Circuit Analysis for Analog Designers
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Lecture - 33
Noise processed by a linear time-invariant system

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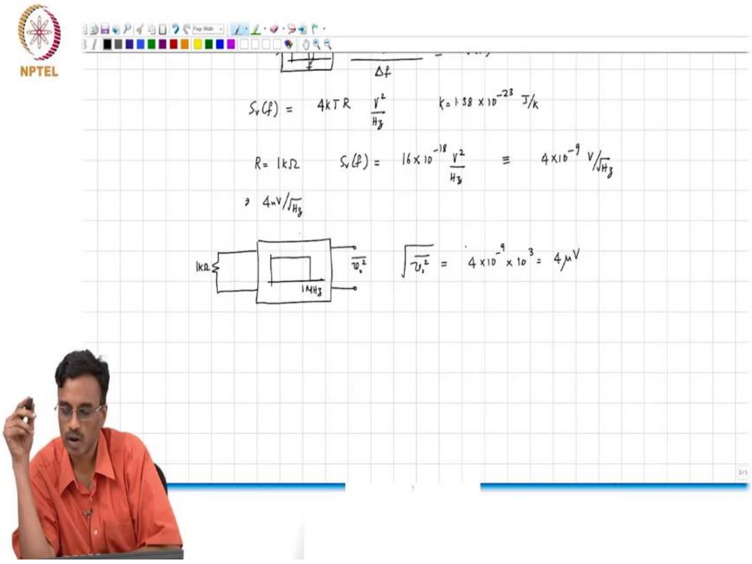

So, as I said this you know 1 kilo ohm and 4 nano volts per root hertz is a good number to keep in mind right and also gives you an idea of you know what these numbers are like ok good, alright.

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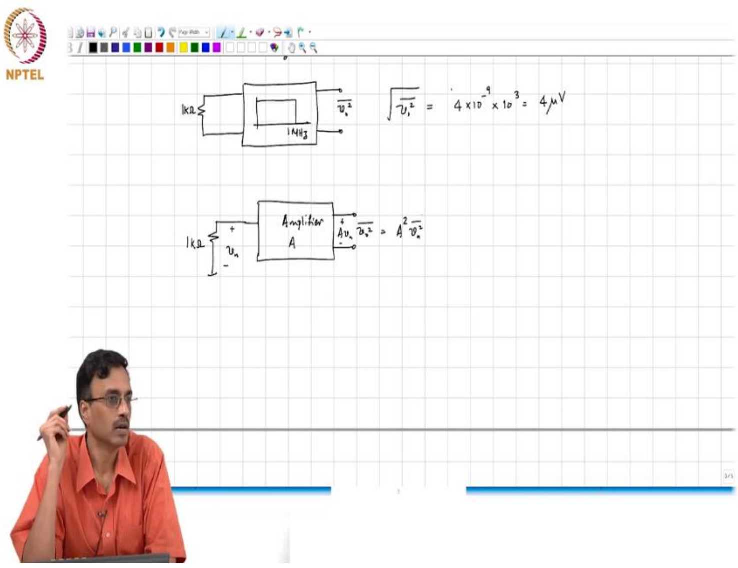

Now let me take give you another let me ask you about. So, let us say this is simply an amplifier with a gain A, correct. What comment can you make about the mean square value of the noise now?

If this voltage is v n, what do you think? The voltage here will be? A times v n.

$$A\ v_n$$

So, what comment can you make about the mean square value of the output? This is simply will be A square times correct, alright. So, that was easy.

