

Circuit Analysis for Analog Designers
Prof. Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 30
Visualization and mitigation of the effect of Q-enhancement

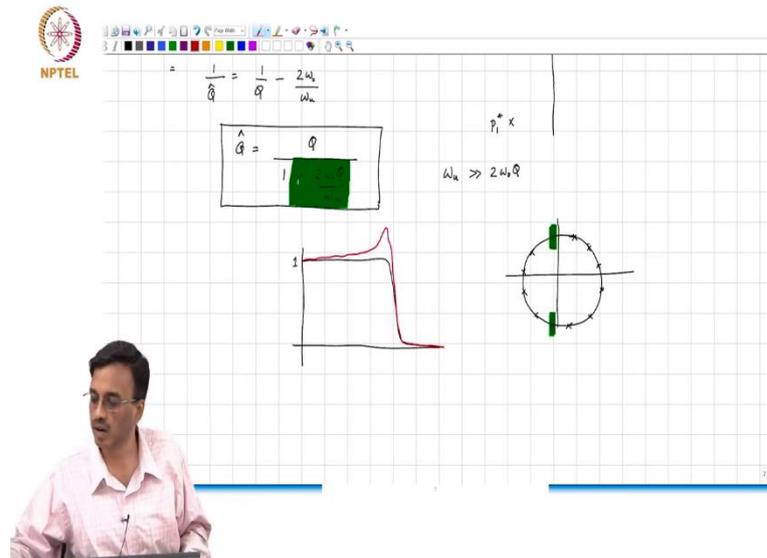
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The whiteboard content includes the following elements:

- Transfer Function:** $H_{\text{non-ideal}}(j\omega) = H_{\text{ideal}}(j\omega) \frac{\omega_n^2}{\omega_n^2 - \omega^2 + j\frac{\omega_n}{Q}\omega}$
- Partial Fraction Expansion:** $\frac{\omega_n^2}{\omega_n^2 - \omega^2 + j\frac{\omega_n}{Q}\omega} = \frac{k_1}{s - p_1} + \frac{k_2}{s - p_2}$
- Residue Calculation:** $k_1 = \frac{\omega_n^2}{2Q} = \frac{\omega_n^2}{2Q} - \frac{\omega_n^2}{\omega_n}$
- Final Residue:** $k_1 = \frac{1}{Q} = \frac{1}{Q} - \frac{2\omega_n}{\omega_n}$
- Boxed Equation:** $\frac{1}{Q} = \frac{1}{Q} - \frac{2\omega_n}{\omega_n}$
- Pole-Zero Plot:** Shows poles at $p_{1,2} = -\frac{\omega_n}{2Q} \pm j\omega_n \sqrt{1 - \frac{1}{4Q^2}}$ and a zero at $s = -\omega_n$.
- Magnitude Response:** Shows a resonance peak at ω_n with a quality factor Q .

So, therefore, if you are trying to realize a high-quality factor biquad then you find that you will have a lot more trouble with q with the gain bandwidth product of the op-amp. For example, if you are trying to realize the Butterworth filter right.

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You have we know that the ideal Butterworth response if I plot it on a, if I plot it should do something like this let us say it is a high order Butterworth filter.

So, it cuts off you know virtually like a brick wall alright. And the omega naught of all the poles is the same for a Butterworth filter and the quality factor as you can as you know right. There will be the sharp cut off is basically the consequence of having high quality pole pair.

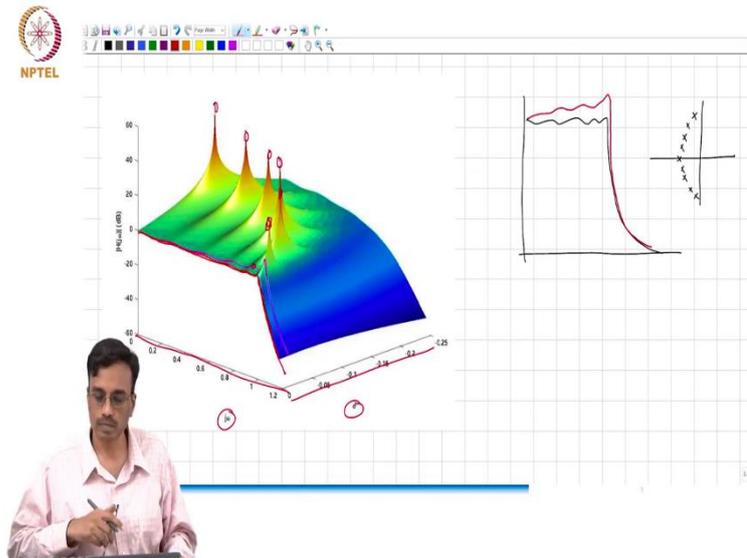
Now, and there are other pole pairs also. So, if you recall the Butterworth poles are basically distributed around a circle right, they something like this blah blah blah ok. So, which of the pole pairs is the most problematic as far as gain bandwidth product of the op-amp is concerned?

