltage and gone, right. Then, well the network does it is thing and these capacitor voltages redistribute themselves.

It is entirely possible as you guys pointed out, that the capacitors all get discharged, right. But at any rate, at t equal to infinity, there is some energy left in the network that energy could be 0, right.

So, let us call that energy at infinity, let us call that E_{∞} . Does make sense? Alright. So, what comment can we make about E_0 minus E_{∞} ?

Remember, power and energy are different things. What comment can we make about $E_0 - E_\infty$. It is positive. Right? Otherwise, you would not be here, ok, alright. That is one that is a great piece of insight. What next is the do we have?

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$E_0 - E_\infty$ which is a positive number, right, which is got dimensions of energy, right. Must all have been dissipated in the resistors. Correct? And, so what comment can we make about the energy dissipated in the registers? What is the power in the register?

I square R, right, the energy is simply nothing but the? Integral of the power. So, it is the $E_0 - E_\infty$ is nothing but, R_k in the kth resistor is $R_k \int_0^\infty i_k^2 dt$. Correct?

$$E_0 - E_\infty = R_k \int_0^\infty i_k^2(t) dt$$

Are this clear people? And what is i_k ? By reciprocity, well, what is the reciprocity saying is saying that the voltage transfer function from v_k to the output is the same as the current transfer function from the output from an impulse I mean injected at the output to the current through this voltage source v_k . So, that is the small h_{k} of t . Correct? So, by the way, what if in all these calculations you have to be very careful about dimensional consistency. Let me ask you a question therefore, what are the physical units of h_k of t ?

Pardon? Ok. Let us see, how many answers I get. No dimension. The input and output quantity are known. Ok, ampere.

Pardon? One over. Yes, Alfred? 1 over second, that is hertz. Come on people. Can we decide? H_k of $j 2 \pi f$ or H_k of f is nothing but is the Fourier transform of h_k of t e to the minus $j 2 \pi f t$ dt, alright.

$$H_k(f) = \int_0^\infty h_k(t) e^{-j2\pi ft} dt$$

So, what are the dimensions of H_k of f ? It is? It is a gain. So, it is, dimensional is unit less, ok. What are the units of e to the minus you know x?

It is a number. So, it is got no units. So, what must be the units of this? This better be? It is better be what? 1 by time, which is Hertz. Correct? So, definitely ampere is not correct. It is Hertz, alright.

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So, this, but. So, when you say if you put delta of t here, you get i k of t which is h k of t, this is definitely not, this is not correct. This is correct, or no? If I put an impulse there, my claim is that, by reciprocity the current in v sub k must be h k of t that is dimensionally inconsistent, that is incorrect. Right? All, you can say is the shape of the waveform is the same as h k of t, but I have to multiply it by what?

h k of t is basically per second, i k of t is? Ampere. So, I have to multiply it by Ampere second. Where is that ampere second coming from?

It is the not the charge from the capacitor, it is basically the impulse. Correct? Alright, ok. The input current, actually it is the input current multiplied by the delta t in the convolution, right. And so therefore, it is incorrect to write this as i k is i k of t as h k, they are related but they are not the they are not the same; one is apples, one is oranges you cannot compare both of them. Correct? You have to multiply h k of t with ampere second to be able to get i k. This is subtle, but you know it is important. Otherwise, you will get results which are dimensionally inconsistent, alright.

So, therefore, this is nothing but, what is $i k$ square times I mean what do you call, what comment can we make about this guy now? Is this related to $h k$ square of t or not? Yes? No?

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Yes. So, this is nothing but, $h k$ square of t dt, right. This divided by, I mean, this times, you have to multiply that $h k$ with ampere second which is coulomb. Correct? So, that is if I divide this by 1 coulomb square, I basically get $R K$ times integral 0 to infinity $h k$ square of t dt. So, but I needed, I mean and this the summation over k is obvious. Correct? Is this clear?

$$\frac{E_0 - E_\infty}{1C^2} = \sum_k R_k \int_0^\infty h_k^2(t) dt$$

So, therefore, what comment can you therefore make about the mean square value of v_o . Guys? Mean square value is nothing but, $2 k T$ times E_0 minus E_∞ by 1 coulomb square, alright.

$$\overline{V_o^2} = 2kT \frac{E_0 - E_\infty}{1C^2}$$

So, let us start with our typical known case which is R and C . Correct? So, what do we do? We need to, what is E_0 by the way? Let us recall, what is E_0 ?

The energy you inject an impulse current into the output node and find the energy at 0 plus. So, basically this is R, this is C, at 0 plus, what do we see? What happens to the impulse current? It all goes into the capacitor. So, what is the voltage? 1 by C, I mean dimensions are we ok? What are the dimensions of 1 by C? I mean 1 by C is what? You, what are the units of 1 by C is? What should it be? It is a voltage. So, it must be volts. So, are we sure we are dimensionally consistent? That, 1 by C, that 1 is a corresponds to 1 coulomb, right, which is contained in that unit impulse. So, when we say 1 by C here that, 1 is, 1 coulomb by? Farad. So, it is indeed? Voltage. Correct? So, E_0 , therefore is what? 1 by 2. Half times C times V square which is 1 by C square, which is 1 by 2 C. Right? And that 1 is, what are the units of that 1? It is 1 by C. People. What are the units of that, I mean, are we dimensionally consistent? Coulomb square. Right? Ok. So, alright.