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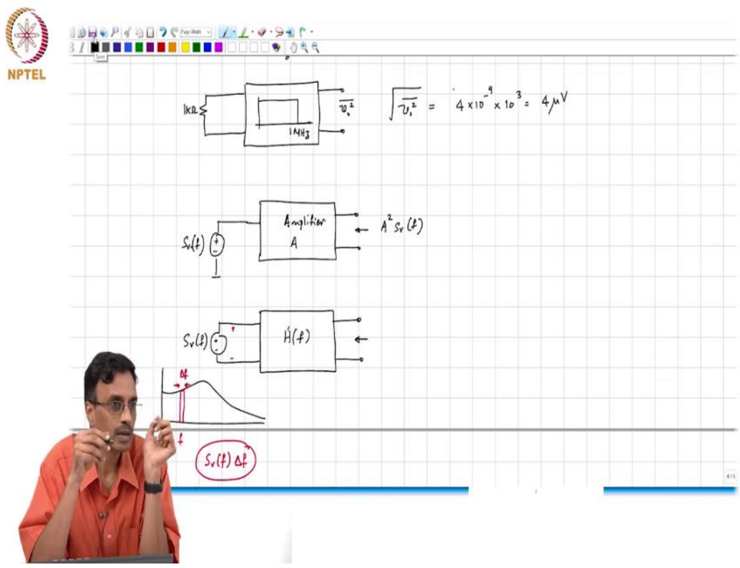

Now, the next thing; if this was a noise voltage source with a spectral density S v n, S v of f, what comment can we make about the spectral density of the noise here? Pardon?

This is simply nothing but A square times the spectral density of the noise input noise. Is enough? Alright, ok.

$$A^2 S_v(f)$$

Now, let us say I have a filter here with a transfer function H of f, I have a noise source. I mean though I keep marking plus and minus right, I mean remember that the mean is 0. So, it does not really matter. S v of f, what comment can we make about the noise spectral density here?

Remember, what is S v of f? It simply let us say that this noise spectral density is doing something like this ok. The noise spectral density remember quantifies at a certain frequency f and if you have a bandwidth of Δf and this is f alright, the mean square noise is what? Here in that bandwidth what is it? Yes. S v of f times Delta f.

$$S_v(f)\Delta f$$

So, that is the mean square noise in that band correct, centered at f. Now, what is the gain of the filter at H at frequency f? Mod H of f, correct. So, this is getting is this spectral density is being passed through an amplifier whose gain is dependent on frequency and at a certain frequency f the gain of that amplifier is H of f. So, what comment can you make about the spectral density at the output or the mean square noise at the output at a frequency f? Do you understand the question?

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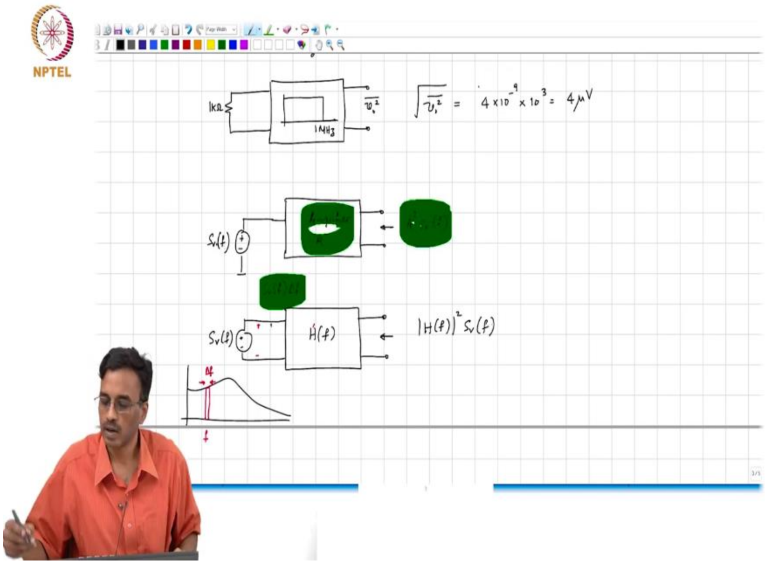

At frequency f the mean square noise here will be S v of f times delta f, ok. And earlier when we had an amplifier whose gain did not change with frequency all of you immediately said that the mean square the spectral density is we say at the output is A square times S v of f. Now, what do we need to do? Well, the gain is changing with frequency. This consists of components at a frequency, f right and narrow bandwidth delta f. The gain corresponding to that frequency is? H of f. So, what comment can you make about the mean square noise of the waveform coming out at frequency f?

