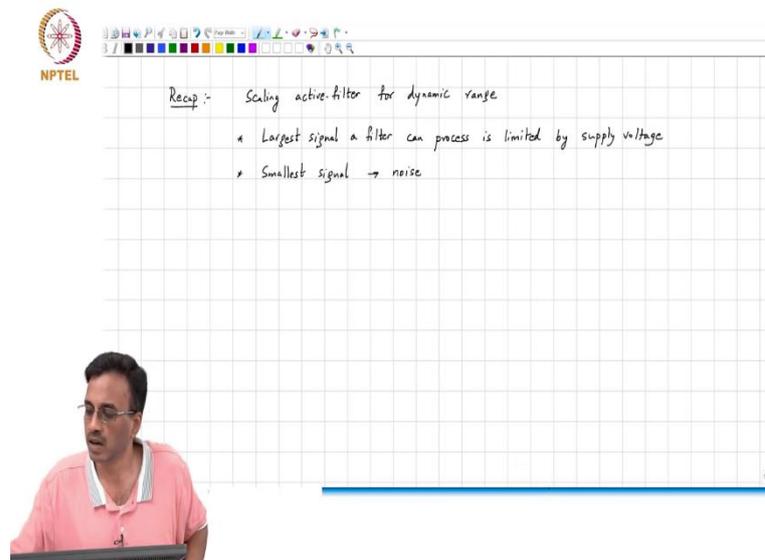


Circuit Analysis for Analog Designers
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Lecture - 27
The finite gain-bandwidth model of nonideal opamps

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Recap :- Scaling active filter for dynamic range

- * Largest signal a filter can process is limited by supply voltage
- * Smallest signal \rightarrow noise

In the last class, what did we discuss? We discussed the problem of scaling an active filter for dynamic range and the intuition is the following. The largest signal that a filter can process is limited by supply voltage and the smallest signal is limited by noise, alright.

And we saw yesterday that the transfer function from the input to various internal nodes in the filter can have frequency response magnitudes that can be widely vary right; whose peak responses can be widely vary. And as a consequence, you know one of the op amps will prematurely limit the maximum signal that the filter can process even though the other op amps are actually quite ok alright.

And therefore, it makes sense to kind of equalize the peak magnitude responses at the outputs of every op amp inside the filter. And, as a result, now you know all the op amps would kind of saturate add for the same maximum input signal star.

Now, we also saw how we would accomplish this thing. This is the one short procedure. First thing is to we simply find the peak magnitudes at every node and we saw how one

could, we saw recipe where you can simply scale the peak response at every node by factor and that is done by changing the value of the integrating capacitor or whatever it is there in the feedback branch.

And, simultaneously scaling all the impedances that sense this node right the output node by the same factor, this way the current that was going into the branch would be the same regardless of what the voltage was alright. So, the next thing that and for the lowest limit as we were talking about yesterday that depends on noise alright.

And the noise is coming from all the elements inside the filter namely the resistors add noise it turns out; and the op amps which internally have transistors also add noise. So, the output of the filter is not merely the input signal which is been filtered, but noise of its own and so we will get to see how this noise behaves when we actually talk about noise right.

I mean we could have means instead of going to filters at this point we could have talked about noise and then talked about filters later, but you know then you know a young crowd like you gets ansty, when you know if you just keep seeing theory without any application and then right you keep wondering you know why am I studying all this, alright.

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- * Largest signal a filter can process is limited by supply voltage
- * Smallest signal \rightarrow noise

V_i V_o

A_1 A_2 A_3

R R R C

So, the next thing we need to worry about is; well, let us quickly draw filter. As we said yesterday, if you are only interested in the low pass response it does not particularly matter whether you put the damping the resistor on the input side of the output side as far as the

transfer function is concerned right. The input output transfer function is concerned, and this is V_i , this is V_o which is a low pass transfer function all the resistors except the damping resistor are R for instance and this is C .

And depending on the Q , the peak transfer functions at the outputs of different op amps can be different. If the Q is you know is very high then it turns out that as you will check for yourselves in one of the assignments. The peak signal of the output of this op amp A_1 becomes much larger than that of A_3 , and then which case you can do some dynamic range scaling and equalize the peaks alright.

The only other aspect that we have not seen so far is the following and that is that we have assumed so far that all the op amps are ideal right. In other words, you know if you apply a differential input, the output first of all you know becomes infinite and not only that, not only does it become infinite it does so, infinitely fast, correct.

