Circuit Analysis for Analog Designers
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Lecture - 28
Effect of finite opamp bandwidth on an active-RC integrator

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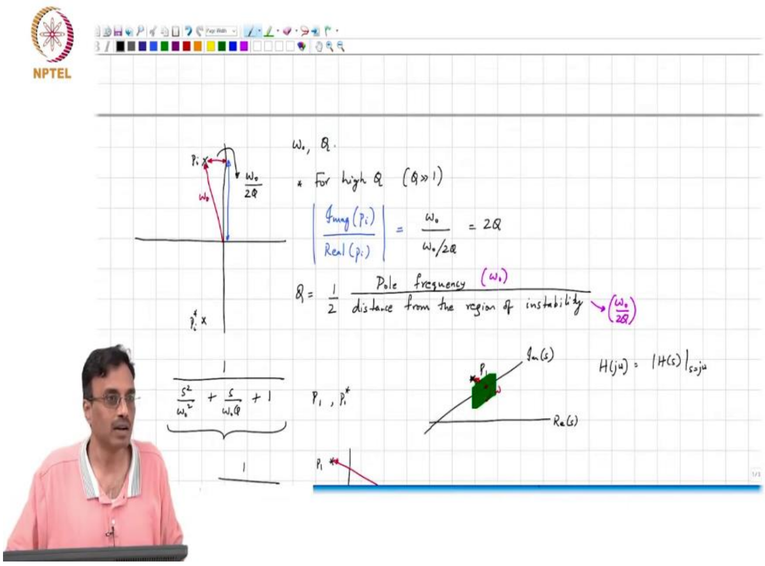

Ok. That is something to bear in mind. The next thing, I would like to draw your attention to is the following. Remember, that when you plot a transfer function of this sort $\frac{1}{\frac{s^2}{\omega_0^2}+\frac{s}{\omega_0 Q}+1}$;

I mean basically this is a complex number s is a complex number. And therefore, this is also a complex number and has both the real part and an imaginary part and let us assume that P_1 and P_1^* are the poles of this system.

So, if I plot the real part of s and the imaginary part of s on a sheet of paper and plot the magnitude on the z axis right, I will basically get a I mean it is basically a three-dimensional plot correct. And, if and the pole is you know somewhere on the x y plane, if I draw the pole that is the pole and that distance happens to be? What is that distance?

It is omega naught by 2Q; and the length of the arrow from the origin to the x is basically omega correct. Now, if I look at a 3D plot, how will the magnitude plot look like?

Student: (Refer Time: 01:53).

Pardon. It will tend to infinity at the pole right. And so, if you and if basically if you have a whole if you have a filter transfer function with the whole bunch of poles, you know at each of these points basically the you will see you know you what you will see is Mount Everest right ok. And, but when you evaluate H of j omega, what exactly are we doing on this picture? If we are evaluating modulus of H of j omega, what would you do on this picture?

You take this 3D plot right the surface and slice it along the j omega axis and look at you know the profile of that slice as seen from yeah you know whichever side that you want alright. So, if you do that, then you will basically see mod H of j omega right. So, mod H of j omega is simply nothing but mod of H of s evaluated at s equal to j omega.

$$H(j\omega) = |H(s)|_{s=j\omega}$$

In other words, you have you travel along the j omega axis and then keep evaluating H of s, a pictorially you take the 3D plot and then slice it on the j omega axis, alright.

So, these are a simply things that you bear in mind right. And, what comment can you make about the; let us consider only a that you had only this pole call this P_1, at what frequency do you think the gain will become very large as you keep going along the j omega axis? At what frequency do you think you should expect to see a peak in the magnitude response?

Pardon. So, this close to omega I mean somewhere in that region and why does it make intuitive sense? At P 1. So, what? But we are not evaluating it at P 1 right.

Pardon. Yeah. Very good right.

