Circuit Analysis for Analog Designers
Prof. Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 26
High-order filters using cascade of biquads, Dynamic range scaling in opamp-RC filters

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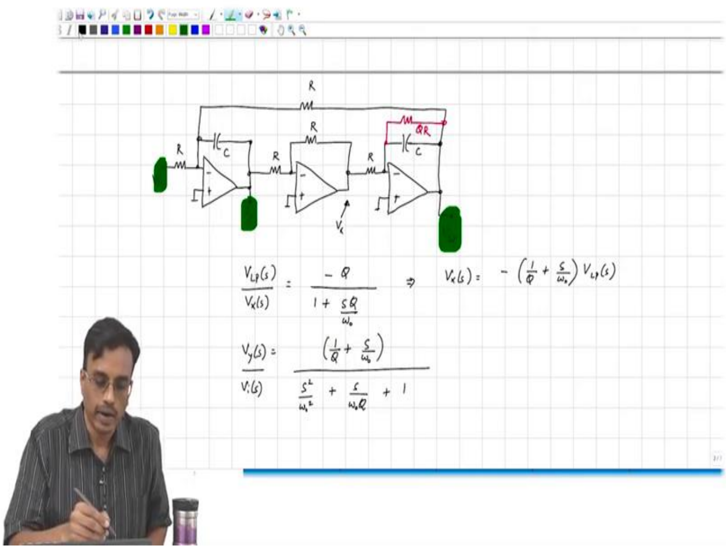

Alright so, basically is all I had to say about the second order biquad right. Now, let us see the issues at crop up when you cascade a bunch of biquads, I mean after all we are interested in realizing a high order transfer function right. And therefore, as we say as we discussed when we try to realize a high order transfer function what we are going to do is split up the transfer function into products of first and second order sections and then just put them in cascade alright and lets its easiest to work with an example.

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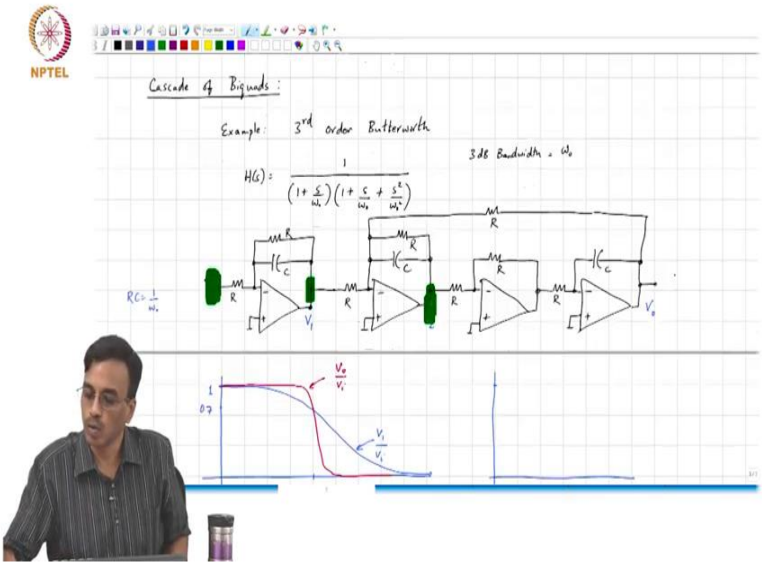

So, let us say we are interested in building a 3rd order right, a 3rd order Butterworth alright. And how does the transfer function look like? H of S is nothing, but 1 over 1 plus S over omega naught times it turns out as we discussed in class the other day its nothing, but S over omega naught plus S square by omega naught square. This is a 3rd order Butterworth filter with a bandwidth 3 dB bandwidth given by omega naught.

$$H(s) = \frac{1}{\left(1 + \frac{s}{\omega_0}\right)\left(1 + \frac{s}{\omega_0} + \frac{s^2}{\omega_0^2}\right)}$$

Now we say ok let us try and you know figure out how to realize this, well we will say we will realize this as one section and this; obviously, is the other section; the first order section well you already know what to do ok. So, we choose RC is equal to 1 over omega naught V i and then we follow it up with our good friend the second order by section which is. And what is q for the 2nd order stage simply 1.

$$RC = \frac{1}{\omega_0}$$

So, quite conveniently choosing all resistors to be the same and all capacitors to be the same gives you a 3rd order Butterworth filter right and the bandwidth being 1 over omega naught, the bandwidth being 1 over RC which is omega naught. So, this is H of S.