Yeah, these are the guys who whose Q is going to get enhanced by the largest amount because that 1 minus 2 omega naught Q by omega u is in the; is in the denominator right ok. So, what do you think the non ideal response should look like?

All the Qs are enhanced, but the highest Q pole pair Q is enhanced by a lot more right. So, what do you think you will see if you basically you would expect that, well the gain everywhere is greater than intended right, and at the band edge it does alright ok.

And you know and while we have discussed all these results for a second order biquad as I mentioned right this is a general result ok, where this holds for an arbitrary transfer function if all the integrators are replaced by you know something else right.

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And let me show you picture of a what happens with a high order filter. So, for example, this is the pole 0 map of a of a filter which is supposed to give what is called an equal equiripple response alright. And, it is a high order, it is a high order filter whose magnitude response is supposed to be like this ok. Those of you are taking analog and digital filters you know you probably heard of Chebyshev filters.

But I mean, that is just another way of approximating the ideal brick wall. And it turns out that again on the S plane the pole 0 map will be you know something like this it just turns out that these this actually is a 1, 2, 3, 4, 5, 6 it is a 12th order Chebyshev filter. So, basically there are a whole bunch of poles like this that is what is going to give you that that brick wall this is the $j\omega$ axis ok.

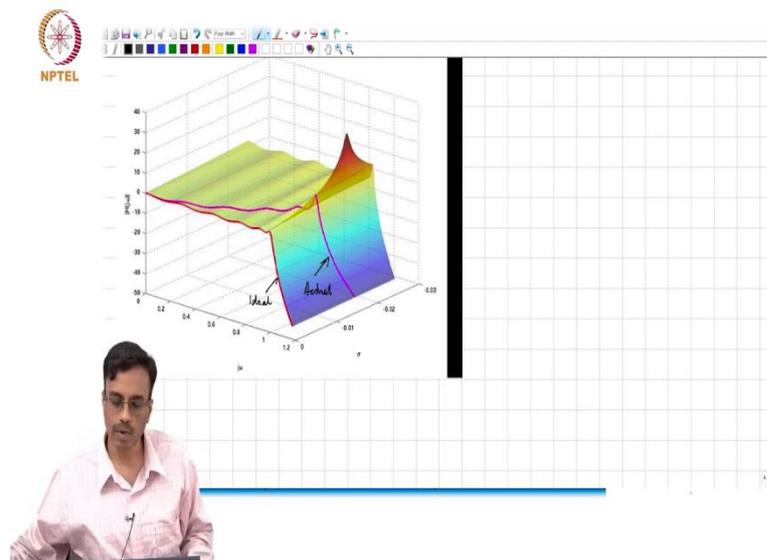
And this is the negative sigma axis right. And this is the 3D plot which corresponds to the magnitude response the I mean; the x y plane is basically the complex plane right. And what do these correspond to? Those are the pole frequencies at which the magnitude response goes to H of s goes to infinity, but we are interested in finding the magnitude response. So, what do we do? We slice this along the $j\omega$ axis and the ideal response that you get is basically the one shown in red here.

I do not know if you are able to see it right that is this guy right. And it does what we supposed to do as advertised. Now, when all the integrators are non ideal what happens? We are going along we are evaluating the same H ideal of s , but along a trajectory which is. You know whatever it is, it is $j\omega$ minus ω^2 over ωu and as you can see well, we now going along this path I shown in the magenta curve right.

And therefore, you can see that you are basically you are basically at the past band edge right. Now, you know kind of scaling the foothills of the mountain right. So, the peak response will, the response will peak. So, basically you will see the response will peak at all frequencies correct ok. Because you know the pole the quality factor corresponding to every pole is increased pole pair is increased.

However, the enhancement is the highest with respect to the pole pair which has already has the highest quality, does make sense?

