Circuit Analysis for Analog Designers

Prof. Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 29
Effect of finite opamp bandwidth on an active-RC biquad

Good morning and welcome to advance electrical networks this is lecture 13; a quick recap of what we were doing in the last class.

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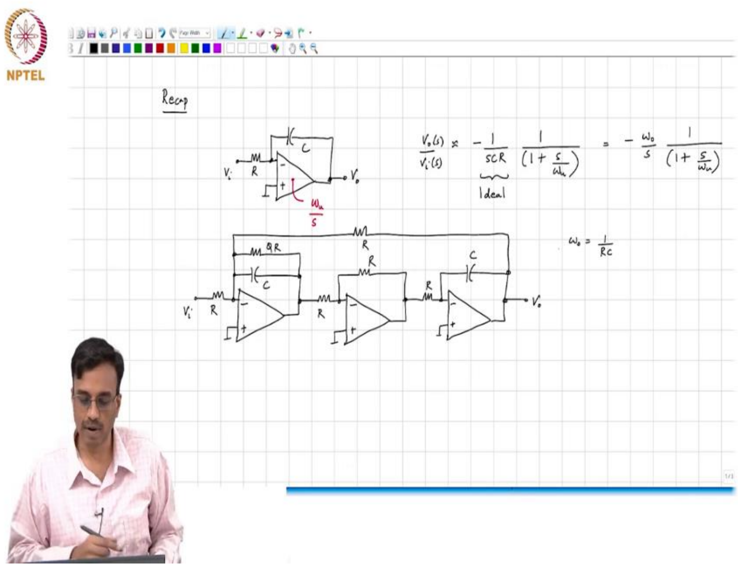

We basically wondering what would happen to the transfer function of a biquad, if the op-amp was not ideal. And the model we assume for the op-amp is basically we assume that the gain of the op-amp is of the form omega u by S and we now know you know why that model comes about.

Where you assume that the op-amp is typically realized using Miller compensation. And therefore, you have a dominant pole and a large DC gain. The DC gain of course, for all practical purposes can be neglected since its so, high. So, all that remains you can is that the op-amp instead of you know having an infinite bandwidth, you can think of it as having a transfer function or the useful frequency range of omega u by S.

Omega u is the gain bandwidth product of the op-amp in radians per second. And when we analyze this integrator with the gain of the op-amp being omega u over S rather than just simply infinity, the integrated transfer function we got was minus 1 over SCR times or this is approximately minus 1 over SCR times 1 over 1 plus S over omega u right.

So, intuitively what this is this equation is telling us is that well this is recall that this is the ideal transfer function that you would get if omega u tends to infinity right. Now, what this is telling you is that, you have this within quotes practical integrator where you have a real op-amp, but not an ideal one. And the output is of course, an integrated version of the input except that there is a there is a delay in the output.

And that delay to first order is simply a time delay of I mean minus 1 over a delay is basically 1 over omega u seconds. And that is as far as the integrator is concerned and remember that the biquad is basically consists of integrate, invert, integrate, this is the damping resistor ok.

And for simplicity we simply use the same valued resistors everywhere and this is has an omega naught which is 1 over RC and. So, this becomes minus omega naught over S times 1 over 1 plus S by omega u and the quality factor is Q.

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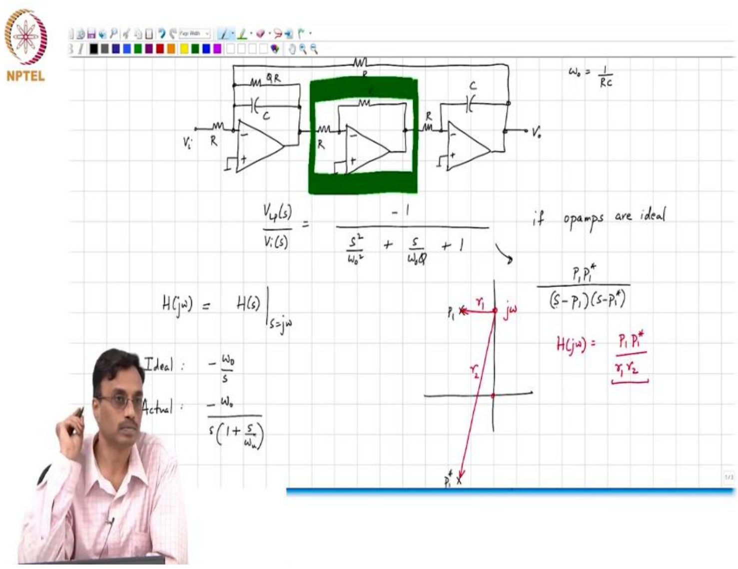