I mean this you know one infinity is bad enough, I mean you know where are we going to get two infinities? Right. Bandwidth is infinity and the gain is infinity. So, of course, in practice neither the gain is infinite nor the bandwidth is infinite alright.

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The slide displays a circuit diagram of a three-stage op-amp system. The first stage is an inverting amplifier with gain A_1 . The second stage is an inverting amplifier with gain A_2 . The third stage is an inverting amplifier with gain A_3 . The overall transfer function is given by:

$$\frac{V_o(s)}{V_i(s)} = \frac{A_{dc}}{1 + \frac{s}{\omega_d}}$$

where $A_{dc} = A_1 A_2 A_3$. A Bode plot shows the magnitude response $|A_{dc}|$ and the phase response $\angle A_{dc}$. The plot is annotated with "Typically $A_{dc} \rightarrow \infty$ ".

And it turns out as you probably seen in you know your earlier classes that a good model to use, I mean we do not want to go into the internals of the op amp at this point because that will take us too far a stray.

So, if this is the differential input voltage of the op amp and this is the output voltage of the op amp. The model for the op amp's transfer function it turns out is V_o of s by V_d of s right is some A_{dc} divided by $1 + s$ over ω_d .

$$\frac{V_o(s)}{V_d(s)} = \frac{A_{dc}}{1 + \frac{s}{\omega_d}}$$

And, this corresponds to you know a two stage Miller compensated op amp which is probably the most commonly used op amp whenever you buy a discrete op amp two stage Miller compensated op amp is the one that is used.

And those of you who have done a course on transistor level circuits should be well aware how this comes about right. That A_{dc} that finite gain is simply the product of the dc gains of the individual stages and ω_d stands for the stands for the dominant right. It turns out that typically A_{dc} is actually very large. So, typically 10^5 , 10^6 something like that.

So, for to make our life simple, we can basically say that for all practical purposes the gain of the op amp is very large. And therefore, you can think of for you know you can think of this as. So, in other words this A_{dc} is extremely large and this is the log scale and this is 1. And, what frequency is this is the bode plot of magnitude plot of the gain of the op amp. At what frequency does the magnitude of the gain go to 1? $A_{dc}\omega_d$. So, and as you all know the ω_d is the very low frequency thanks to Miller compensation. And so, for the useful I mean except at very very low frequencies. You can see that, there is no difference between this plot and the gain of the gain of an amplifier with this transfer function correct.

How does the transfer function of this look like? Well, that will look like this; all the way here the only difference is that. Well, it keeps going to infinity at dc and so over the useful I mean over a large part of the frequency range, you can see that this approximates the characteristic of the op amp; the transfer function of the op amp rather.

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Smallest signal \rightarrow noise

Typically $A_{dc} \rightarrow \infty$

$A_{dc} \omega_d = \omega_u$

And this is therefore, this is called also called the gain bandwidth product right. Let me call that omega u by s alright.

$$\frac{V_o(s)}{V_d(s)} \approx \frac{A_{dc} \omega_d}{s}$$

$$= \frac{\omega_u}{s}$$

Now, the question is well, what happens I mean what is the obvious question now? If the op amp is ideal the transfer function, we get here is would be what? 1 by or minus 1 by s square by omega naught square plus s by omega naught Q plus 1 alright.

$$V_o = -\frac{1}{\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1}$$

And, so what is the question that we would like to ask now? Yes.

Well, you know evidently the op amp is not what we thought it is different from our ideal model. So, a natural question to ask, would be; what does it do to the transfer function? We should expect you know it that there will be some change right for one. What comment can we if the op amp was ideal what comment can be made about the order of this system? Second order right.

Now, we replace the op amp with, we thought the op amp was ideal and had infinite bandwidth, but it unfortunately turns out that the model for the op amp is actually it has a frequency dependent gain given by $\frac{\omega_u}{s}$. So, now, the question is what is the order of this so called filter now. What is the order of the transfer function? What do you think?

Well, it is you know every op amp is going to now get an additional pole. So, this would be a fifth order transfer function right ok. The only hope is that, if ω_u tends to infinity right. I mean so, the I mean as ω_u tends to infinity, the transfer function will tend to a second order one meaning. Remember, that a fifth order transfer function will have 5 poles correct as ω_u becomes very large, we will see what that large is you know going forward.