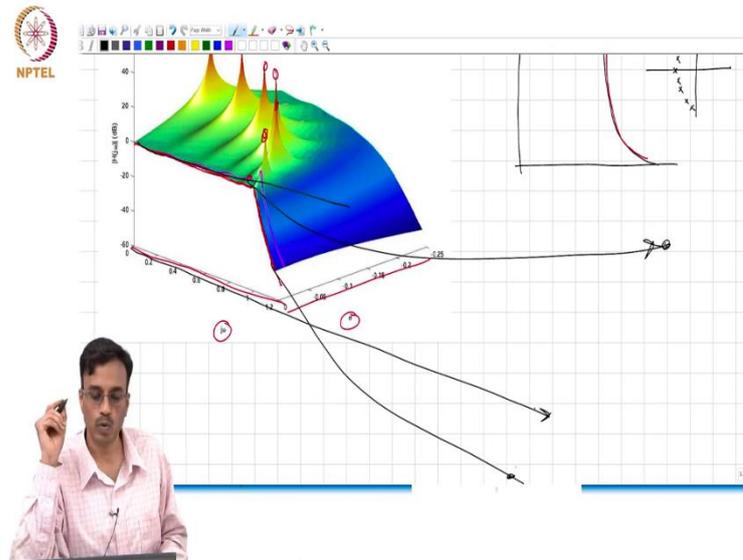
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And here is a better view. So, as you can see will you get a better view now, right? So, this is the ideal transfer function, ideal magnitude response and this is the actual one with non ideal op-amps ok. Now, looking I mean, using this as a visualization can you we comment or what happens of course, the at the past band edge you will see significant peaking in the magnitude response.

What you think will happen in the in the stop band? Are we doing worse or are we doing better or are we doing you know the same? Deep in the stop band what do you think will happen to the attenuation? Remember, deep in the stop band what we are evaluating is.

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In the ideal case you know we are going along the $j\omega$ axis like this and evaluating the surface you know somewhere here right, ok.

Now, what are we doing? We are going along and that cut there which kind of does you know keeps going somewhere there on the in the S plane right. What comment can you make about you know the response there versus the response here? Which is farther away from the poles?

Pardon. No, think carefully, you come back here right. We are interested in I mean the stop band response again is gotten by finding the distance from a point somewhere along this that curve to both P_1 and P_1^* right. As ω tends to infinity where is you know for or rather for a large ω if you are remaining on the $j\omega$ axis you would be somewhere here, if you went off like this you would be somewhere right. So, which distance is larger?

The non ideal case right. So, what comment can you make about this stop band response?

At very high asymptotically as the frequency response I mean, frequency as you know approach is infinity when you have a non ideal op-amp ok the attenuation is more. But I mean, is that is a surprising or it is not surprising?

Yeah, well the op-amp is totally dead right. So, you know in other words the op-amp is not fast at all and the gain is it is again is so small compared to 1.

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The slide contains the following content:

- NPTEL Logo** and a toolbar at the top left.
- Transfer Function:** $H(s) = \frac{P_1 P_1^*}{(s - p_1)(s - p_1^*)}$
- Frequency Response:** $H(j\omega) = \frac{P_1 P_1^*}{j\omega - p_1} \frac{1}{j\omega - p_1^*}$
- Ideal vs. Actual:**
 - Ideal: $H_{ideal}(s) = \frac{1}{s}$
 - Actual: $H_{nonideal}(s) = H_{ideal}(s) \left(1 + \frac{s}{\omega_u}\right)$
- Pole-Zero Plot:** Shows a pole at the origin (0) and a zero at $-\omega_u$ on the real axis. The imaginary axis is marked with $j\omega$.
- Bode Magnitude Plot:** Shows the magnitude response. At low frequencies, the magnitude is $20 \log(1/\omega)$. At high frequencies, the magnitude is $20 \log(1/\omega) - 20 \log(\omega/\omega_u)$, showing a roll-off of -40 dB/decade. The corner frequency is ω_u .
- Equations:**
 - $H_{nonideal}(j\omega) = H_{ideal}(j\omega) \left(1 + \frac{\omega^2}{\omega_u^2}\right)$
 - $\frac{\omega_u}{s} = \frac{\omega_u}{s} - \frac{\omega_u^2}{s^2}$

That the in I mean, the integrator as you can see the integrators magnitude response is at high frequency was in the ideal case only going as omega naught over S. Now, it is at high frequency it is attenuated by a further factor of 1 by 1 plus S over omega u. So, it seems reasonable that the attenuation at high frequencies is much smaller alright ok.

So, the question now is ok well we know what happens. Now, we need to figure out what to do to what do people do to fix it right.

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$\hat{Q} = \frac{Q}{1 - \frac{2\omega_0 Q}{\omega_u}}$

* Predistortion \rightarrow Rely on how good ω_u is estimated to be.

