Circuit Analysis for Analog Designers
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Lecture - 34
kT/C noise in a sample-and-hold circuit

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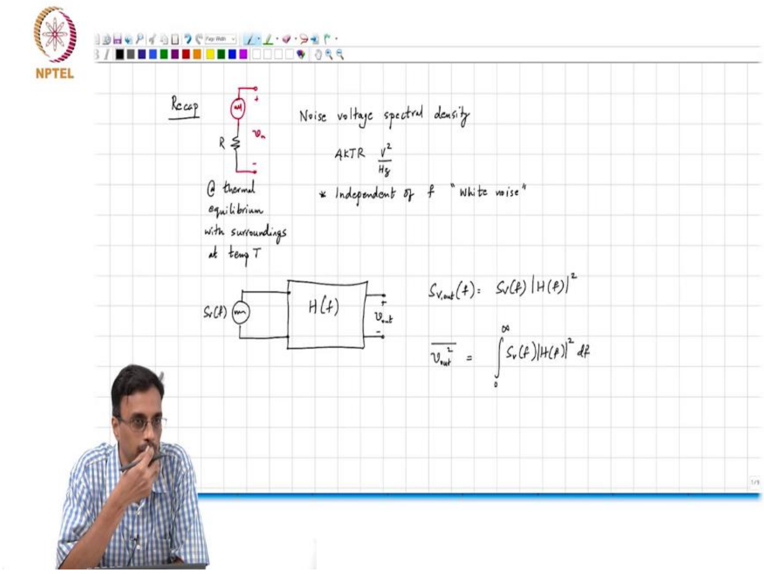

In the last class we started discussing about noise and here is a quick summary of what we learnt in the last class. So, every resistor in thermal equilibrium with surroundings at an absolute temperature T is associated a noise voltage source in series with it and the noise voltage spectral density corresponding to this noise source is 4kTR volt square per hertz.

And as we discussed the last time around this spectral density is independent of frequency. In other words, this is often what is called white noise and the reason is simply the following I mean optically if you have all colors the resulting color is actually appears white to us, right.

So, and colors in optics are basically because of light of different frequencies. So, in a similar vane if you have noise corresponding to all frequencies in equal strengths you know it is called it is called white. Obviously, if the noise spectral density is not uniform with frequency, then, it stands to reason that this is called colored noise, ok.

So, then we saw that if you have a noise source with a noise spectral density given by S v of f and it is processed by a transfer function H of f and the output noise spectral density is simply the input noise spectral density multiplied by the squared magnitude of the transfer function, alright.

$$S_{v_{out}}(f) = S_v(f)|H(f)|^2$$

And the total noise is simply the integral of the noise spectral density and the output which happens to be is that clear people, alright.

$$\overline{v_{out}^2} = \int_0^{\infty} S_v(f)|H(f)|^2 \, df$$

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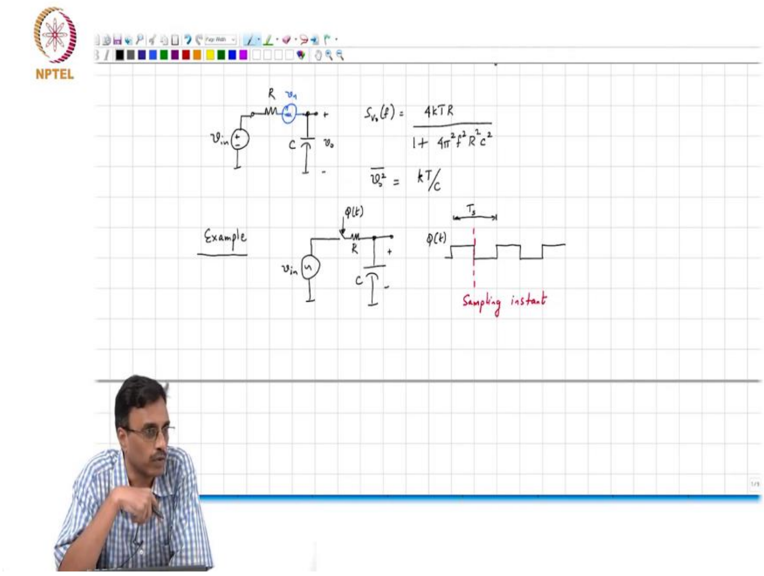

And we did our first calculation of this noise spectral density assuming we have a first order R C network let us assume that we have is an input and then R C network. The output consists of V in filtered by the first order R C transfer function. It also consists of noise whose spectral S v o of f is nothing but 4kTR by 1 plus 4 pi square F square R square C square.

$$S_{v_o}(f) = \frac{4kTR}{1 + 4\pi^2 f^2 R^2 C^2}$$

And the mean square value of the noise as we calculated the last time was, what was it?

k T over C, right where k is Boltzmann's constant.

$$\overline{v_0^2} = \frac{kT}{C}$$

And we also saw the intuition behind this apparently surprising result. Even though the noise source it is self depends on the resistance the total integrated noise apparently is independent of the resistance and we reconcile that by recognizing that while it is true that the noise spectral density increases with resistance; the bandwidth of the transfer function from the noise source to the output also experiences a change in the opposite direction.

