Circuit Analysis for Analog Designers
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Lecture - 35
Noise in RLC networks

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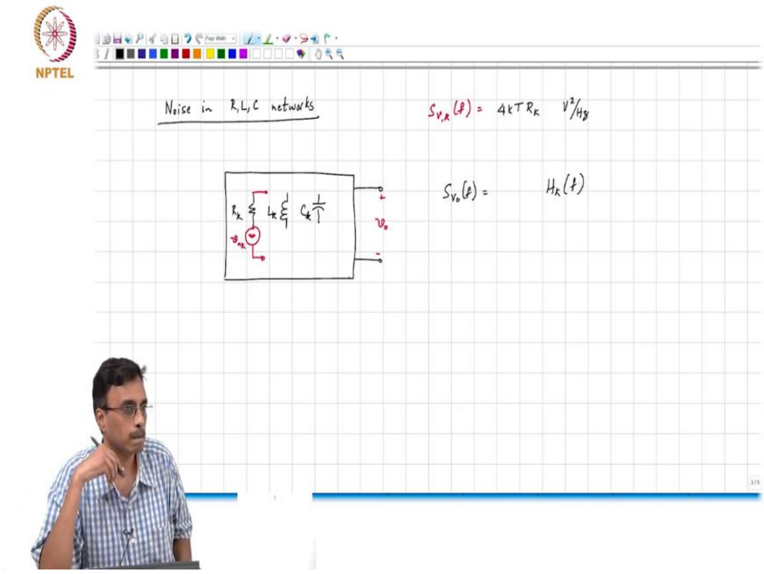

The next thing I is to talk about Noise in RLC networks and this occurs you know quite often in practice. So, in other words we have already seen what happens with noise in a resistor it turns out as I was mentioning the other day capacitors and inductors are noiseless. So, if you have an RLC network what it basically means that you have a network with any number of Rs Ls and Cs ok. And every resistor is associated with a noise source, which we denote by v n sub k, (v_{n_k}) and this is the output, correct.

So, what is S v n let us call this S v, k of f is what? It is simply 4 k T times R sub k the units being volts square per hertz alright.

$$S_{v_k}(f) = 4kTR_k \, \frac{V^2}{Hz}$$

So, it is seeming there is I mean it is obvious that now if you have a whole bunch of resistors and you have this black box and then you get two terminals out the voltage if you put a voltmeter across these two terminals what do you expect to see?

What would you expect to see? You would have voltmeter across those two terminals? You will have; obviously, noise because well you know there are a whole bunch of noisy elements inside the box and. How will we find the noise spectral density at the output?

Well, it is a very straightforward, what do we do I mean step-by-step approach is simply find the transfer function from every noise source to the output let us call that H sub k of f correct and that.

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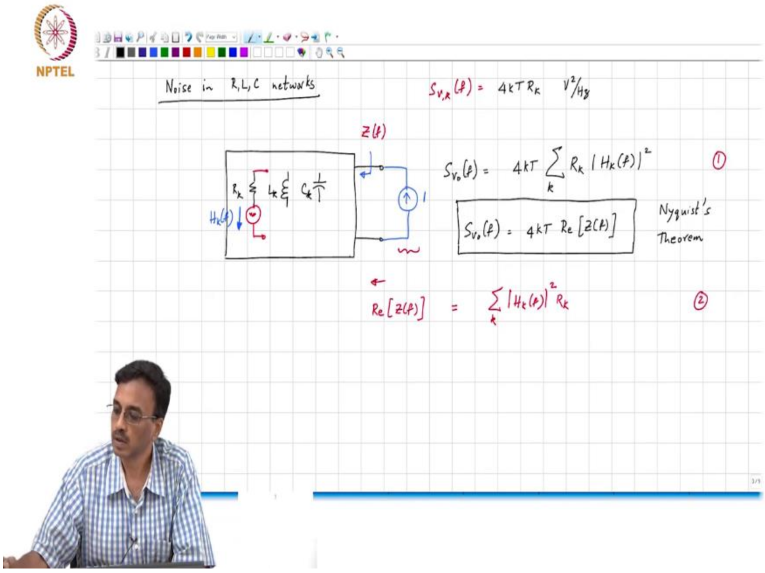

And what is the spectral density corresponding to the kth noise source its 4kT R sub k times mod H sub k of f the whole square alright.

$$S_{v_o}(f) = 4kT\, R_k |H_k(f)|^2$$

This would be this will be the in English. What does this stand, what does this mean what is that quantity there? That 4kT R sub k H k of f the whole square what does that represent?

The noise spectral density at the output due to the? Due to the kth resistor right and we also discussed that the noise from you know different resistors independent. So, if you want to find the noise spectral density due to all the resistors it is simply What? It is the sum over all resistors. So, simply sum over k R k H k f of whole square alright ok.

$$S_{v_o}(f) = 4kT \sum_k R_k |H_k(f)|^2$$

So, well there is one aspect of the whole problem that we have not exploited yet, what is that? Well, this is true for any network right. If you have multiple sources, you find the noise spectral density at the output due to each source and you add them; what is new there is nothing new here right. There is one aspect of the problem that we have not exploited yet and what might that be?