Now, I like to draw your attention to a couple of points. So, what comment can we make about the frequency response from V i to let us call this V 1. At omega naught the what is the DC gain magnitude I am only interested in the magnitude. So, the DC gain is 1.

At omega naught it will be something like 1 by root 2 which is like 0.7 right. And it looks like this and I mean this is probably not really accurate picture, but you get the idea. And what is the response at the output? What is the response of the output?

Well, the overall response is much flatter and sharper it probably does something like this ok. So, that is $\frac{V_o}{V_i}$ and this is $\frac{V_1}{V_i}$. For what comment can we make about V_2 here?

It is band pass that is correct and is it its band pass from here to here correct ok, but what comment can you make about the comment I am interested in the transfer function from here to V i to V 2.

What comment can we make about the transfer function from V 1 to V o to V 2?

Its band pass so, that is nothing, but S by omega naught remember lets recall what the band pass transfer function was. It simply nothing, but s by omega naught divided by the denominator polynomial.

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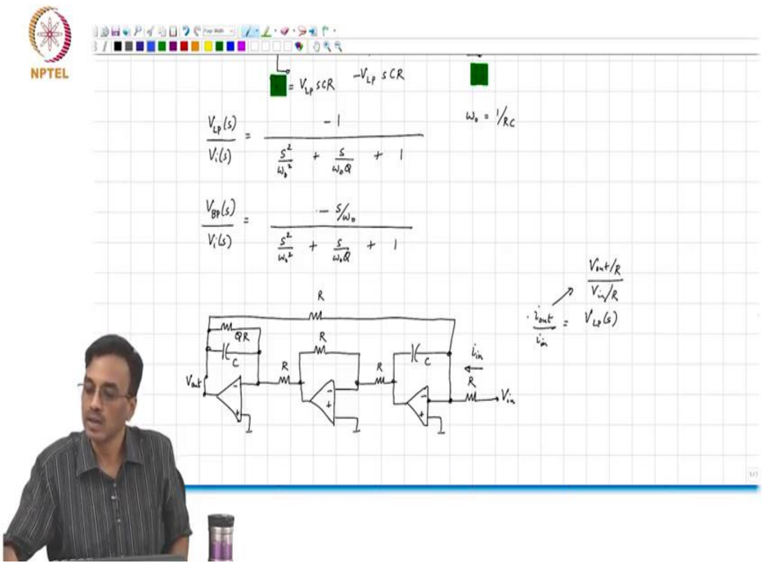

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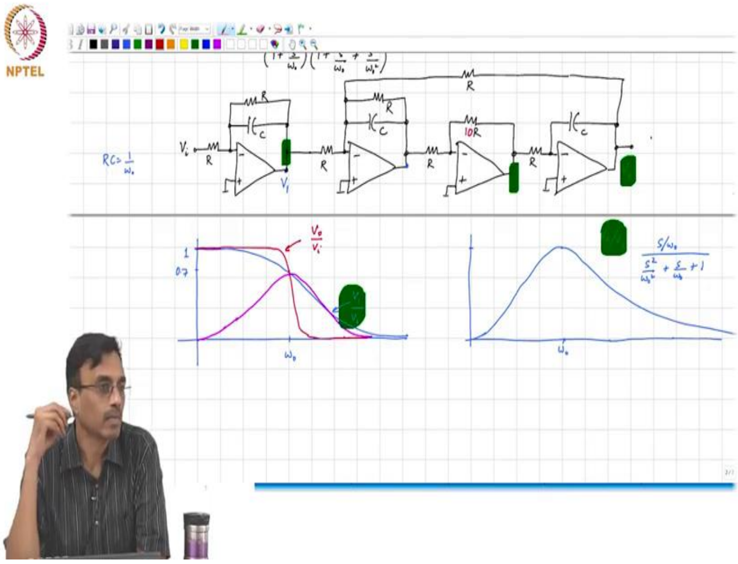

So, it is simply S by omega naught divided by S square by omega naught square plus S by omega naught plus 1.

$$\frac{\frac{s}{\omega_0}}{\frac{s^2}{\omega_0}+\frac{s}{\omega_0}+1}$$

So, what comment can you make about the peak gain or the gain at omega naught? Ok, at very low frequency how will the band pass response look like? At DC it is 0, but at low frequency how will it go from 0?