The transfer function that the low pass transfer function that we get is V LP of S by V i of S as we discussed is minus 1 over S square by omega naught square plus S over omega naught Q plus one provided, provided what?

$$\frac{V_{LP}(s)}{V_i(s)} = -\frac{1}{\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1}$$

If the op-amps are ideal, the question now is oops you know our op-amps are not ideal therefore our integrators are not ideal ok. And then what will this do to our biquadratic transfer function. As we were discussing yesterday this is truly now a fifth order transfer function right our only hope is that omega u is so, high that in the frequency region of interest it still can be approximated by a second order transfer function.

And we also saw yesterday that if you interested in the magnitude response, the magnitude response is simply or the frequency response for that matter is simply you go you travel along the j omega axis in the S plane right

$$H(j\omega) = H(s)|_{s=j\omega}$$

and what do you do, if the transfer function can be represented as S minus P 1 into S minus P 1 star right.

There are 2 poles and the DC gain is 1. So, this must be P 1 P 1 star correct.

$$\frac{P_1 P_1^*}{(s - P_1)(s - P_1^*)}$$

So, I mean why do we do this? Well, this gives us a nice way to interpret H of j omega. So, let us say this is P 1, P 1 star is you know somewhere here and this is j omega alright. And to find H of to find the complex number H of j omega, well you draw the vector j omega minus P 1, which basically is this guy here and j omega minus P 1 star, which is that guy there ok.

And so, if you call this vector r 1 and then if you call this vector r 2 then mod H of j omega is H of j omega is simply P 1 P 1 star by r 1 r 2 right;

$$H(j\omega) = \frac{P_1 P_1^*}{r_1 r_2}$$

we have seen the I mean you have seen this in your earlier classes multiple times. So, in effect when we are I mean when we plotting H of j omega what we are actually doing is starting at the origin and keep going along the j omega axis at each point on the j omega axis you basically draw these arrows r 1 and r 2 and compute this is this clear? Right.

So, now for the time being just assume that this op-amp is ideal so, that the inversion is obtained for free right without any bandwidth limitation. It turns out that and those of you have taken analog IC design would know this already, it turns out that most you know IC's today the signal path is always realized in what is called fully differential fashion right.

How many of you are familiar with what fully differential means? How many of you have taken analog IC design? Ok and the others are taking it. So, you will find out. So, whenever you realize circuits in fully differential fashion you know one of the advantages of doing that is that you know apart from the fact that it is immune to common mode disturbances and a whole lot of nice stuff like that.

Another aspect is that inversion is simply inversion is obtained for free right it simply obtained by in interchanging the wires right. So, this our assumption that or the this inverting amplifier basically let us assume at least for the time being that it is ideal is not a bad one.

So, if that is the case then the only difference between the ideal one and the actual one is remember the ideal integrator is minus omega naught by S.

$$-\frac{\omega_0}{s}$$

The actual one is minus omega naught over S times 1 plus S over omega u.

$$-\frac{\omega_0}{s\left(1+\frac{s}{\omega_u}\right)}$$

So, if you stare at these two expressions so, you know what do you think you see.