We have the network consists only of R L and C and therefore, what we have not exploited is. You know once you have a network like this you know there is also there is you can one aspect is that it turns out to be reciprocal and. So, basically that is what we are going to exploit next and so to see that. So, let us say this is a current the current i alright, what comment can you make about the transfer function from here to the current in the kth resistor? Well, if this is i this will be or rather let us say this is a phasor of angle 1 angle 0, what will be the phasor here? This phasor is simply H sub k of f times 1.

Is this clear correct alright? So, the next thing I would like to draw your attention to is the looking an impedance here let us call that Z of f alright. So, what comment can we make about the energy supplied or the power supplied by this current source into the box? What comment can I mean if you have a current driving into an impedance Z of f right ok. So, what comment can we make about the energy supplied by the current source into this box?

Pardon. i square times the real part of?

Very good. So, basically the power going in is basically i square fortunately that happens to be 1 times real part of Z of f right. So, where is all the power inside the box being dissipated?

It is being dissipated in the resistors and well, do we know the current through the resistors? Right that is nothing but H k of f. So, what is the power dissipated in the kth resistor?

Yes.

Mod H k of f the whole square times R k. So, what is the total power dissipated in all the resisters? It is simply the sum over all k and that must be equal to that make sense people alright.

$$Re[Z(f)] = \sum_k |H_k(f)|^2 R_k$$

So, now, can you stare it these two equations and tell me what conclusion you can draw from this?

Pardon.Yeah. So, what comment can we make about the output noise spectral density? So, S vo of f is simply put one and two together you get 4 k T times the real part of Z of f alright. And this is what is called Nyquist's theorem I guess it must be called one more of Nyquist's theorems right and.

$$S_{v_o}(f) = 4kT\ Re[Z(f)]$$

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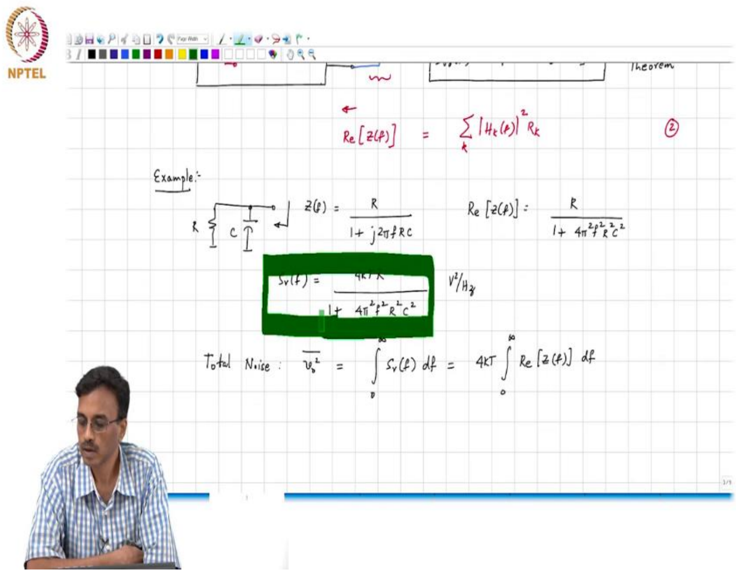

So, for example, let us come back to our familiar example here we already know the answer and alright. So, what comment can we make about Z of f? R by 1 plus j 2 pi f RC, real part of Z of f is? R by, yes people. 1 plus 4 pi square f square R square C square ok. So, S v of f is simply 4 k T R by 1 plus 4 pi square f square R square C square.

$$Z(f) = \frac{R}{1 + j2\pi fRC}$$

$$Re[Z(f)] = \frac{R}{1 + 4\pi^2 f^2 R^2 C^2}$$

$$S_v(f) = \frac{4kTR}{1 + 4\pi^2 f^2 R^2 C^2} \;\; V^2/Hz$$

Well, this and this is volts square per hertz and we knew this answer already right. Where earlier we had actually calculated the transfer function from the noise source to the output and done the Math it turns out to that yeah, we could have done this way. It is clear people? Ok.

So, alright so, what comment can we make about the total noise? That is v o square is simply the integral of S v of f df and which is simply nothing but 4 k T integral. Integral what people?

Real part of Z of f and as we have already seen the real part of Z of f df is this. And if you integrate this, we did the integral the last time around and we found it to be k T over C alright.

$$\overline{v_o^2} = \int_0^\infty S_v(f)df = 4kT \int_0^\infty Re[Z(f)]df$$

Now, it turns out that in a lot of practical situations we are only interested in the total mean square noise ok. In other words, so far, I mean let us kind of backtrack a little bit and see what we have done so far to determine the total noise.

What have we done? [FL] Yes, can you re remind me what we what we need to do?

