Circuit Analysis for Analog Designers
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Lecture - 32
Introduction to noise in electrical networks

Basically, the next topic that motivates our next topic, which is you know we need to understand how you know we know that the individual elements namely the resistors and transistors etcetera are noisy. We need to understand when we put all these elements together what happens to the output noise of a network ok. So, that is the topic that we will study next. Once we study noise, we will be in a position to come back to filters for a brief while and look at what the lowest.

What do you call signal you can put into the filter will be, in order to achieve a certain accepted signal to noise ratio at the output of the filter.

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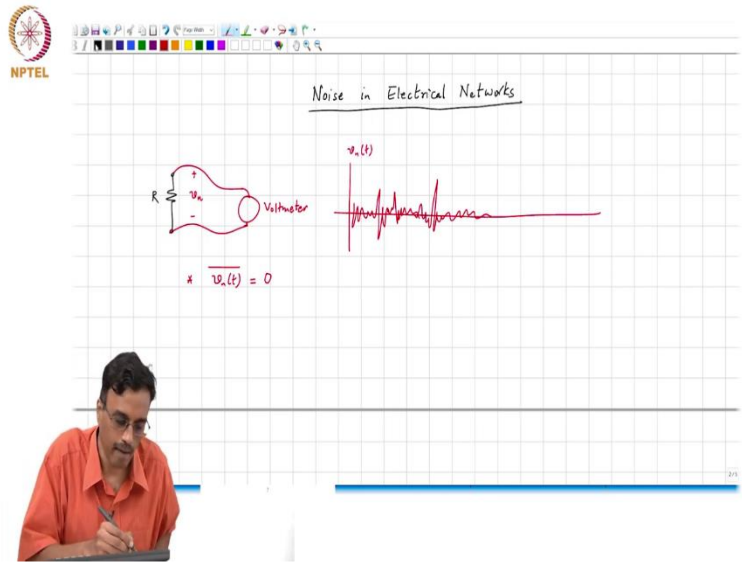

So, alright how many of you have looked at, I mean have studied noise in earlier, which course?

And what do you call, how many of you have looked at, have been introduced to noise from a communication class point of view? 1, 2 ok; the BTechs have not seen noise at all ok. So, for those of you who have seen it in a communication class right you know we are not going to go into long theorem proof type results here. Our aim is you know given a circuit can I predict how much noise there is at its output and what the characteristics of that noise are.

So, the you know and again the idea is to work with practical circuits and be able to understand how to calculate noise, what to do to make to get better noise performance and that sort of thing. So, I mean to get to this stage in communications you would go through several classes of what a random process is, you know what ergodicity is and you know what auto correlation is and then you know Wiener-Khinchin theorem and all that stuff ok.

It is a before you come to you know the calculations that we do right, we are going to skip all that right and you're all that I am interested in this course is to give you a minimum bare bones tool set needed to be able to make useful noise calculations in circuits alright.

So, first I am going to mention some facts of life, there is nothing we can do about these things alright. So, it turns out that you know if you had a resistor in thermal equilibrium with its surroundings with an absolute temperature t, it turns out that if you put a fictitious volt metre across the resistor right, it turns out that the voltage is not you would ideally think that the voltage should be 0 right. No current is flowing through the resistor after all.

So, you would think the voltage is 0, but it turns out that you will see a very you know you will see a random wave form, it is almost impossible for me to draw the draw a random wave form, alright. So, but you will see some random wave form ok. Without knowing anything else what comment can we make about the mean of this random wave form? 0, why I mean why is it, why should be, how did you kind of say 0 all in unison? I cannot hear you.

Power? What conservation? Where?

Well, there is a voltage so there is definitely power.