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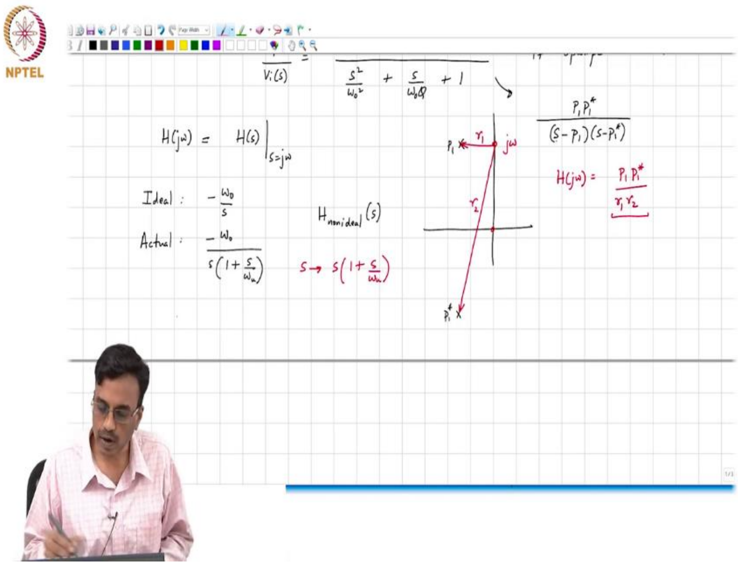

In other words, how will you express the non ideal transfer function in terms of the ideal one?

Stare at these two all that we have done is replace the ideal integrator with a non-ideal one. So, the question is you know is there a simple way of figuring out what the non-ideal transfer function will be in terms of the ideal one.

Well, all that we are doing if you can stare if you stare at it carefully is replace omega naught by S with omega naught by S times 1 plus S by omega u which is equivalent to saying. In fact, there is a simpler way of thinking about it, its simply replacing S with?

Replace S with S plus 1, S times 1 plus S over omega u correct.

$$s \rightarrow s\left(1 + \frac{s}{\omega_u}\right)$$

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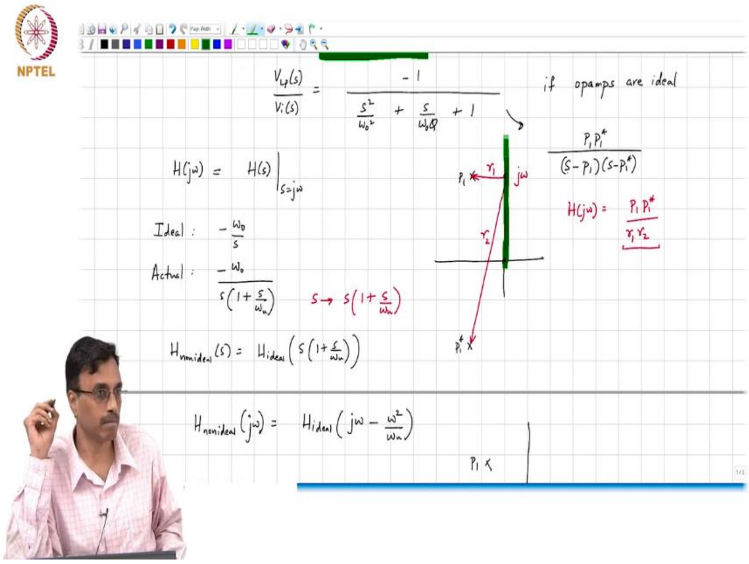

And therefore, H non-ideal of S is simply equals simply yes. So, now, how do you express the non-ideal transfer function in terms of the ideal one? H ideal of S times 1 plus S over omega u correct does make sense,

$$H_{non-ideal}(s) = H_{ideal}\left(s\left(1 + \frac{s}{\omega_u}\right)\right)$$

And while you know this you know we are talking about a biquad this also applies to any higher order filter; because you know if every integrator is modified according to this relationship. Then it is simply replacing which whatever transfer function you have if the integrators are messed up according to this.

Then it is simply a matter of replacing S with the new transfer function or the non ideal transfer function is simply the ideal transfer function with S replaced by S times 1 plus S over omega u right. And what are we interested in? We are interested in finding H non-ideal of j omega right ok. And how is that related to H ideal of j omega therefore? We know how H non ideal of S is related to H ideal of S how do you relate H non-ideal of j omega to H ideal of j omega? Yes.