Very good, it is simply. So, therefore, the output noise spectral density is simply mod? Mod H of f the whole square times? S v right, it is also very straightforward ok.

$$|H(f)|^2 S_v(f)$$

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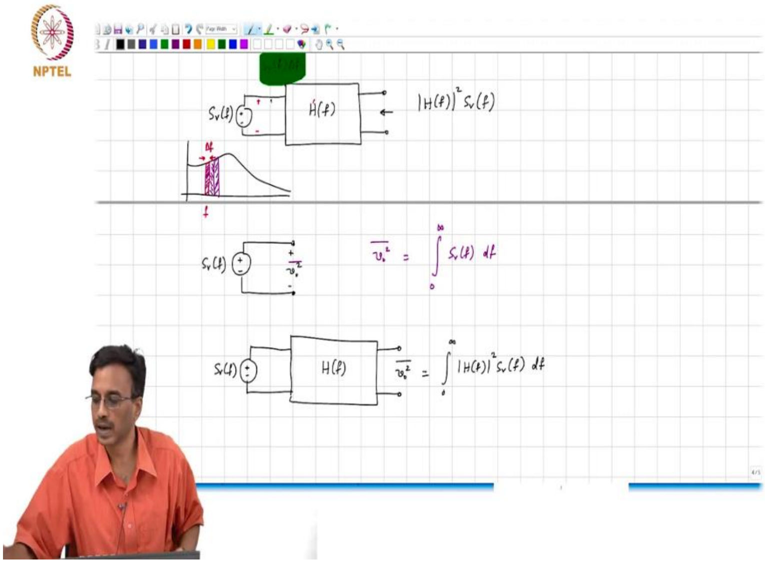

So, this is and what comment can you make about if you have a noise source with spectral density S v of f? What comment can we make about that mean square noise of the noise source? In other words, if I put a filter there whose bandwidth is infinite, what do you expect the mean square noise to be?

Well remember that the mean square noise if you put a band pass filter of bandwidth 1 hertz or delta f hertz is? S v of f times delta f correct. So, if I put a filter with the band; if I do not put a filter at all what would be the total mean square noise?

It is. It is simply nothing but well it is the mean square value of this component plus the mean square value of that guy plus the mean square value of this chap and so on. So, this is nothing but integral S v of f d f. From? 0 to infinity alright, ok.

$$\overline{v_0^2} = \int_0^\infty S_v(f)df$$

So, alright ok, now we just put everything together that we have seen together. So, well let us say you have a noise source with the noise spectral density S v of f, this is H of f, what is the output what is the mean square output noise?

What is the output noise spectral density? Pardon. Mod H of f times S v of f. So, what is the mean square output noise?

Integral 0 to infinity mod H of f the whole square times S v of f. It will make sense people?

$$\overline{v_o^2} = \int_0^\infty |H(f)|^2 S_v(f) df$$

Right? You can see all these I mean you know the random process guys will you know will throw a fit if you know they see derivations like this, but you know if we have to sit and do stochastic processes and do this we I mean you know by the time we build our circuits will be dead right.

So, ok. So, these seem intuitively reasonable ok and this is what we are going to use. This is good enough to for us to make useful calculations like the one we will do right now, ok.

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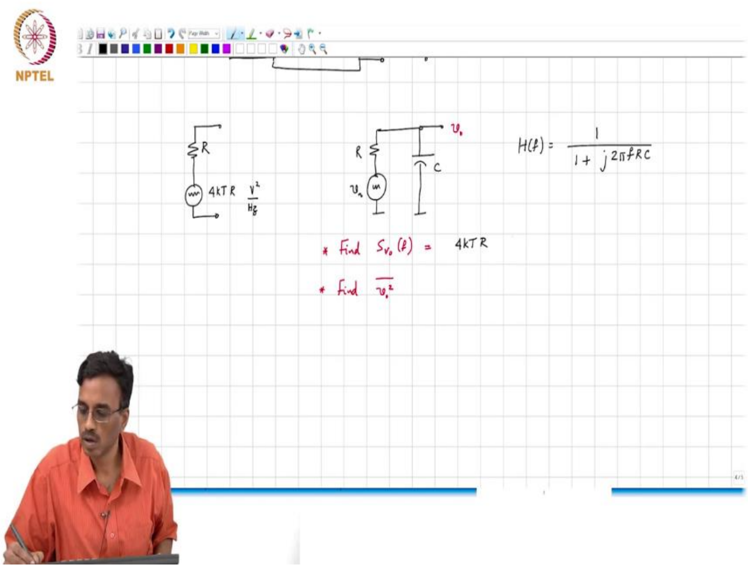

