

**Circuit Analysis for Analog Designers**  
**Prof. Shanthi Pavan**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Madras**

**Lecture - 31**  
**Transconductance- Capacitance integrators**

Few miscellaneous things that I wanted to mention regarding filters; so, if you recall so far, we have looked at you know basically trying to realize a second order filter using op amps resistors and capacitors and the motivation to use active elements has been to remove or eliminate the use of inductors ok.

For one inductor are big and bulky the lossy the you know whole bunch of things. So, the op amp and you know the associated circuitry basically is trying to emulate the function of an inductor in the sense that it is accomplishing integration just like an inductor and I you know.

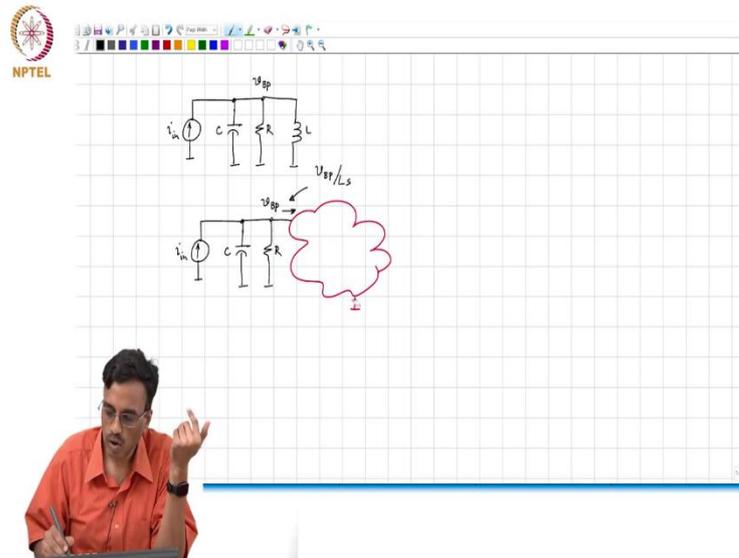
So, as far as the circuit is concerned it you know it does not know whether this is a real inductor or you know a synthetic one this is you know within quotes an artificial inductor. Using an op-amp and you know R and C are not the only ways of realizing or emulating an inductor using active elements there is also other ways of doing it and you know I will discuss one such way today.

(Refer Slide Time: 01:41)

The slide displays a circuit diagram on a grid background. The diagram shows an input current source  $i_{in}$  connected to a parallel combination of three branches: a capacitor  $C$ , a resistor  $R$ , and an inductor  $L$ . The output current is labeled  $i$ . The NPTEL logo is visible in the top left corner of the slide area. In the bottom left corner, there is a small video inset showing a man in an orange shirt, presumably the lecturer, speaking and gesturing with his hands.

And with that we will wind up our discussion on active filters the if you remember we started off with our quest for realizing a filter using  $i_{in}$  and this R L C and the low pass output basically corresponded to the inductor current  $i$  right. And then yeah and then we went about trying to realize inductor using op amps R and C yet another way exists and if we do not want to use the inductor here right.

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So, let us say let us call this voltage  $v$  ok and what volt I mean what is that transfer function from  $i$  into  $v$ . It is second order band pass right. So, you know let us call that  $v_{BP}$  and what we want to do is be able to realize the same functionality without having to use the inductor. In other words, we are going to remove this guy and we are going to put in some black box and rather a red box here ok.

And what is the whatever we do this red box must whatever magic it does what comment can you make about the current drawn by the red box. If the red box is trying to do the same thing that an inductor would do, what comment can we make about the current that is pulled by that magic box. No, it is low pass, but what is it in terms of  $v_{BP}$ ?