Yeah well, I mean the easiest way to understand this is if the average was not 0, then we would be able to I mean there is no need for us to sit here, we could all be making power and then selling it, correct ok. So, you know sounds too well to be true and remember in life anything that sounds too good to be true is, is always too good to be true right. And therefore, the average better be 0, no matter you know what else is true the average better be 0. So, first fact of life the voltage across the resistor v n of t on average is 0 alright.

$$\overline{v_n(t)} = 0$$

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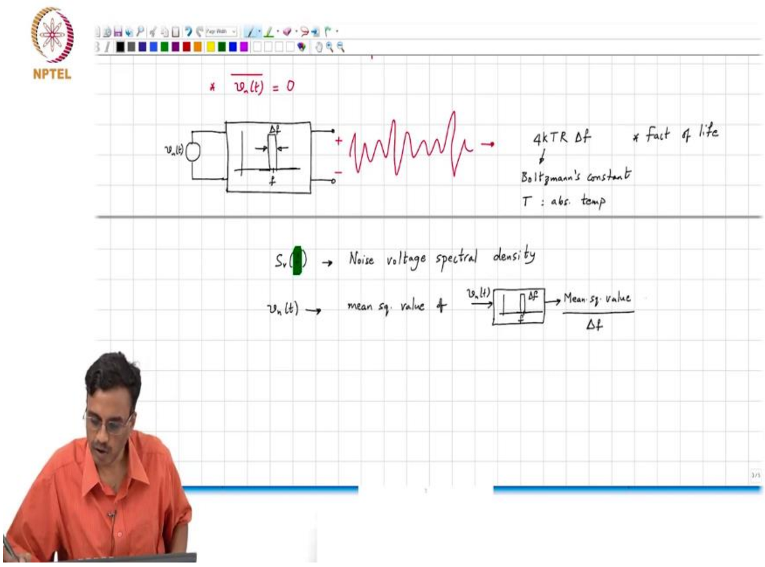

The next fact of life is this is $v_n(t)$, the next fact of life is that if you take this $v_n(t)$ you recorded that $v_n(t)$ right pass this through a bandpass filter, an ideal brick wall band pass filter centred at a frequency f right. And had has a bandwidth of a small bandwidth Δf right. What do you think the bandpass filter will do? This is an ideal band pass filter, perfect brick wall with a bandwidth of Δf and centred at f. What will the bandpass filter do?

It will? It will remove all; I mean this is a random waveform this is some wave form. So, we did not treat as some spectrum correct. You pass take this waveform and then pass this through a bandpass filter, the bandpass filter is only going to pick up those components which are centred at f and in the neighbourhood of Δf around f ok.

So, if you look at this wave form on a scope what do you think this will look like? It will look like a?

I mean it; I mean it cannot look like a sine wave of f, of frequency f; I mean if you have to get the I mean it cannot be a sine, a sine wave with frequency f, because if the output is a sine wave, pure sine wave then this is a linear time invariant filter. So, the input must also be a must be a sine wave right, but the input is kind of randomly varying. So, what does it, because it is a bandpass filter the output will look like a sinusoid, but its because its being driven by a random input, the magnitude and phase of that sinusoid the amplitude and phase of the sinusoid will keep varying. So, you will basically say see a sinusoid like waveform whose amplitude and phase are varying alright.

So, you can measure the mean square value of this waveform, this is a wave form it is a free country. So, I can nothing is preventing me from measuring the mean square value of this wave form right. And if you do, if you so make the measurement, it turns out that mean square value will be 4 kTR times delta f.

$$4kTR\,\Delta f$$

Again, this is what do you call a fact of life ok, and those of you are interested in figuring out why this is so, feel free to go and it is got to do with you know quantum mechanics and black body radiation all that fun stuff right ok.

So, what is the you know kind of strange about this formula? I mean do you find something strange at all? By the way 4 is 4, k is Boltzmann's constant, T is absolute temperature, R of course is the resistance value and Δf is the bandwidth to the filter. So, what do you find you know as somewhat puzzling about this?

It does not have f at all. So, basically this is telling you that no matter where the centre frequency of the bandpass filter is the output mean square noise always happens to be 4 kTR times Δf alright. So, the 4 what we now define what is called the noise voltage spectral density and as I said those of you have not done noise before there is no need to get intimidated right ok. All that this is saying is that if I take this noise source, pass it remember this noise voltage spectral density is a function of f the frequency at which you are making the measurement right.