So, let us start applying the stuff that we have seen to real circuits. So, basically every resistor is accompanied by a noise source. The bottom line is that every resistor is accompanied by a noise source whose noise spectral density is? 4 kTR volt square per hertz.

You can think of it as you know as there being a voltage, so I mean there is a voltage source in series with every resistor and the noise spectral density of that voltage source is this. Now, let us try and see if we can make some calculations. So, this is let us take our first simple circuit. This is a resistor and a capacitor in parallel, right.

So, this is v n, this is a voltage source right ok. It just happens to be a noise voltage source. So, this is a voltage source. What comment can you make about the transfer function from the voltage source? We are interested in finding the noise spectral density at the output as well as the total mean square noise at the output.

So, how do we how do you propose that we go and find out the noise spectral density at the output? So, we want to find $S_{v_o}(f)$ and find the mean square noise $\overline{v_o^2}$. So, how do you propose that we find $S_{v_o}(f)$? Well, that voltage v n is there, it I mean you know there is an RC filter. So, what comment can you make about the transfer function from v n to v o?

So, the transfer function H of f is 1 over 1 plus j? 2 pi f R C.

$$H(f) = \frac{1}{1 + j2\pi fRC}$$

So, the output spectral density is the? Mod H of f the whole square times the input spectral density. The input spectral density is nothing but 4 k T R alright times. What should we do?

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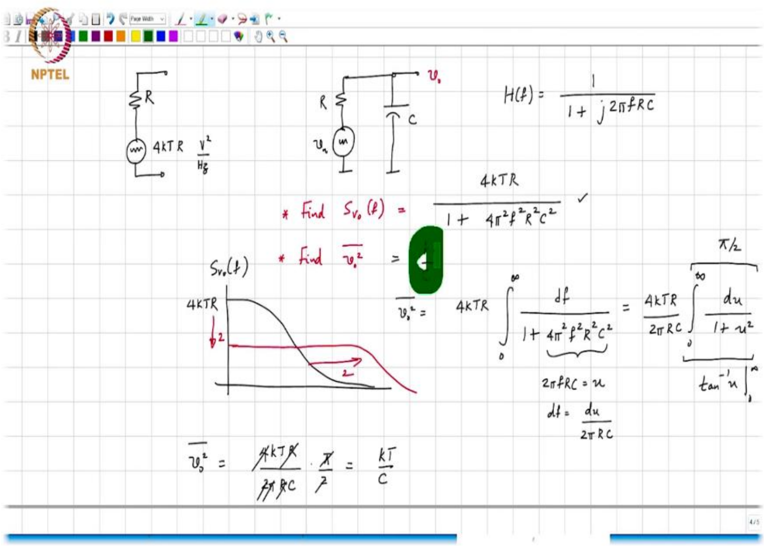

1 over. Divided by 1 plus 4 pi square f square R square C square ok.

$$S_{v_o}(f) = \frac{4kTR}{1 + 4\pi^2 f^2 R^2 C^2}$$

And how does this look? What will be the spectral density be at DC?

Ok. At DC it will be? 4 k T R and why does it make intuitive sense?

At DC the capacitor is open, the transfer function is 1. So, the low frequency output noise spectral density will be the same as that of the input. And why does the shape make sense? Well, it is a low pass filter, right. At high frequencies the capacitors behave like a short. So, the high frequency components of the noise are getting attenuated by the filter and that is what the filter is supposed to do alright. So, that is clear.

Now, the next thing is to find the mean square noise at the output, the total mean square noise at the output. And how will you figure that out?