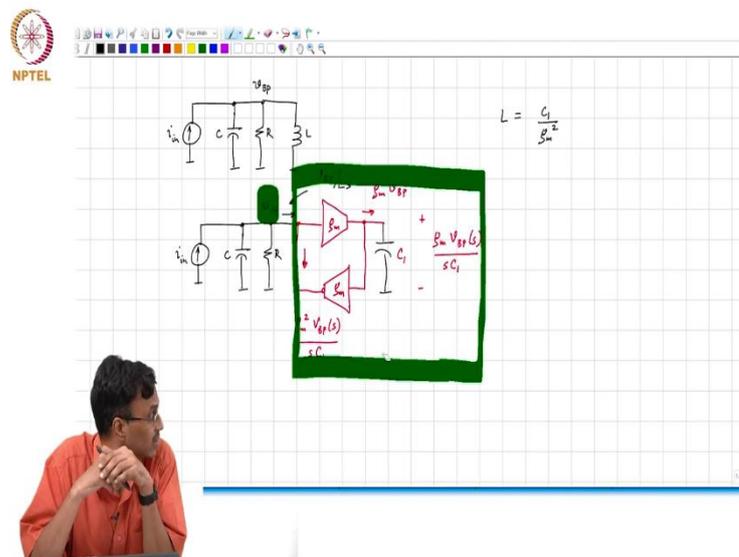
It is nothing but this current better be  $v_{BP}$  over  $sL$  correct.

$$\frac{v_{BP}}{Ls}$$

So, in other words you have to draw a current which is the integral of which is proportional to the integral of  $v_{BP}$  right. I mean the one way of doing that would be to simply put an inductor there, but the whole idea was to avoid the inductor in the first place right. The only other inductor integrator that I mean so, the since we need integration the only other element, we know that can integrate is a capacitor and as usual you know unfortunately a capacitor; however, integrates. A capacitor integrates current and it generates a voltage whereas, here what do we need?

We need to integrate a voltage and generate a Current. So, what do we do?

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Well, what do you I mean you can basically our capacitor here can only integrate current alright, but we need we needed to, but we need to integrate voltage. So, what we do we can convert the voltage we have into a current, correct? So, that the capacitor can integrate it and the capacitor will generate a voltage, but we need a current. So, we can convert the capacitor voltage back into a current that is the basic idea. So, we need to convert this voltage  $v_{BP}$  into a current. So, what kind of controlled source would you use we have a voltage we want a current. So, what kind of controlled source would we? A voltage controlled current source is what you can use.

So, for example, let us call that  $g_m$  and this current is nothing but  $g_m \times v_{BP}$  correct and if this capacitor is if I call this  $C$  if I call this  $C_1$  for instance what comment can you make about the voltage here.

It is nothing but  $g_m v_{BP}$  of  $S$  by  $SC_1$  right and as we expect the voltage across the capacitor is indeed an integral of the input voltage  $v_{BP}$  correct alright,

$$\frac{g_m v_{BP}(s)}{sC_1}$$

But what do we need to do? We need to pull out a current, I mean what is the it is a voltage I mean what we have in red is a voltage controlled current source. So, what is the current being drawn from  $v_{BP}$  at this point in time. No current is being drawn right, but what current must we draw?

What current must we must we draw?  $v_{BP}$  over  $SL$  right. So, what do you think we should do? Do we have  $v_{BP}$  by  $S$  somewhere, where? Voltage across the capacitor is has the same form except that it is a voltage we need a current. So, what comment can you make now? You need a voltage controlled current source, what voltage is that voltage controlled current source supposed to sense? It senses the capacitor voltage right let us say this is also  $g_m$  correct ok. And what should I do to the output of the  $g_m$ ?

What is the I mean what am I supposed to do am I supposed to push current or draw a current? Draw a current. So, this must be an inverting  $g_m$  and therefore, must look like alright. So, what is this current therefore?  $g_m$  square  $v_{BP}$  of  $S$  by  $SC_1$  alright

$$\frac{g_m^2 V_{BP}(s)}{sC_1}$$

And so therefore, what effective inductance is this. So, this what do you call this box here is effectively a contraption that mimics the functionality of an inductor and what is the value of inductance that it achieves?