Very good it is basically simply replace S with j omega and therefore, H ideal of j omega minus Omega square over omega u correct.

$$H_{non-ideal}(j\omega) = H_{ideal}\left(j\omega - \frac{\omega^2}{\omega_u}\right)$$

So, how do we interpret this equation?

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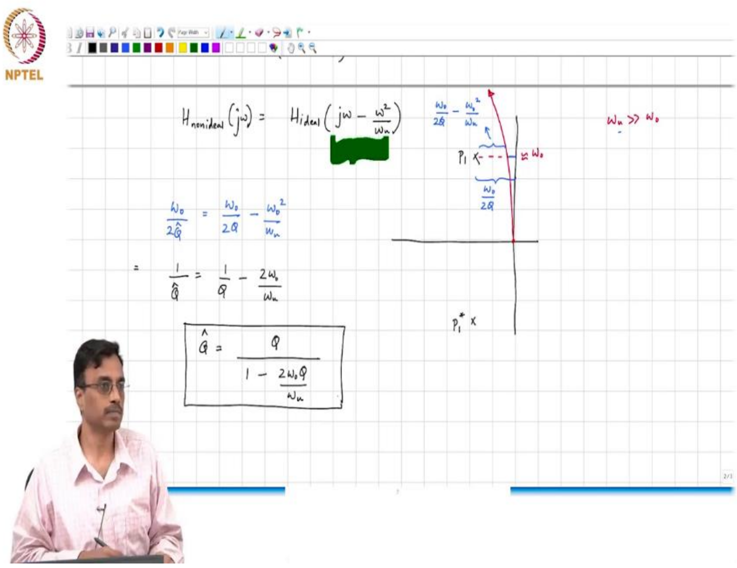

Remember if we had the original transfer function right, this is P 1 and this is P 1 star to find H ideal of S H ideal of j omega we would go along the j omega axis along this path as shown there, correct? Now, the nonideal magnitude response or frequency response what is this equation telling us? We now instead of traveling along the j omega axis you go along a path which is j omega minus Omega square by omega u right and how does that path look like?

Ok, well and remember that omega u and let me also you know draw your attention to the fact that, this is approximately omega naught ok alright. And how does this how do this path j omega minus omega square by omega u look like and omega u remember is much greater than omega naught. So, how does this path look like? At DC it will be 0, right. As you keep increasing omega what happens? It deviates from the j omega axis does it go into the left half S plane or the right half S plane?

Left half S plane and at the trajectory you are taking I am grossly exaggerating here. So, that you can see it, but it takes some trajectory like that alright ok. And the let me also draw your attention to another fact what comment can we make about that part the x co-ordinate of P 1? What is that distance?

Omega naught by 2 Q and what is the definition of Q? It is omega naught which is the radius corresponding to P 1 divided by the distance to the j omega I mean how close it is to the region of stability to the edge of stability and that is omega naught by 2 Q. So, the ratio of the 2 is just times 1 half, which is Q that is the definition of Q right.

Now, what was happened now? As you keep going along the red path, I shown here what comment can you make at what point is the gain the highest? At what point on that red path would the magnitude response peak or become very large?

Technically its perpendicular, but this thing is going so, what do you call the slow I mean because omega u is much greater than omega naught right. It is a technically a parabola, but you know for all practical purposes that it is almost you know parallel to the x axis to the y axis right. So, to first order therefore, the peak in the transfer function right or the distance to the edge of the region of instability is now is it smaller or larger?

It is smaller right so; therefore, do you think the filter is within quotes more stable or less stable now? Less stable and intuitively that makes sense right we have a negative feedback loop in that loop now there is extra delay; because of finite bandwidth of the op-amps and as a result the feedback loop becomes is tending to instability. So, what is the distance now? What I just look at it and tell me give me a simple I just use geometry and tell me what this distance is now?