Well, it is simply 4 k T R integral 0 to infinity 1 over 1 plus 4 pi square f square R square C square df ok and well you use 2 pi f R C as u and df is nothing but du by 2 pi R C. So, this is nothing but 4 k T R over 2 pi R C integral 0 to infinity du by 1 plus u square. So, I do not know how many of you still remember the integral. What is it?

$$\overline{v_o^2} = 4kTR \int_0^{\infty} \frac{df}{1 + 4\pi^2 f^2 R^2 C^2}$$

$$2\pi fRC = u$$

$$df = \frac{du}{2\pi RC}$$

$$\overline{v_o^2} = \frac{4kTR}{2\pi RC} \int_0^{\infty} \frac{du}{1 + u^2}$$

Tan inverse u alright and that has to be evaluated between limits of 0 and infinity which basically is. What is that? Pi by 2. So, the mean square noise after all this is 4 k T R times pi over 2 just like in your JEE or GATE exam all the everything cancels out 2 2 4 R R. What do you get? k T by C alright.

$$\overline{v_o^2} = \frac{4kTR}{2\pi RC}\frac{\pi}{2} = \frac{kT}{C}$$

So, what is this telling us that the mean square noise is all important quantity k T over C; now you know we kind of little bit puzzled and this integral seems pretty complicated right perhaps we made a mistake. How was it possible that the mean square noise is?

I mean, after all the noise is coming from where? You know resistor and, but the total integrated noise has got. No R in it at alright and that would sound to me like you know maybe we made a mistake right, but then I trust your judgment on you know and I trust your competence with integral calculus. So, it appears right. So, k T by C. So, now, we got to figure out well the answer you know turns out to be K T by C, why does this make sense right and they can say what can I say comes out of the math right ok.

Question is there you know some intuition behind the result right. So, as you can see if I reduce the resistor what is happening to the spectral density of the noise produced by the resistor?

Let us say we reduce the resistor by factor of 2, what happens? Well then all the noise added by the resistor is smaller. I mean the spectral density of the noise added by the resistor is small right, but there is another important thing happening. What is that?

Well right, the bandwidth is doing has become wide right. The spectral density has gone down by a factor? Let us say I reduce the resistance by factor of 2. What has happened to the spectral density of the resistor of the noise? This gone down by a factor of 2, but what has happened to the bandwidth? Well, the bandwidth is also gone up by a factor of alright. So, while it is true that reducing the resistor as the beneficial effect of reducing spectral density of the noise source, well you know the total noise is a you know does not change because the bandwidth over which we it has to be it makes a contribution is now larger right.

Now, that does not mean that this factor and that factor exactly cancel. In this case it just so turns out that you know the reduction in spectral density exactly cancels out the increased (Refer Time: 17:36). Does this make sense, right? So, that is the intuition behind this k T by C alright.

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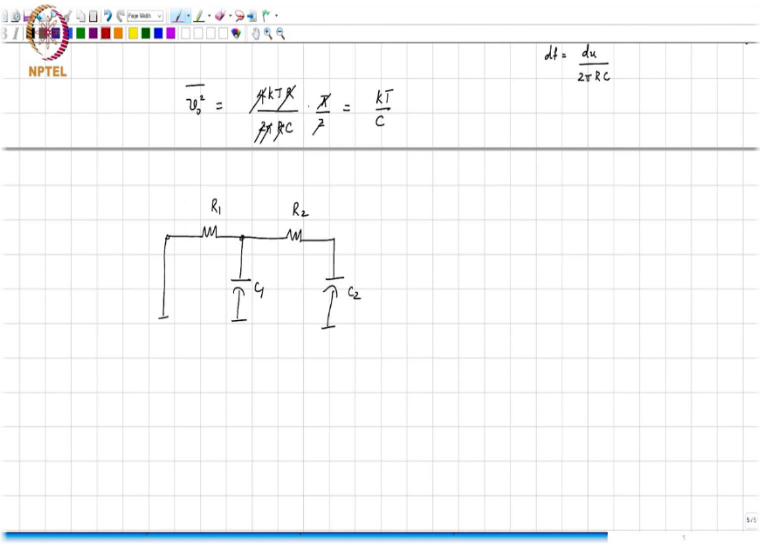

So, now all of you look sufficiently bored how about doing. This is C 1, this is C 2, this is R 1, this is R 2 right and of course, we are not going to do this in the class alright.