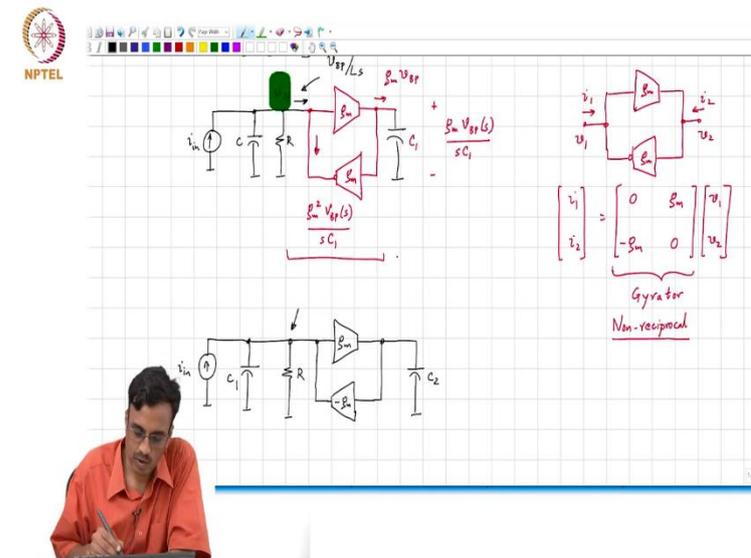
Well, we wanted  $v_{BP}$  over  $sL$  and what we are getting is  $g_m$  square  $v_{BP}$  by  $SC_1$ . So, the inductance  $L$  is nothing but  $C_1$  over  $g_m$  square right ok.

$$L = \frac{C_1}{g_m^2}$$

And all that this box is doing is I mean the contraption in red all that it is doing is taking the capacitor voltage and making it look like a current and taking the input voltage and making it look like the capacitor current. So, this way it is simply interchanging the roles

of voltage and current ok, which is exactly what the differences between a capacitor and inductor. What is the difference between a capacitor and inductor? The only difference is that well you know both of them are integrators one integrates voltage and generates a current the other integrates a current and generates Voltage

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And this guy here our friend here all that he is doing is simply interchanging the roles of voltage and current across the two ports alright and this this has got. So, this element I mean these two ports by itself has got a special name. So, if you call this  $v_1$  and  $v_2$  and this is  $i_1$  and  $i_2$  how can we write the y matrix of this guy i 1 i 2 is yes people.

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & g_m \\ -g_m & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

The first row. Think carefully. The first row and what is the second row right flipping the signs of both of them will also do just as fun correct and. So, this is a special element or a special two port this is called the gyrator and the gyrator basically is interchanging the roles of voltage and current at one port you know if you look at the other port basically you know port one's voltage looks like port twos current and vice versa ok.

So, within you know the appropriate scaling factors with the right dimensions ok. Do you think the gyrator is reciprocal or non reciprocal and the sun rises in the east?

So, yeah. So, why is there so much hesitation? Is it reciprocal is not reciprocal? What is the necessity if the matrix has to be reciprocal if the bi matrix has to be reciprocal what should you have? Why?

Yes Vaibhav. Yeah. So,  $y_{12}$  must be equal to  $y_{21}$  right, what do we see here?

Well apart from the negative sign everything seems ok, but you know what is not equal is not equal right ok. Is not like your exams where you say I missed the minus sign and then you know it is the same thing you understand. So, this is a non reciprocal element make sense alright. So, now, let us alright. So, let us continue with us. So, this is R, I am going to call this  $C_1$  and I am going to call this  $C_2$  this voltage is basically yeah you need a voltage control current source here.