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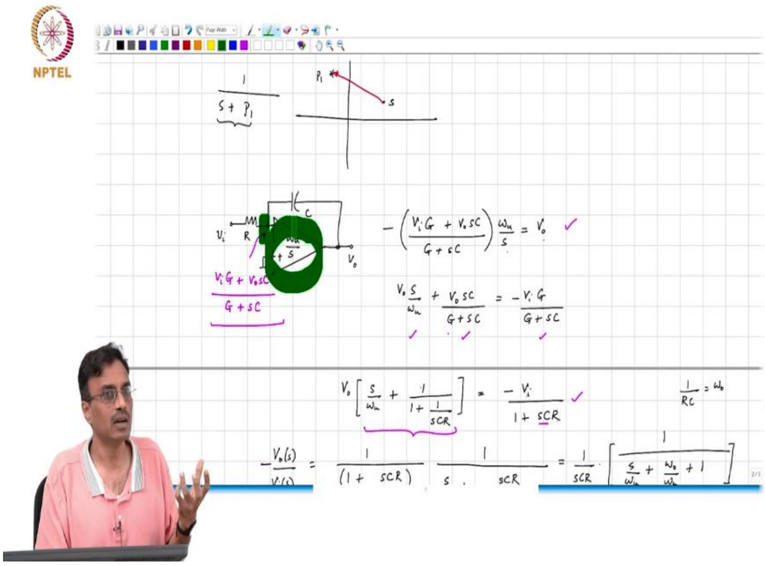

So, remember that as a you know this is again from whole zero plots in your network systems class or electric circuits and networks class. If you want to evaluate the, if you have a transfer function and you want to evaluate H of j omega at some s, what would you do? You basically draw arrows from, is this familiar to you guys?

So, let us say you have a transfer function 1 over s plus P_1 and you want to find a H of s at some frequency s in the complex plane, what do you do? This is the complex plane, looking from the top. Let us say you want to evaluate this is say some P 1, if you want to find H of s what would you do at this point? Let us say you want to find H of s where s is that point marked there, what would you do?

You will draw vectors from Origin? From s; this is nothing simply the length of the complex number. Right. So, that is this guy here right. And you do it to all the poles, and then you do it all the zeros. And, then the magnitude response is the ratio of the vectors all going from that point to all the zeros, divided by the vector going to all the poles right. I hope this is familiar ok and. So, therefore, if you are evaluating stuff on the j omega axis, it must follow that at every point on the j omega axis you would draw these vectors from that j omega to all the poles right. And, in this region which pole has the maximum influence on the magnitude response?

P 1, why? Very good, that is because the at this j omega for instance the arrow that goes from that point to the pole is the shortest and the response is 1 over the length of that arrow;

and therefore, in that region you can see that the length you know changes most rapidly, and therefore, you should expect to see that that pole has the maximum influence on the magnitude response in that when you evaluate the frequency response over that region. Does it make sense? Alright ok. So, with all this background information, let us see what happens to the behaviour of our integrator.

So, let us say this is R and this is C; this is V i and this is V o alright. So, what is the you know, what is the ideal response? Of course, is minus 1 by sCR. And now, the gain of the op amp is omega u over s ok. So, what comment can we make about the transfer function? There is no other way except to simply evaluate it. So, what do you do? What is this voltage? Please sit and evaluate it.

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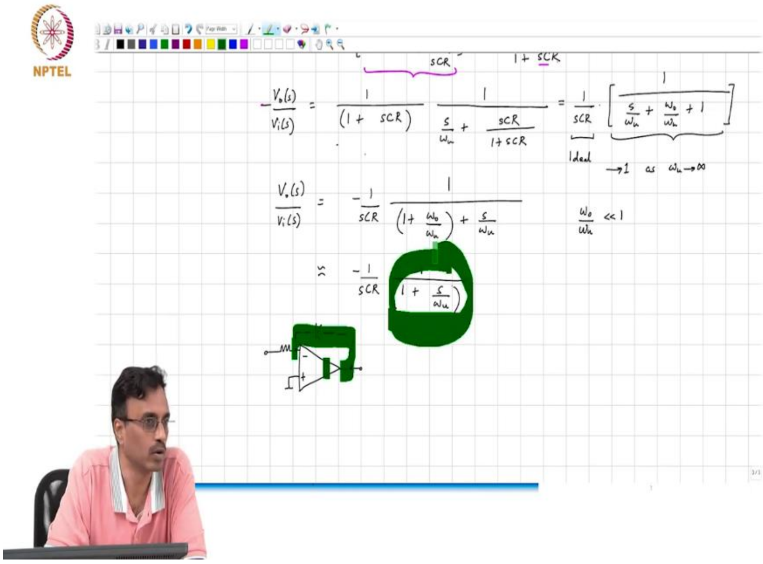