Remember while you stare at this equation and say Q hat is Q divided by 1 minus 2 omega naught Q by omega u alright.

$$\hat{Q} = \frac{Q}{1 - \frac{2\omega_0 Q}{\omega_u}}$$

So, what do you think you can do to fix this?

Well, you know the easiest thing to do is to say give me omega naught omega u is infinity right. Then that as you know is you know is the technological limitation I mean, evidently you know the transistors we make can't you know work at infinite speed. The other thing as he suggested is to say well, we know that if we shoot for Q, you will get you will get a Q hat which is actually higher. So, you know what is a common sense thing to do? What is the common-sense thing to do?

Well, you basically shoot for a Q which is lower I mean, you know I mean, using this equation you know what Q to shoot for. So, that with finite gain bandwidth the response is the enhanced Q is you know is exactly what you want ok. What comment can you make about you know is that what comment can you make over the robustness of such an approach? The omega naught is omega naught times Q is fixed right, what varies in practice is the unity gain bandwidth of the op-amp can which can change with temperature and process and all this other stuff right.

So, if you are counting on the ω_u , so that this factor you're coming the fact that your pre compensating for Q enhancement basically means that you know that this factor is reliable right.

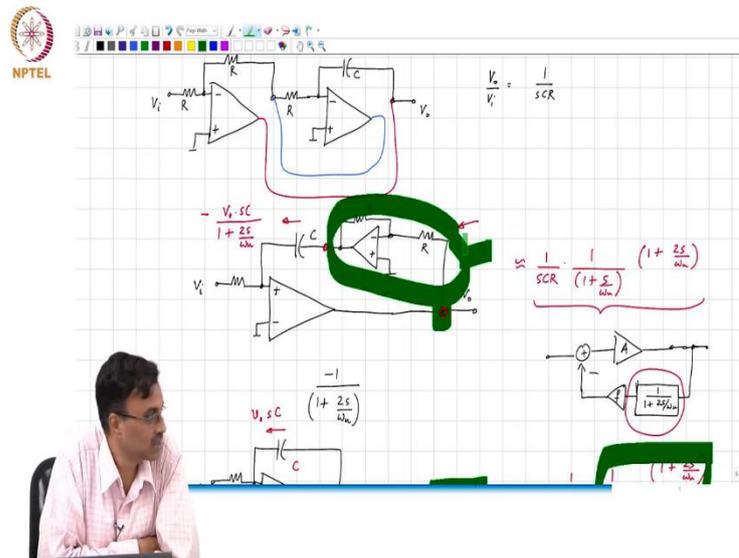
That is only possible if the ω_u does not change our temperature and so on, in practice you know I mean, you know you get what you get ok. So, if ω_u varies well the Q that you get also varies and therefore, the response also will be varied. There been other clever ways of trying to fix this problem right.

So, this way of adjusting the Qs apriori for the gain bandwidth of the op-amp that is called pre distortion of the filter transfer function right. And you know the idea is very simple we know that if we shoot for this you get something like this. So, the logical solution would be to say ok well you know let us presume that I am shooting for this ok then Q an enhancement will cause this to go and become the black curve alright.

So, that is one way of doing it and you know the problem with pre distortion is basically pre distorting the filter transfer function is basically that you now rely on how good ω_u is estimated to be. Does this make sense people? Now, the next thing that I like to draw your attention to the following I mean, this is one approach. And; obviously, you know it is ok, but you know there is an issue with robustness.

There is again as I said there is been you know whole lot of work arounds for this problem, I will discuss just one simply because we have already seen you know this aspect as far as your when we did MNA.

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Remember, that we said when we are discussing the modified nodal equations, we said that, well in a circuit with ideal op-amps. If you look into our biquad this is R, this is R, this is R, and this is C. And if this is a op-amps both the op-amps are ideal the integrated transfer function will be V_o by V_i which happens to be minus plus 1 over SCR.

When we are discussing modifying nodal analysis, we said that if you have a network with ideal op-amps right you can well we break this connection you can break this connection right, connect this here and connect this here and Sorry so.

This must connect to here and this must connect here alright. If you do this the transfer function does not change correct ok. So, if your kind of I am going to skip the connections and just show you the final result it turns out to be, I just simply redrawing the picture. This is V_i this is C this is R this is R and this is V. And what should we do? As far as remember we have to figure out the negative feedback signs and then it turns out that this is the inverting amplifier.

So, you need plus and minus here to get negative feedback around this op-amp. So, this is this also is an inverting integrator a non inverting integrator if the op-amps are ideal. So, but when the op-amps are real there is no I mean, there is no reason to believe that this will give you the same transfer function as the earlier one right. And it turns out and I will give you the intuition behind that it turns out that this gives you an integrated transfer function which is got some interesting property.