(Refer Slide Time: 14:18)

Gyrator  
Non-reciprocal

Transductor-Capacitor  
or Gm-C filters

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{g_{m_1} g_{m_2}}$$

$$\frac{V_{LP}(s)}{V_i(s)} = \frac{1}{LC}$$

This is  $v_i$  let us call this  $g_{m_1}$ ,  $g_{m_2}$ ,  $g_{m_3}$  ok and well you yeah you already know that this voltage is a band pass transfer function right. What comment can you make about this transfer function? Why? That volt I mean the voltage there is analogous to the inductor current. So, this is  $v_{LP}$  alright ok and you know that a resistor can be realized using a voltage controlled current source, what do you think you can do? How can you use a voltage controlled current source to realize a resistor.

Connect the output to the input  $g_{m_4}$  let me call that minus ok and this therefore, represents a second order like what realized only using voltage controlled current sources and these

gms are often called transconductors and. So, these filters basically are what are called transconductor capacitor filters which is yet another way of which is yet another way of realizing you know a second order filter transfer function without using inductors explicitly alright are also called Gm-C filters ok. Now, what is the low pass transfer function? What is the omega naught? It is very easy to do if you know if you go understand the correspondence between the LC network and this one.

Pardon. No, that is not right.

Yes Karthi, what do you think it is? gm. What gm there is gm1 gm2 gm3 gm4, which gm?

Square root gm2 gm3 by Square root C1 C2 and why does that make sense? Remember that omega naught is 1 by square root LC correct and what is L?

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Square root of L, what is the inductance that is emulated? Yes C2 over gm2 gm3 very good and the capacitance is evidently C1.

(Refer Slide Time: 18:27)

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Gyrator  
Non-reciprocal

Transconductor-Capacitor  
or Gm-C filters

$$\omega_0 = \sqrt{\frac{g_{m1} g_{m2}}{C_1 C_2}} = \frac{1}{\sqrt{\frac{C_2}{g_{m1} g_{m2}} \cdot C_1}}$$

$$Q = \frac{C_1 g_{m1} g_{m2}}{C_2 g_{m2}}$$

$\frac{V_{lp}(s)}{V_i(s)} =$  \_\_\_\_\_

So, 1 over square root LC is basically square root of gm2 gm3 over C1 C2 ok.

$$\omega_0 = \frac{1}{\sqrt{\frac{C_2}{g_{m_2}g_{m_3}} C_1}} = \sqrt{\frac{g_{m_2}g_{m_3}}{C_1C_2}}$$

what about the Q? Again, it is the easiest thing to do is to simply imagine the correspondence with the L C network.

Pardon. C. C 1 g m 2 g m 3 divided by? g m. 4 it is correct well it does not seem dimensionally correct Q must be dimensionless, correct?

Yes. So, (Refer Time: 19:42) what is the formula?

(Refer Slide Time: 19:47)

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Gyrator  
Non-reciprocal

Transconductor-Capacitor  
or gm-C filters

$$\omega_0 = \sqrt{\frac{g_{m_2}g_{m_3}}{C_1C_2}} = \frac{1}{\sqrt{\frac{C_2}{g_{m_2}g_{m_3}} C_1}}$$

$$Q = \frac{1}{g_{m_4}} \sqrt{\frac{C_1 g_{m_2} g_{m_3}}{C_2}}$$

$$\sqrt{L/C} = \sqrt{\frac{C_2}{g_{m_2}g_{m_3}C_1}}$$

1 by g m 4 Square root C 1 g m 2 g m 3 by C 2 right this seems at least dimensionally consistent is this correct sanity check is what?

$$Q = \frac{1}{g_{m_4}} \sqrt{\frac{C_1 g_{m_2} g_{m_3}}{C_2}}$$

When g m 4 tends to 0 what should you expect the quality factor to become?