$$-\left(\frac{V_i G + V_0 sC}{G + sC}\right)\frac{\omega_u}{s} = V_0$$

$$\frac{V_0 s}{\omega_c} + \frac{V_0 sC}{G + sC} = -\frac{V_i G}{G + sC}$$

$$V_0\left[\frac{s}{\omega_u} + \frac{1}{1 + \frac{1}{sCR}}\right] = -\frac{V_i}{1 + sCR}$$

$$-\frac{V_0(s)}{V_i(s)} = \frac{1}{(1+sCR)} \frac{1}{\frac{s}{\omega_u} + \frac{sCR}{1+sCR}} = \frac{1}{sCR} [\quad]$$

Yes, can we put this in the form 1 by sCR times some other transfer function; and the idea is that if omega u tends to infinity this should tend to 1, ok?

1 divided by s by omega Q Plus 1 Student: Omega u CR. Let us use omega u; remember that 1 by RC is nothing but omega naught that is the center frequency of the ok. So, fine go ahead. Omega naught by omega u.

Student: So, that omega naught on no omega u by omega naught is equal to plus s s divided by omega u plus omega naught by omega u plus omega naught by omega u minus 1 (Refer Time: 13:06).

Minus 1. What is happening people? You cannot get minus 1, I think there is an.

No. I am getting more answers than there are people. Alfred, what do you think is the correct answer?

Your right; I want somebody to confirm because we do not want to go in to do a whole lot of math and then find out that the basic thing is wrong, can somebody else confirm that it is correct?

Overall minus should be there no. I mean this is equivalent to putting a minus here if that is what you guys are saying. Let us go through the steps, let us see where we have made a mistake if we have.

$$-\frac{V_0(s)}{V_i(s)} = \frac{1}{(1+sCR)} \frac{1}{\frac{s}{\omega_u} + \frac{sCR}{1+sCR}} = \frac{1}{sCR} \left[\frac{1}{\frac{s}{\omega_u} + \frac{\omega_0}{\omega_u} + 1}\right]$$

This voltage is nothing but? Is this correct? Let us go step by step, is that correct? It simply, it is a potential divider correct. So, the voltage in the middle is simply V i times this conductance, G is nothing but 1 by R times plus V o times sC which is the conductance of that arm divided by the sum of the conductance’s right; that multiplied by minus omega u by s must be equal to V naught. So, this seems ok alright. And now, I will just simply take over s over omega u to that side.

And then move the V naught times sC by G plus sC which is a negative sign I will move it to the right side it just becomes positive. So, this is fine, this is fine and this must be equal to minus V i times G by G plus s, it is correct? Ok. So, now, I just group these terms which is s over omega u plus 1 over 1 plus G by sC which is nothing but 1 over sCR correct. And, this is minus V i over 1 plus sCR. So, this is also ok.

Yes ok. So, V o by V m you know V o by minus V i is nothing but 1 by 1 plus sCR times 1 over this becomes a denominator here 1 over s over omega u plus sCR by sCR plus 1 ok. Now, all I am saying is that let us try and isolate this 1 by sCR so that whatever else we have ok hopefully become you know a constant as omega u tends to, it should tend to when as omega u tends to infinity, what would we expect to get C for the transfer function?

Minus 1 by sCR right. So, this is the ideal transfer function and the rest of it is simply a factor, which causes a deviation from the ideal transfer function. And therefore, as omega u tends to infinity, we should make that that should go to 1 alright ok. So, what is that factor is what I am trying to get from you people. So, what is that factor now?