This is basically saying mean square value of the wave form of wave form that appears when you take v n of t pass it through a bandpass filter of bandwidth Δf and a centre frequency f and find the mean square value here right. And divide this by; well, if I make the filter you know bandwidth narrower what would you expect for the mean square value at the output?

What do you expect? If I make Δf - 0 what do you expect at the output?

You get nothing correct. So, clearly the mean square value must be dependent on the bandwidth of the bandpass filter. So, the mean square value divided by delta f ok, is some measure of how much power there is in that wave form at that frequency right. So, this is what is called a the mean square value I mean so the mean square value divided by delta f is basically the this is the voltage noise spectral density as a function of f.

$$\frac{Mean\ square\ value}{\Delta f} = S_v(f)$$

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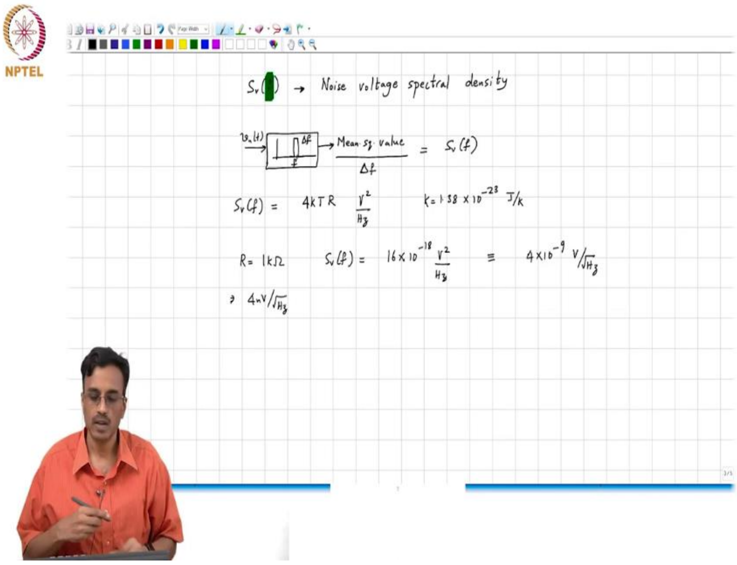

In English all that it means is that, if you take a voltage, I mean a noise voltage with this spectral density, you pass it through a bandpass filter centred at f and a bandwidth Δf, you should expect that the mean square value of the voltage at the output of the bandpass filter is simply $S_v(f)$ times Δf. That is all that there is student ok.

Now, so, now, that we have learned some jargon let us try to apply it. So, what comment can we make about the noise voltage spectral density of a resistor? Regardless of where you put the bandpass filter we seem to be measuring a mean square value of 4 kTR times Δf right. So, this basically is 4 kTR and what will be the, what are the dimensions of this what are the units of this?

Volt square per hertz right. Remember that k is, what is Boltzmann's constant? 1.38 into 10 power Minus 23 right and unit - Joules per kelvin alright. Is this a, is this at room temperature or at some high temperature.

$$S_v(f) = 4kTR\ \frac{V^2}{Hz}$$

$$k = 1.38 \times 10^{-23}\ {}^{J}/_{K}$$

I mean it is a constant right. I mean constant is supposed to be constant regardless of temperature right, that is why it is called constant alright. So, so and to put some numbers so that you get a feel for this, it turns out that if you put R equal to 1 kΩ what comment can you make about $S_v(f)$? Please do the math yes.