$$E_0 = \frac{1}{2} C \frac{1}{C^2} = \frac{1}{2C}$$

What is the E infinity? What happens to this voltage as t tends to infinity? Well, the capacitor completely discharges. So, what is E_0 , infinity? 0, alright.

$$E_\infty = 0$$

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$E_0 = \frac{1}{2} C \frac{1}{C^2} = \frac{1}{2C}$
 $E_\infty = 0$

$\overline{v_c^2} = 2kT \left[\frac{1}{2C} - 0 \right] = \frac{kT}{C}$

$E_0 = \frac{1}{2C_3}$
 $E_\infty = 0$

$\overline{v_o^2} = 2kT \left[E_0 - E_\infty \right] = \frac{kT}{C_3}$

And so, what is the mean square noise? It is? It is well 2 k T times, 1 over 2 C minus 0, right.

$$\overline{v_0^2} = 2kT \left[\frac{1}{2C} - 0 \right]$$

At 1 by coulomb square and this coulomb square gets cancelled, right. we remember it is 0 minus E infinity by 1 coulomb square, once you do that, there is a coulomb square in 1 over 2 C. So, that gets cancelled. So, dimensionally we are consistent. So, what is this? This is nothing but k T over C, alright.

$$\overline{v_0^2} = \frac{kT}{C}$$

Now, let us try let us trivial example. So, this is $R_1, R_2, R_3, C_1, C_2, C_3$ and we are interested in finding the mean square noise across, say, C_3 . Alright. So, as usual, we inject an impulse here. So, where does that impulse current go? At t equal to 0 plus.

All the current goes on to? C_3 . So, what is E_0 therefore? Or, E_0 by 1 by, E_0 by 1 coulomb square is nothing but, 1 over 2 C_3 . E_∞ is? Well, at t equal to 0 plus there is a voltage on C_3 . Right? After a long time, what will happen? I mean all the voltage across C_3 will discharge through the resistors, what about the voltages across C_2 and C_1 ?

Well, yeah. So, at t equal to infinity, the steady state voltages across all the capacitors are? Is 0. So, what is energy at t equal to infinity? 0, alright, ok.

$$E_0 = \frac{1}{2C_3}$$

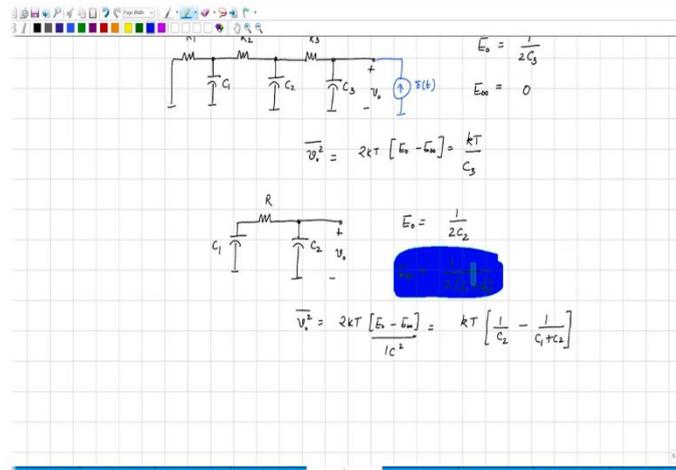
$$E_\infty = 0$$

So, what is the mean square noise? It is 2 k T times E_0 minus E_∞ , which k T over C_3 , alright.

$$\overline{v_0^2} = 2kT [E_0 - E_\infty] = \frac{kT}{C_3}$$

So, it basically shows that you do not need to, I mean, it is wasteful to kind of find the entire spectral density and integrated and throw away all that information. If you are only interested in the total noise, it is much easier to you know use some technique like this, ok.

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As a final example, let us say this is the output noise, you are interested in. So, what is E_0 ? E_0 is nothing but, $\frac{1}{2C_2}$. What is E_∞ ? What is the voltage at T equal to infinity?

Pardon? $\frac{1}{C_1 + C_2}$ because that, t equal to infinity, all you need to do is simply replace, the resistor with a short because no current flows through the resistor t equal to infinity. So, whatever charge there was in C_2 must be shared between C_1 and C_2 . And therefore the, voltage at t equal to infinity is, $\frac{1}{C_1 + C_2}$. So, what is the energy at t equal to infinity? $\frac{1}{2(C_1 + C_2)}$.

$$E_0 = \frac{1}{2C_2}$$

$$E_\infty = \frac{1}{2(C_1 + C_2)}$$

Does make sense people? Ok. And therefore, the mean square noise is, $2kT$ times E_0 minus E_∞ by 1 coulomb^2 and that is, $2kT$ times or kT times, $\frac{1}{C_2} - \frac{1}{C_1 + C_2}$, ok.

$$\overline{v_o^2} = 2kT \frac{[E_0 - E_\infty]}{1C^2} = kT \left[\frac{1}{C_2} - \frac{1}{C_1 + C_2} \right]$$

I mean this just is that we will be to illustrate the fact that, it is not necessary that all the capacitors discharged to 0, right. This shows there is an example of a network where the capacitors do not discharge, ok. And, if we need to check if C_1 goes to 0. C_1 goes to 0 or infinity? So, C_1 goes to infinity, it is like having a short circuit there, you should reduce to kT over. Alright.