Plus Omega naught by omega u plus. Pardon Minus V naught by V i right ok. So, can somebody else confirm? Very good ok. So, hopefully we have our math right V o by V i of s therefore, is minus 1 over sCR times this transfer function which is 1 plus omega naught over omega u plus s over omega.

$$\frac{V_0(s)}{V_i(s)} = -\frac{1}{sCR}\frac{1}{\left(1+\frac{\omega_0}{\omega_u}\right)+\frac{s}{\omega_u}}$$

Intuitively, what would you expect for omega u versus so, I mean versus omega naught? What do you think we should have? Well, you know you basically are trying to make an integrator with a unity gain frequency of 1 by RC, which is omega naught ok; and this is working on the principle of negative feedback. So, at all frequencies of interest, where is our integrator? Ok, at all frequencies I mean this will work like a good integrator if this is that voltage is at which if that is a virtual ground that will only happen if the gain of the integrator is you know ideally infinite in practice very large. The question is over what range of frequencies should that gain be very large? Ok, and what comment can you make?

Well, at least at frequency is much greater than omega naught up to frequency is much higher than omega naught, you expect that the op amp has got a sufficiently large gain. So, that virtual ground voltage is sufficiently.

The reason why that virtual ground voltage is small is because the gain of the op amp is very high correct. So, if you want this to work like a good integrator over a frequency range you know up to at least up to omega naught and; obviously, including frequency is much higher than omega naught then over that range of frequencies you must ensure that the gain of the op amp, which is what? What is the, what would be the gain of the op amp at a frequency omega naught?

What would it be? The magnitude of the gain would be omega u over omega naught right, that number better be? Much larger than 1, correct. So, the therefore, the concept in the argument therefore, is that omega naught by omega u without even you know going further must be a number which is, what comment can we make about omega naught over omega u in comparison with 1?

Much much smaller than 1 correct. So, therefore, you can approximate this as minus 1 by sCR correct divided by 1 plus s over 1 ok.

$$\frac{V_0(s)}{V_i(s)} \approx -\frac{1}{sCR}\frac{1}{\left(1+\frac{s}{\omega_u}\right)}$$

So, I mean does this make why does this make intuitive sense? I mean of course, this comes out of the math, but why does this make intuitive sense?

Alright. So, alright at high frequency which of these elements and what happens to the capacitor?

The capacitor is a short circuit right. So, what kind of what comment can we make about the loop gain around the op amp? The capacitor is a short circuit.

What happens? What comment can we make about the feedback factor?

It is 1 right. If the feedback factor is 1 right, what comment can you make about the closed loop pole? The open loop pole is omega u by I mean the forward amplifier so, to speak has got the transfer function omega u by s. The feedback factor is 1. So, what comment can you make about the closed loop bandwidth?

It is simply the unity gain bandwidth of the op amp alright; and that is what this is telling us. It is telling us that at very high frequency basically the behaviour must you know eventually have a pole at omega u, is it clear people? Alright. So, in English what this means is that well the op amp is not infinitely fast right. It takes some time to react and as a result what comment can we make about the integration? If you had an ideal integrator if you put a step, you should get a?

What kind of transient? If you put a step in an ideal integrator, what will you get at the output?

You will get a ramp, alright. Now, if you have an integrator like this ok well where it has got some integration, but it is also got an extra pole right, what comment can you make about the output response when you put in a step?

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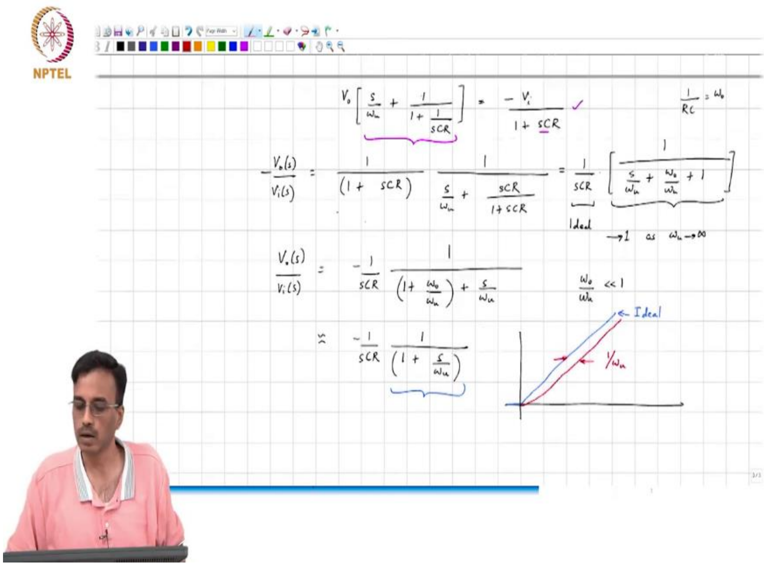

So, I am going to draw the negative of the waveform to avoid the minus signs. So, ideal integrator is you put a step and you get a ramp alright. What is the slope of the ramp?