Very good. So, we have gone and found the transfer functions from each noise source to the output right then, it must be noise somewhere, no? The transfer function is got nothing to do with transfer function is transfer function.

So, we need to multiply we need to find the transfer function from every noise source to the output right. Multiplier find the magnitude squared of that transfer function multiplied by the spectral density of the noise source and then that will give us the output noise spectral density due to one noise source.

And you know you find the sum over all the noise sources right, that will give you the noise spectral density at the output. Now, you have to integrate this noise spectral density across the entire frequency range 0 to infinity to be able to get that total mean square noise. Is this clear people? Right.

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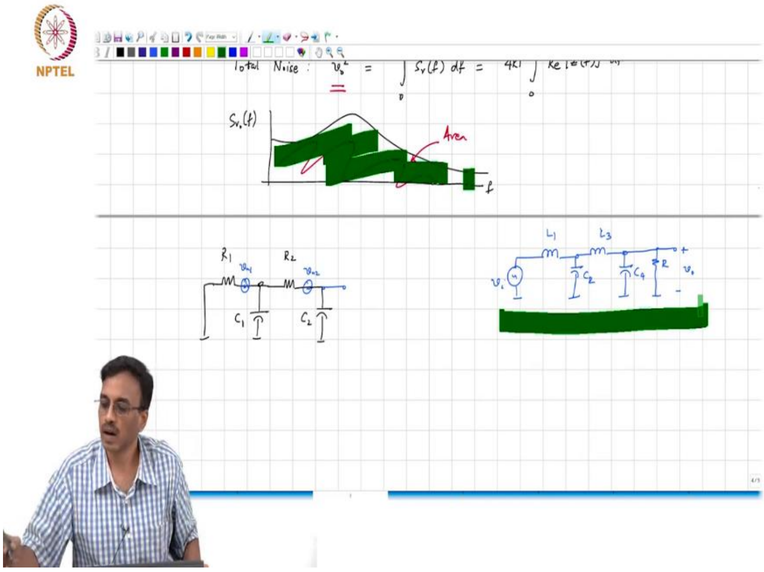

So, I mean in other words, if you find if you found the total noise spectral density let us say like this you have basically finding the area under the curve right. So, this is S v o of f and this is f and basically what you are doing is finding the area here. Now, does somebody see this as being kind of round I mean long winded and perhaps unnecessary. See we are only interested in finding the total area under the curve right, the exact nature of the curve is irrelevant. So, it does not make sense to find the exact spectral density which is a lot of work right. Why is it a lot of work?

We had to calculate the transfer function from every noise source to the Output alright and then find the square transfer function and you know I mean just in case I mean since most of you have this you know why is this so difficult look on your face ok. So, let us call this R 1, C 1, R 2, C 2 alright the transfer function from this noise source to the output we will have it is a second order denominator right. So, the denominator magnitude square will be a? Will be a? Fourth order polynomial right and likewise the transfer function from v n 2 to the output let us say this is the we are interested in finding the total noise across C 2 that, will also be a fourth order polynomial. And then you need to find the integrals of these fourth order polynomials one over those fourth order polynomials across all frequency right. And this is only for a? Second order network let us say you had you know 5 capacitors and 4 inductors and then you have all of a sudden you have a ninth order transfer function correct, which means the squared magnitude response will have 18th order polynomial in the denominator right.

So, all the tricks you learnt in your JEE and get coaching classes, which basically right are of no use ok, alright. So, but the fact remains that oh well I mean. So, here is an analogy right let me ask you a question with respect to the circuit on the right, what is the order of the transfer function? It is a Fourth order transfer function. Let us say I want to find the DC gain from v i to v o at this v o, what comment can we make about the d the DC transfer function from v i to v o? 1 right. How do you do I mean can you give us some insight into the dramatic speed with which you were able to get the answer?

Excellent, right I mean so, this is common sense what you would do is basically say oh well the capacitors are open at DC the inductors are shorts this is the answer is 1 right. You could also do it this way which is not very uncommon by the way in exams right you could find the whole transfer function fourth order transfer function and then put S equal to 0 and get the answer to be 1 ok. So, this finding the area under the curve right to find the area under the curve to write the magnitude response square it and integrate it from overall frequencies right, is to do a whole lot of work to get the details of the spectral density at the output.

And then throw away 99 percent of that information and since you are only interested in the area under the curve right. That is exactly equivalent to finding the transfer function I mean corresponding to this network if somebody ask you what the DC gain was is to find h of s and plop then plop in S is equal to 0. And be very happy that oh most of the terms vanish correct ok.

So, again fortunately it turns out that there is you know I mean if you are only interested in the total area under the curve or if you are only interested in the total mean square noise it turns out that it seems reasonable that you do not have to work as hard to go and find the actual spectral density right, which basically means that you are working very hard to get a lot of information and then throwing most of that information away because you are only interested in the area. I mean all of you understand that you know there is a lot more information in the exact shape of the curve than in the than in the area. Are this clear people?