When g m 4 becomes 0 it is like having an L C network in parallel L and C in parallel without any damping resistance and therefore, you would expect the Q to become infinite so, that makes sense. And the quality factor remember is for a parallel R L C tank is nothing

but R by square root L by C and. So, R is evidently 1 by g m 4 and square root of L by C is nothing but square root of C 2 over g m 2 g m 3 divided by C 1.

$$\sqrt{\frac{L}{C}} = \sqrt{\frac{C_2}{g_{m_2} g_{m_3} C_1}}$$

And therefore, if you go and do the math you basically you will get.

(Refer Slide Time: 21:07)

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Gyrator  
Non-reciprocal

Trans-conductor-Capacitor  
or Gm-C filters

$$\omega_0 = \sqrt{\frac{g_{m_1} g_{m_2}}{C_1 C_2}} = \frac{1}{\sqrt{\frac{C_2}{C_1} \cdot \frac{C_1}{g_{m_1} g_{m_2}}}}$$

$$Q = \frac{1}{g_{m_2}} \sqrt{\frac{C_1 g_{m_1} g_{m_2}}{C_2}}$$

$$\frac{V_{op}(s)}{V_i(s)} = \frac{(g_{m_1}/g_{m_2})}{\frac{s^2}{\omega_0^2} + \frac{s}{M_1 R} + 1}$$

So, this is the quality factor. So, we have s square over omega naught square plus S over omega naught Q plus 1 and the question now is, what is the DC gain? How would we do this? Pardon.

Open all capacitor. So, let us say you apply a DC voltage we this is we already know that this is a band pass node. So, what is the DC voltage here? What is the DC current?

v i times? g m 1 ok the volt, what is the voltage at this point? At d c.

What is the DC voltage there? It is a bandpass node by definition what does it mean? That voltage is 0 correct. So, if that voltage is 0 what is the current flowing through this branch?

That current is 0, this current is 0. So, where is all this current flowing?

What is the current flowing through the output of  $g_{m3}$ ? This current is  $v_i$  times  $g_{m1}$  correct, if that is  $v_i$  times  $g_{m1}$  what must the voltage be here so that the output current is  $v_i$  times  $g_{m3}$ ? So, this is  $v_i g_{m1}$  over  $g_{m3}$ . So, what is the DC gain?

$g_{m1}$  over  $g_{m3}$ , does it make sense people?

$$\frac{V_{LP}(s)}{V_i(s)} = \frac{g_{m1}/g_{m3}}{\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1}$$

And likewise, you can go and calculate the band pass transfer function a couple of observations that I would like to draw your attention to. When we talked about the active RC biquad what comment did we make about its input impedance how did they?

It was a resistor correct it was the resistor going to the virtual ground of the integrator, what is the output impedance?

Pardon, where were we sensing the output where is the output of the active RC biquad. At the op-amp output right and if it is a the output impedance is therefore, 0. So, it did not matter that the input impedance was not infinite because the output impedance was 0 and this way we could cascade by quadratic sections without loading affecting the transfer function right. Now, here what do you think our situation is?

What is the output impedance? Pardon I mean is it 0 or nonzero that is all I am looking for. It is non zero right it is if you actually look at it carefully it turns out to be a C in parallel with an L and R series, but it is not 0 what comment can we make about the input impedance however.

Input impedance is infinite. So, what comment can we make about cascading these biquads is that feasible or it is not feasible?

Yes no? It is feasible because while it is true that the output impedance is not 0 right. The input impedance of the structure is infinite. So, if you copy and paste and connect the next section is not going to load the previous section because it is not drawing any current and therefore, the  $g_m C$  biquads can be cascaded just like active RC 1s alright that was that is. So, you know one thing to bear in mind.

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$$V_o(s) = \frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1$$

$$Q = \frac{1}{\beta_0} \sqrt{\frac{C_1 \beta_0}{C_2}}$$

Low speed

Open loop integrator

The next thing the obvious question that should arise is the following now what is you know what is the what is an obvious question that now arise is you have seen two ways of making filters. So, what is the obvious question?