It is simply omega naught by 2 Q minus that much right and so what is that? It is omega naught by 2 Q minus Omega naught square by Omega u, correct.

$$\frac{\omega_0}{2Q} - \frac{\omega_0^2}{\omega_u}$$

Now, if you had a fictitious biquad with ideal op-amps what Q would it have if the distance to the j omega axis was the same? Do you understand the question? Suppose we had a second order biquad with ideal op-amps right ok, what Q would it have if its distance to the j omega axis was the same which is namely omega naught by 2 Q minus omega naught square by omega u, how would we find that out?

So, basically, we have a fictitious biquad we do not know its quality factor, but we know that the distance if you denote the quality factor by Q hat, then the distance from that P 1 of that biquad to the j omega axis would basically the omega naught by 2 Q hat alright.

$$\frac{\omega_0}{2\,\hat{Q}} = \frac{\omega_0}{2Q} - \frac{\omega_0^2}{\omega_u}$$

So, what is how is you know what is how is Q hat related to Q?

So, which basically means 1 over Q hat equals 1 over Q minus 2 omega naught square over 2 omega over omega u ok what is the sanity check does it does this pass sanity check?

$$= \frac{1}{\hat{Q}} = \frac{1}{Q} - \frac{2\omega_0}{\omega_u}$$

As omega u tends to infinity, we expect that Q hat basically tends to Q. So, what is Q hat? What is Q hat in terms of Q?

Can you express that as you know Q times something? So, that when that something when omega u tends to infinity that something tends to 1.

Very good, very good. Q divided by 1 minus 2 omega naught Q divided by omega u.

$$\hat{Q} = \frac{Q}{1 - \frac{2\omega_0 Q}{\omega_u}}$$

Very good alright ok. So, Q hat I mean so is Q hat higher than Q or smaller than Q?

That again makes sense right because you know. So, in other words the moment the op-amp is got a finite gain bandwidth product because is the excess delay added by the op-amps right. The quality factor appears to have been enhanced right indicating you know in English that you know the poles are moving closer to the j omega axis right. Pushing the filter transfer function closer to instability right.

And the Q hat I mean the Q is enhanced according to this relationship right. So, if you want you know a quality factor which is close to what you intended right what comment can you make about the gain bandwidth product of the op-amp? I mean of course, you know large is of course, the first order 0th order statement we can make right can we what is the meaning of large now?

No, think carefully. What must be much smaller than 1, if I mean what is the meaning of large omega u? So, this factor $\left(\frac{2\omega_0 Q}{\omega_u}\right)$ must be smaller than 1.

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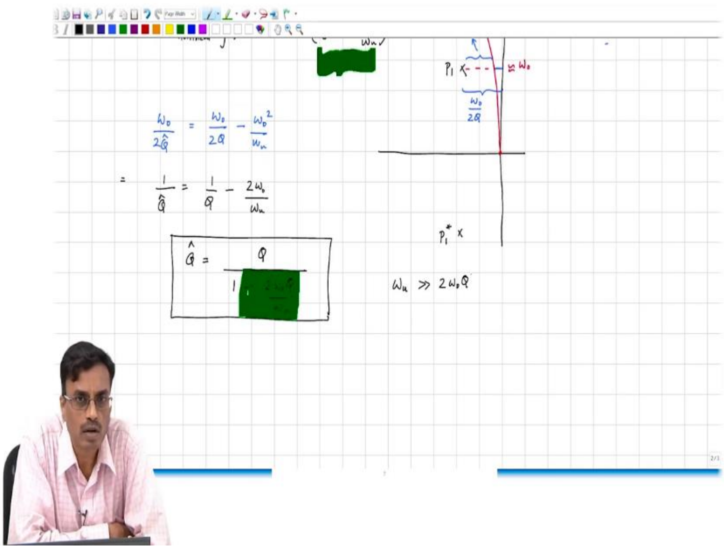

Much smaller than 1. So, the large the notion of what a large the gain value product of the op-amp must be large enough, what that large enough means is that omega u must be much greater than 2 omega naught right.

You know we might have simple mindedly assume that omega u being much greater than omega naught is probably all that it takes; you can see that the constraint is much more is much tighter right if. So, basically, we expect to see omega u to be much greater than 2 omega naught, does makes sense people?