And in this case, it just so, happens that the two of them simply cancel out, right. So, the total the mean square noise is $\frac{kT}{C}$. Now what are the you know apart from just being a mere curiosity it turns out that this result has quite some importance in practice. And here is an example where it plays a role. As we were mentioning as we were talking about in the early part of the semester, we like to process signals digitally which basically means that you take a signal you sample it and you quantize it.

So, that you have a digital representation of the input and you know one way of doing this is to basically sample the input that you want to digitize and then, once you sampled it you kind of look at that sample and then quantize it, right. So, quantization obviously takes some time. So, you need some place to store that input sampled value and storage of something can only be done on elements that can that have memory, right.

And, so it is either an inductor or a capacitor right and for reasons of size it is often a it is often a capacitance, right. And so, for example, here is the most simple-minded example one can think off. So, this is a switch right which is periodically operated let us call this signal phi of t, $\varphi(t)$.

So, for instance this is an example $\varphi(t)$, this is the sampling period yes. When the switch when $\varphi(t)$ is high the switch is closed, when $\varphi(t)$ is low the switch is opened and therefore when the switch is closed what happens to the voltage across the capacitor.

Well, when phi is high, the capacitor directly comes across the voltage source and therefore, it tracks the input voltage, right. When the switch is open well, whatever charge is there on the capacitor is trapped and therefore, what comment can we make about the sampling instant at what instant are we actually looking at the input voltage, I cannot hear you.

Yeah. So, what is there a precise instant of sampling? It is the falling edge of $\varphi(t)$, right. So, this is the sampling instant, alright. Of course, life is not as simple as this in reality there is a whole bunch of non idealities the first one being that no switch is idle and it turns out that every switch will have associated with it a resistance, alright.

And so therefore, the bottom line is that well the output will track the input, but you know there will be a small delay corresponding to the R C time constant right, but more importantly the resistance also adds the resistance also associated with a noise source correct.

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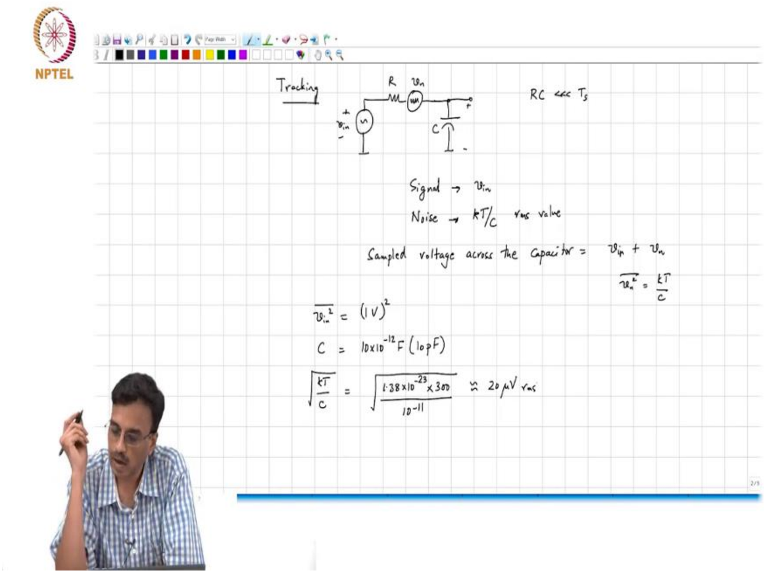

So, during the tracking phase how does the circuit look like? Well, there is the input, alright. And so, what comment can you make about the voltage across C I mean and by the way the R C time constant what comment can we make about the R C time constant in relationship with T s.

Pardon. I mean that R C time constant must be extremely small compared to half T s, because you want the input to track, I mean the voltage across the capacitor to track the input as closely as possible. So, R C is much much much smaller than T s, alright.

$$RC \lll T_s$$

And, so therefore during the tracking phase what comment can you make about the volt total voltage across C?

It will of course, consist of v_{in} right ok and I remember at the sampling instant namely the falling edge of the of this waveform phi of t. You not only have the input plus you have.

A component due to the terminal noise of the switch right and what will be the so, what comment can you make about the signal is basically V in whatever it is, ok. What about the noise? Yes, what comment can we make about the noise, is 0 mean that is correct what else.

It is a random waveform that is correct ok, it is got 0 mean and. It is got some mean square value and what is the mean square value? k T by C, right, alright. Please understand that when we say the mean square noise is k T by C it does not mean that you know you add square root of k T by C to the output, right. It just means that, if you build a million such circuit is or if you look at the noise associated with the voltage across the capacitance a thousand or a million times right; you will find that the means square value of that noise is k T by C does not mean that at every instant of time we every time you measure it is square root k T over C is this clear, right. So, what comment can you make and so, this is what the voltage across the capacitor would be during the track phase right and then, the whole phase comes when the whole phase comes what happens?