But if ω_u becomes very large, then we should expect that it reduces to, if we set ω_u tending to infinity, we should expect that this fifth order transfer function should reduce to a second order one. Therefore, if you keep increasing ω_u right, you know 3 out of those 5 poles right must be tending to ; will tend to should be expected to tend to infinity, because as ω_u tends to infinity this must reduce to a second order one which is equivalent to saying 3 of the poles have gone to infinity correct.

So, if ω_u I mean our intuition is telling us that if ω_u is sufficiently large where we still need to figure out what the meaning of that sufficiently large is alright. We should expect to see that you know well at least in the regions of interest of our filter namely you know below ω_u and maybe few decades above ω_u , we should expect to see that you know we should be able to approximate that as a second order transfer function you understand ok.

So, that is the spirit in which the analysis that we see going forward has to be seen right it is you know remember that this notion of quality factor etcetera all pertains to Q pertains to a second order pole pair right.

So, it does not make sense to say you know talk about what is the what would be the Q of my filter we now have a fifth order transfer function right. But we still do talk about the Q of our filter, assuming that the ω_u of the op amps is so, high that in the frequency region of interest, the behavior largely looks like second order behavior, in which case you can talk about its Q, you understand alright.

So, that is one thing that I would like to I would like to clear. The next thing that I would like to talk about is you know what before we go and I mean how do you think we go and saw you know analyze this problem, what would we do?

What do you suggest? Well, when the op amp was ideal you know you wrote down all the virtual ground equations blah, blab, blah, you can analyze the circuit you can get your transfer function ok. Now, tough luck the op amp is not ideal. So, how do we go about I mean finding the new transfer function? What would you do?

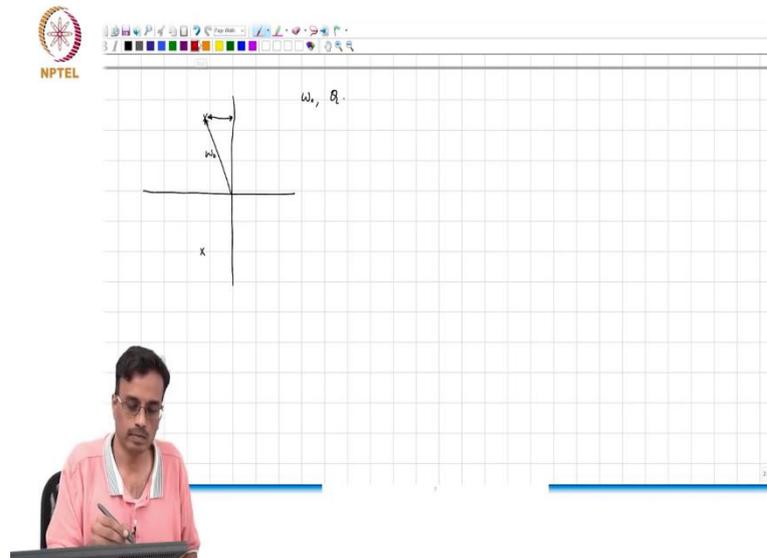
So, well one thing you could say is well I will wait from a neighbor to do the math and then you know I will copy from him right, but unfortunately it looks like your neighbor is also you know assuming the same thing right.

So, you know we are all in the same boat. So, what do you think I mean what is the dumbest thing that you can do? Yeah. Apart from waiting for your neighbour to finish. What do you think you could do?

Pardon. Well, I mean there is you know there is no rocket science here right. You know you think of the op amp as $\frac{\omega_u}{s}$ and go through the math right. I mean and you know probably is going to run into a couple of pages of algebra right, but in the end you will have you will have the transfer function alright. But, that kind of analysis you know you can do right, but you what have you not exploited? Well, we have not exploited the fact that you know typically $\frac{\omega_u}{s}$ must be that ω_u must be, you know is very large, we I mean you know in some sense right. So, you know the and we did not exploited the fact that in the region of interest of the filter namely omega naught and its neighborhood right. The behavior still looks like second order is the op amp is sufficiently fast alright.