Remember if this op-amp is non ideal what comment can you make about the transfer function from V_o to what is fed back? That is from this point to this point what comment can you make about the transfer function there?

Pardon. Yeah, I know where is the pole? At what frequency is, what is the transfer function? Minus 1 divided by 1 plus. No, think carefully. $2S$ over ω_u you have seen this in your earlier classes the feedback factor around the op-amp is half.

$$-\frac{1}{\left(1 + \frac{2S}{\omega_u}\right)}$$

So, the gain I mean, the bandwidth will be one-half the unity gain bandwidth. So, the transfer function in the feedback path is minus 1 by 1 plus $2S$ over ω_u alright.

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The slide contains several diagrams and equations. At the top left is the NPTEL logo. The main content is divided into two parts. The upper part shows an op-amp circuit with a feedback capacitor C and a resistor R . The transfer function is given as $-\frac{V_o sC}{1 + \frac{2s}{\omega_u}}$. To the right of this is a block diagram of a feedback loop with a gain A and a feedback factor $\frac{1}{1 + \frac{2s}{\omega_u}}$. The lower part shows a similar op-amp circuit with a feedback capacitor C and a resistor R , with a transfer function of $-\frac{1}{1 + \frac{2s}{\omega_u}}$. There are also some handwritten notes and a small block diagram showing a feedback loop with a gain A and a feedback factor $\frac{1}{1 + \frac{2s}{\omega_u}}$.

I will you know, so in the feedback path, let me draw your attention to our standard negative feedback system right. So, this is A and this is the feedback factor is f right and let us say you had some transfer function from the input to the output.

Now, if I put in a pole here of 1 by $1 + 2S$ over ω_u what comment can you make about how the closed loop transfer function will change?

It will be? It will show up as a 0 . So, the transfer function will be, so whatever transfer function you had earlier correct it will now have a 0 because the feedback path has been,

the feedback path has been? How does this compare with the original integrator? What is the difference between this and that? The current here is approximately V naught times S C . Now, what is the current here? Yes, what is it? It is V naught times S C times 1 over 1 plus I mean, apart from a negative sign there is basically 1 plus $2S$ over ω_u correct.

$$-\frac{V_o s C}{1 + \frac{2s}{\omega_u}}$$

So, what comment can you make about the transfer function that you get?

So, effectively in the feedback path you have inserted a pole and as he pointed out the closed loop transfer function is simply proportional to 1 over the feedback factor times the loop gain by loop gain plus 1 right.

So, at sufficiently you know to first order therefore, the transfer function of this integrator is 1 over SCR times 1 by 1 plus S by ω_u which is what you would get if this character was not around right, multiplied by 1 plus $2S$ over ω_u right.

$$\approx \frac{1}{sCR} \frac{1}{\left(1 + \frac{s}{\omega_u}\right)} \left(1 + \frac{2s}{\omega_u}\right)$$

So, this is an approximation in your assignment you are going to prove it ok. So, what is this what is this giving you? How does it compare with the original integrator, with the non ideal op-amp? This had minus 1 over SCR times 1 by 1 plus people.

Darwin, what is it? I cannot hear you. S by ω_u alright ok. And this guy is now one integrator is this there are the other the combination of inversion and integration which was giving us a non inverting integrator earlier right is 1 by SCR times 1 plus S over ω_u times 1 plus $2S$ over ω_u alright. So, what do you see?

Therefore, well if you evaluate, we have we have a cascade of you know two integrators and what you call a damped integrator and a non inverting integrator. Earlier, you would I mean, you would not get these factors now in one of the integrators you get this factor in the other one you basically get this factor. And what comment can you make about when you multiply the two of them together?

To first order you basically see that you know $1 + S$ by ωu whole square is approximately $1 + 2S$ by over ωu you have $1 + 2S$ by ωu in the numerator. And therefore, the two of them the first order cancel right. Another way of thinking about it is that there is this creates phase lead you putting a lag in the feedback loop, in the feedback factor causes lead in the closed loop transfer function.

And therefore, the two of them the lag of this; the lead of this basically cancels the lag of the other integrator and you basically get something which is to first order you know independent of ωu right. And the ωu therefore, and what do you call? Because the phase lead and the phase lag are caused by the op-amp the same kind of op-amp it follows that well if the op-amp gain band would change a little bit.

Well, the phase lead changes the phase lag also changes by the same amount and then two cancels right. So, to first order is cancellation of gain band product effects, is this clear? Alright.