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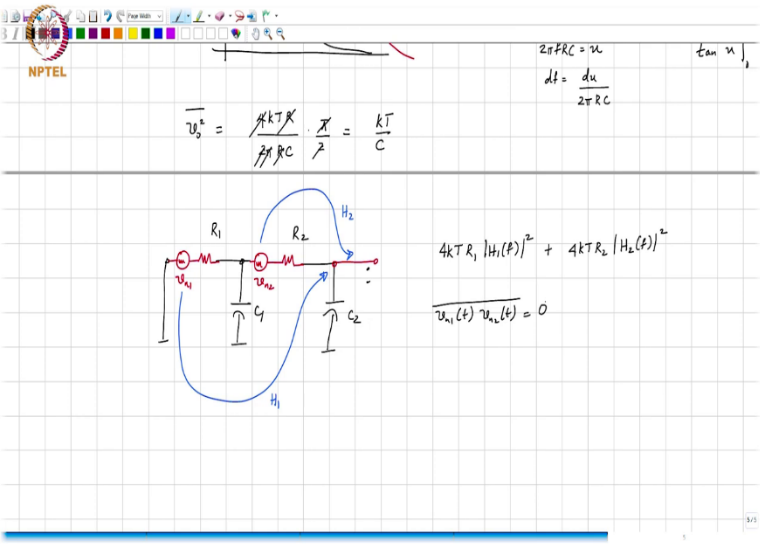

So, ok. So, you have two resistors. So, you have two noise sources and let us say we are interested in finding the mean square noise across I mean delta is finding the spectral density and the mean square noise across C ok. What do you what are we going to do?

Nothing you know fantastically new right. What do we do? We need to find the transfer function from v n 1 to the output. Let us call that H 1, the transfer function from v n 2 to the output let us call that H 2 alright. And so, what will be the total what will be the noise spectral density at the output? What is the noise spectral density at the output, can I get a clear answer?

So, the spectral density of v n 1 is 4 k T R 1 times Mod H 1 of f the whole square plus 4 k T R 2 mod H 2 of f the whole square, right.

$$4kTR_1|H_1(f)|^2 + 4kTR_2|H_2(f)|^2$$

We are adding powers ok that is only true if the cross terms right are; I mean see v n 1 will result in some output noise here, v n 2 will result in some other output noise waveform ok and what we are saying is that the mean square noise of the sum is the same as the sum of the mean square noise of the individual components that is only true if average value of v n 1 of t times v n 2 of t this average is ok. And that it turns out that is indeed true.

$$\overline{v_{n_1}(t)v_{n_2}(t)} = 0$$

Physically it is because well this noise is because of random movement of carriers in the resistor in the conductor right. There is no way carriers and one conductor can know what is happening in the other container right.

So, it is like saying you know if you have a whole bunch of classrooms right, every class is noisy right, but the noise in one class is completely unrelated to the noise that is happening in another class because different people are sitting there you know hopefully yacking about different subjects, right. So, what you hear is basically no correlation between you know that and these are independent ok.

So, that is very convenient because all that we need to do is we need to find the transfer function from these noise sources to the output I mean yeah you know you are already exposed to doing this because you can write the MNA matrices and boom correct and then once you find the transfer functions you basically find the modulus square.

Then you know you integrate and then right. So, if you have 400 resistors and you know and you know 300 capacitors what will you do? Well, you know the same thing you know 400 times correct ok. So, you need to find. So, this is where you know if you want to find the mean square noise at the output of the network you can see now that you need to find transfer functions from Multiple input sources to the Same output right and this as you know I mean if you could do this by super position right and you know that is as we have seen it takes a very long time simply because you are doing the same old calculations over and over again. So, what you know what is now that you know everything about reciprocity and red joints what do you think you will do? Why do you think those concepts are useful? To you find H1 and H 2 and H 300 you do not need to do it one source at a time, you just excite the output with the current source and find, evaluate the whole network just once and find the currents through all the individual voltages right ok.