And when would you tend you know why is why do we study too right if one of them is better then you know why bother studying the other correct right and it. So, turns out that ok. So, remember when we talked about the active R C biquad we said that I mean the active R C integrator we said that the nice thing about the active R C structure is that it is insensitive to parasitics correct what comment can we make regarding about that now.

Well, if there is a parasitic at this node what happens to what is the unity gain frequency of this integrator? It is  $g_m$  by  $C$ . So, it now it becomes  $g_m$  over  $C$  plus  $C_p$  alright

$$\frac{G_m}{C + C_p}$$

And the problem with  $C_p$  is that its often you know not very well predictable ok and therefore, the unity gain frequency of the integrator can is not defined robustly which means that the filter corner frequency and the filter shape is also not you know that well defined ok.

So, if the  $g_m C$  is got a problem, I mean has got this problem, then the obvious question is why bother with  $g_m C$  filters at all, right and it turns out that remember the all the nice

things about the op-amp RC filter are predicated on the fact that we have an op-amp which behaves like an ideal one right if the virtual ground I mean if this node is not really close to 0 then all the benefits of active RC you know of an active RC integrator go for a toss. So, that is an issue. So, what we do therefore, is you know if the frequencies are such that you can design a very good op amp and how do we know what how good an op amp we need we discussed that yesterday.

We have to choose the unity gain bandwidth to be substantially higher than two times  $\omega_n$  times Q alright.

So, the op amp you know better be you know really fast and evidently technology the process technology with which you make the filter basically it places a limit on how fast your op amp can be right and the fastest the best filter or the highest frequency filter you can make is the highest frequency op amp you can make divided by two times  $\omega_n$  times Q divided by another large number right.

We wanted remember that  $2 \omega_n$  times Q of the filter that you are trying to realize divided by the unity gain bandwidth of the op amp must be a number which is much smaller than 1 right ok. So, the so traditionally op amp RC filters were within quotes for low-speed applications right, but you know what the meaning of speed is you know changes pretty fast. So, you know 30 years ago, 40 years ago you could barely make audio filters with active RC with op amp RC techniques.

Now, that audio has become you know several hundreds of mega hertz right if you are on IC and the transistors are blazing fast then you can make a blazing fast op amp which means you can make a very high frequency filter. The gm on the other hand it turns out that it is you know a voltage controlled current source what is the simplest voltage control current source you know a single transistor in principle can act like a voltage controlled Current source;

So, there is no complicated feedback nothing none of that stuff is involved. So, this is an open loop structure right. So, these filters can be can achieve frequency responses that are much higher in bandwidth right, but because this is open loop you know the linearity of it turns out that the linearity of the voltage to current conversion is most of the time significantly inferior to the linearity of this integrator ok.

So, you know if you want a high frequency filter with and you can tolerate some non-linearity then these are the this is the only choice right. If you want to design if you want a very a very highly linear filter, but the frequency of operation is somewhat low then you make this choice right if you want to make a very high frequency filter and have extremely high linearity you sign up to do a PhD right ok. So, so that is the trade I mean that is the trade off between both of these yes Jishnu.

Yeah, but an op amp needs you know several trans conductors to make it work correct a two-stage op amp for example, has at least two stages of gain and all the associated parasitic right. Whereas at least in principle the g m is simply a single transistor the output parasitic capacitors of the transistor do not bother you because they could be part of the integrating capacitance right.

Because those parasitic appear in parallel with C I mean it is not robust, but it integrates you understand. So, the so there are several applications which you know need very high speeds, but do not need do not need high linearity and in those applications, you know the trans conductance capacitance architecture is what is used does it make sense very good.

So, this is all I had to say about filters right and you know one aspect of filters that we discussed was that they need to be you know the signals inside the filter must not be. So, large as to cause saturation right and then you know, but what happens when the input signals are very small and as we discussed it turns out that all active elements and resistors basically also add noise.