16 into 18 volt square per hertz right

$$S_v(f) = 16 \times 10^{-18} \frac{V^2}{Hz}$$

And this is often also expressed the square root of this is also expressed in what is called nano volts per I mean volts per root hertz. So, this as you can see is 4 into 10 power 9 volts per root hertz ok.

$$S_v(f) = 16 \times 10^{-18} \frac{V^2}{Hz} \equiv 4 \times 10^{-9} \frac{V}{\sqrt{Hz}}$$

So, a good, a quick number to remember is that a 1 kiloohm resistor has a noise spectral density of noise voltage spectral density of 4 nano volt per root hertz ok.

$$\Rightarrow 4 \frac{nV}{\sqrt{Hz}}$$

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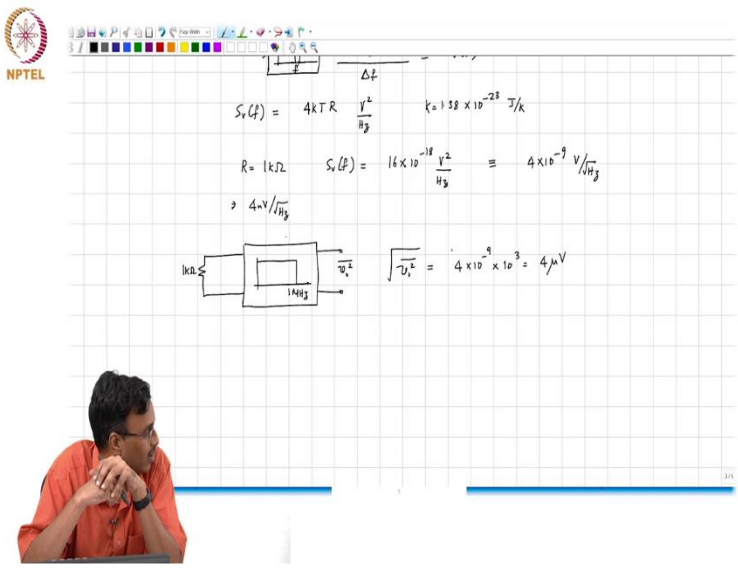

So, since we are still in early stages, let us you know kind of see how to make some calculation. So, let us say I take the voltage waveform across a 1 kiloohm resistor and I have a low pass filter of 1 megahertz bandwidth. How much, what is the mean square value of the voltage that you see there? How do we do the math? We know that the spectral density is 4 nano volts per root hertz, the band width is the RMS value at the output will be simply 4 nano volt per root hertz times? 10 power 3 right which is 4 micro volts.

$$RMS = \sqrt{\overline{v^2}} = 4 \times 10^{-9} \times 10^3 = 4\mu V$$

So, the RMS noise of a 1 kilo ohm resistor in a 1 megahertz bandwidth is 4 micro volts, right ok. So, this basically prompts the next question, if I make it if I make the megahertz a gigahertz what will happen? We can multiply by 30 right if I make the gigahertz a terahertz what happens?

Pardon.

4 volts. Yeah, so if I make the tera terahertz you know 10 billion terahertz, what will happen? If I make it infinity what will happen?

It will be.

I mean the formula is telling us that it will be infinite correct. So, what is your comment on that, what do you think? I mean I do not have to measure it right, if I mean this is just telling you that if I just if I do not have the filter at all.

No, I do not have a filter at all I just have a resistor that is all. It is telling me, this math is telling me that the RMS value can go to infinity right that basically means that you know air around it should have broken down and you know basically we would be seeing sparks everywhere, that clearly does not happen. So, it must be something wrong with this right. So, that is what is called the ultraviolet catastrophe and like you know.

So, this all as I said all comes back eventually to black body radiation and I do not know if you still remember your high school physics. But you know this the h nu and you know by e to the h nu by kT minus 1 and so on. I do not know if you remember black body radiation spectrum right, it goes up and then it, kind of eventually falls off. But it turns out that the as far as circuits are concerned, we are always working with frequencies which are at best few 10s of gigahertz and up to those frequencies it is as good as being constant right.

At ultraviolet frequencies; obviously, the spectral density is not constant and beyond that it actually falls off. So, the integral is actually finite. So, do not go right away and then start you know open a start-up saying we will we will generate energy right by using resistors ok; alright.