1 by RC. This is the ideal one right ok. Now, but we have this extra character here. So, what comment can you make about the response? It is like taking this and passing it through a low pass filter 1 by 1 plus s by omega u right, and that therefore, will be will

cause an output which basically does something like this right ok. And, what will be the delay? 1 by omega right. So, in other words the integrator which was ideally supposed to give you the integrated output waveform right away, correct. Now, has a within quote small delay right and that delay is because well the op amp is not fast enough alright ok. Before, we go ahead and start doing the math on what this delay does to our transfer function, intuitively what should you expect?

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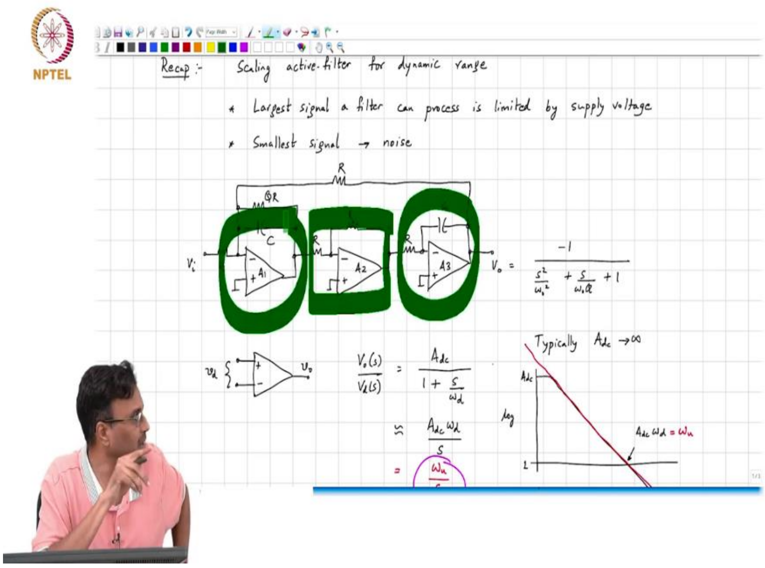

What do you think will happen to the transfer function of this filter? If every integrator I mean let us assume for the time being that this op amp is ideal so that you do not worry about the delay there will be delay there also, but ok; the integrator both these integrators you know they do integrate, but they add a little bit of delay. So, what comment can you make about the poles or the close of this system and why?

I mean yeah eventually everything is because of non-ideality the amplifier right, but I am asking you. Pardon. Very good. So, basically as you can see here this is a feedback system right. You have you know an integrator and then you have another integrator and then the output is fed back right. If you have a feedback system and then you introduce now each of these building blocks has got some delay.

So, if you have a feedback system and then you introduce delay into the feedback loop, what comment can you make about the nature of the closed loop transfer function? If you have a feedback loop and you add a lot of delay into it, what will happen?

It will tend to become you know it will tend to become unstable or it will become closer to getting unstable right. So, in the parlance of poles and quality factor and so on, what comment can we make about what should we expect before doing the math? If we add, if our op amps are non ideal and have some delay associated with them, the integrators also have delay associated with them.

They are now embedded inside this negative feedback loop. Earlier, without delay the quality factor of the poles was something and what is the quality factor the interpretation of the quality factor? It is how close those poles are to the edge of you know instability right. Now, because there is delay, what happens? The does the system tend to become more stable or more unstable?

It is inching towards instability, if you go on increasing the delay, we are almost sure that this will become unstable. So, the poles therefore, must be inching closer to the edge of instability. So, what comment can we make about the quality factor?

We should expect the quality factor to increase. Does it make sense people? Alright.