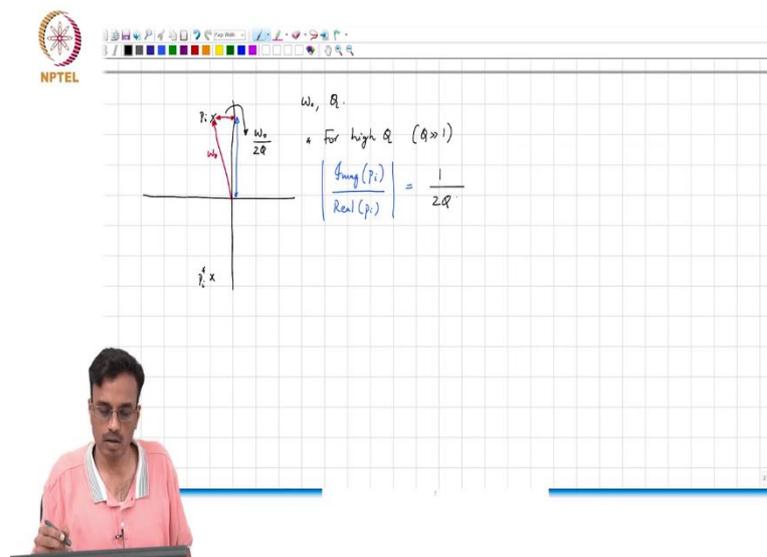
So, let us try and see if we can kind of you know do some kind of hack to be able to estimate what happens, when the op amp is got finite gate bandwidth alright.

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Now, before I go there, let me draw your attention to some background information right. Let us say you have a second order pole pair right with some omega naught and Q. What comment can we make about omega naught I mean in this picture you know what is omega naught? The radius of these two will; obviously, lie on a circle this is the magnitude of that is omega naught and what is Q? What is this, what is this distance?

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Apologies, we draw a vector there the magnitude of that vector is omega naught alright. And what is Q? Remember, the quality factor Q quantifies?

Pardon. How close the poles are to the $j\omega$ axis right. So, what is this distance?

Yes. Please do the math and let me know, yes.

It is ω_0 over $2Q$ alright.

$$\frac{\omega_0}{2Q}$$

And, if Q is very is and you know for high Q that is Q much larger than 1, what comment can you make about the length I mean that and the hypotenuse?

Pardon. It is almost equal to ω_0 . So, the I mean which is why you know you often say that the real part actually is square root of it is ω_0 times square root of $1 - 1/4Q^2$. But, for a sufficiently large Q that $1/4Q^2$ is so small that you know the ordinate of that complex number the imaginary part is simply ω_0 and the real part is magnitude of the real part is ω_0 over $2Q$ alright.

So, part imaginary of the pole divide by real part of the pole is nothing but, yes people come on, this is the pole P_i . This is P_i^* . What is the ratio of the imaginary part to the real part for high Q ?

The imaginary part is basically ω_0 by $2Q$ and the real part is ω_0 .

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The whiteboard content includes:

- NPTEL logo
- Hand-drawn diagram of a pole p_i in the complex plane. The real part is on the negative real axis, and the imaginary part is on the positive imaginary axis. The distance from the pole to the origin is labeled ω_0 . The real part is labeled $\frac{\omega_0}{2Q}$.
- Text: ω_0, Q
- Text: For high Q ($Q \gg 1$)
- Equation: $\frac{\text{Imag}(p_i)}{\text{Real}(p_i)} = \frac{\omega_0}{\omega_0/2Q} = 2Q$
- Equation: $Q = \frac{1}{2} \frac{\text{Pole frequency } (\omega_0)}{\text{distance from the region of instability } (\frac{\omega_0}{2Q})}$

So, the ratio of the, oh sorry, the imaginary part is omega naught and the real part is omega naught by 2Q and therefore, this is 2Q alright. So, this is just something to bear in mind right.

$$\left| \frac{\text{Imag}(P_i)}{\text{Real}(P_i)} \right| = \frac{\omega_0}{\omega_0/2Q} = 2Q$$

In other words, the distance of the pole to the x axis I mean to the y axis of the g omega axis which is the border of instability right is I mean divided by the radius of the pole right; corresponding to the pole is basically the quality factor right, multiplied by 2 ok.

So, in other words you interpret the quality factor Q as distance from the region of instability right. This is they should be in the denominator divided by pole frequency. Does it make sense? This is nothing but omega naught, this is nothing but, what is the distance from the pole frequency to the you know how far should you go?

This is omega naught over 2Q right. So, the ratio of the two divided by 2 gives you the Q.