The switch is open suddenly, right. So, whatever voltage is there on the capacitor that voltage is I mean that charge is trapped right it has got nowhere to go. So, what you are actually sampling is V in plus the sample voltage across the capacitor is nothing but V in plus some noise voltage; where the mean square value of the noise voltage is simply k T over C, alright. For example, let us assume that the rms value of the input signal that needs to be digitized is 1 volt alright and C is 10 picofarad, alright. So, what comment can we make about square root of k T over C, please somebody do the math and tell me? 20.Ok alright. So, alright so, ok.

$$\overline{v_{in}^2} = (1V)^2$$

$$C = 10 \times 10^{-12} F \ (10\ pF)$$

$$\sqrt{\frac{kT}{C}} = \sqrt{\frac{1.38 \times 10^{-23} \times 300}{10^{-11}}} \approx 20\ \mu V$$

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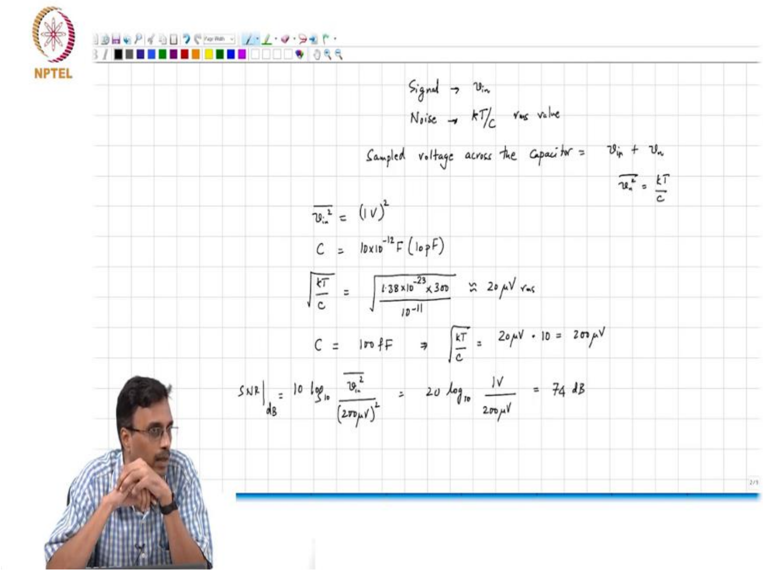

Now, if C was instead of being 10 picofarad was a 100 femtofarads, what will happen ah to this value? This is how much times how many times smaller? Times square root of?

These 10 powers minus 13 and that is 10 power minus 11. So, that is basically factor of 10 was it is now it is 200 micro volts, ok.

$$C = 100pF$$

$$\Rightarrow \sqrt{\frac{kT}{C}} = 20\mu V * 10 = 200\mu V$$

So, what comment can you make about this in the last instance, what comment can you make about the signal to noise ratio? Which is simply the ratio of the mean square value of the signal to the mean square value of the noise, which is and it is often expressed in dB, because this is power it is 10 log which is 20 log, 20 log to the base 10, 1 volt divided by 200 micro volts and that is how much? It is 74 dB, alright ok.

$$SNR|_{dB} = 10\log_{10}\frac{\overline{v_{in}^2}}{(200\mu V)^2} = 20\log_{10}\frac{1V}{200\mu V} = 74dB$$

And so, in other words this is only a factor of less than it is about 3 between 3000 and 4000, right. So, 74 d B is between 3060 I mean 1060 dB you know 1080 dB, 70 is somewhere between 60 and 80 and therefore, it is somewhere between is geometric mean of 1000 and 10000 it is roughly around 3500 or so.

So, basically you say I mean the practical importance of this is that, if you use small capacitor to sample, alright. You will be stuck with a noise voltage on the capacitor right which is large, ok. In this case for instance, you know you already made an error of the order of one part in 4000, right.

So, if you're if the resolution of your A-to-D converter that you are trying to realize is much higher, then this is a very bad choice of capacitor to use, right. So, if you are trying to resolve a voltage to better than say one part in a million right one part in a million is 100 and 20 dB right, ok.

And, if you do that then this is you know grossly inadequate because, the moment you sample the signal already you committed a crime right which you cannot recover from, alright. Because you are adding some random thermal noise, I mean some random voltage in addition to the voltage you wanted a sample that is what do you call dependent on that capacitor value, is this clear? So, what is the moral of the story?

If you want to resolve or if you want to build a sampler or consequently an analog-to-digital converter with higher and higher resolution right based on sampling the voltage on a switch using you know on a capacity using a switch then, you see that the value of that sampling capacitor better be sufficiently large. So, that the error you make when you sample the input voltage on to that capacitor is way smaller than what you are trying to resolve, right ok. So.