S square terms will become? Negligible so it basically does something like this. At omega naught the gain is? 1 right so it does something like this and then it does it goes to 0 at omega naught is equal to at omega equal to infinity this is nothing, but V 2 over V 1 that is the band pass transfer function, but what I am interested in is not V 2 over V 1, but V 2 over V i. So, what comment do you think you can make?

So, what do we do to get the magnitude response at of V 2 over V i? We simply multiply this curve with this curve and therefore, you will expect to see something like this where the peak will now only be. Actually, the peak will not occur at omega naught it will get it will shift slightly to the left.

But I mean that shift will be so, small that in this point does not matter. Because intuitively we have multiplied a curve which is doing this with a curve which is doing this so, the maximum of the peak will shift to the left alright. So, what I want to point out is the following right one so, the peak. So, as you sweep frequency input frequency what do you notice here the peak swings at the outputs of each op amp are different alright.

So, and why are we interested in the peak swings remember that the op amp being you know ideal and the virtual ground being a short and all that is only valid under the assumption that the op amp is in its you know within codes high gain region of operation right.

And that is only valid as long as the outputs of the op amp do not get too close to the positive and the negative supply rails alright. So, something that we have to contend with is that you know as you if you go on increasing the input amplitude you will eventually reach a point where the op amp saturates.

In which case the virtual ground is no longer virtual ground and you know there is no longer a transfer function the system is become non-linear right. So, what we want to do therefore is we want to avoid, we would like to avoid op amp saturation right ok. And we would like to avoid op amp saturation for an input signal which is as large as possible right and the reason behind that is the following.

Let us say that because of some weird choice of resistor values here right. The peak gain of one of these op amps was much higher than the peak gain of others right. And to illustrate the point what I am deliberately going to do is I am going to gain this node up by a factor k let us say a factor 10 ok. If I just make that resistor 10 times larger, what comment can we make about the transfer function of the biquad, will it change or will it remain the same.

It will. The question is what will be the transfer function of this of this biquad?

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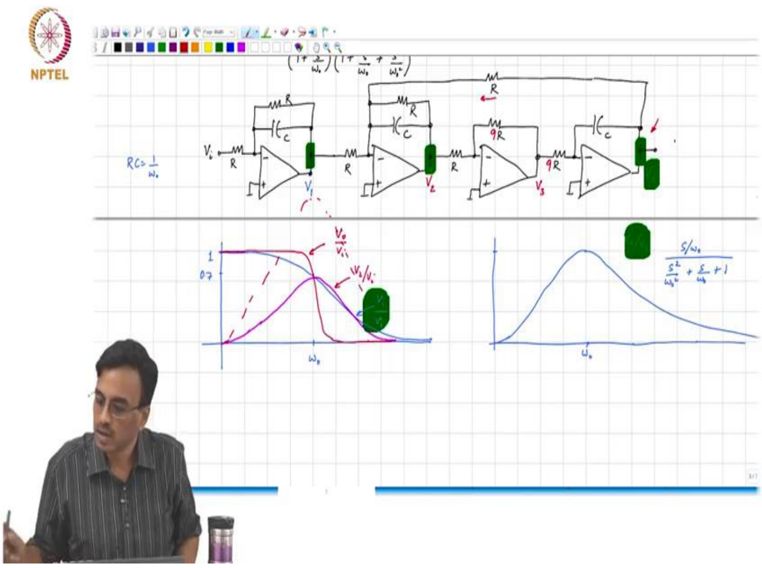

When I go and change that resistor by let me just make this round number let me make it a nice number let me make it 9R ok, hopefully this gives you a clue right. What comment can we make about the transfer function of this biquad? Now, will it remain the same or will it change? Or the omega naught and q of the biquad will it change or will it remain the same? Ok, let me before I put that 9 what are the transfer function from here to here?

1 upon SCR. Now what is the equal now what is the transfer function? 9 upon SCR so, it is like changing it is equivalent to taking that original integrator and changing one of the capacitors by reducing the capacitor by a factor of 9 right ok alright. So, now, will that mean that the closed loop I mean the poles will move or will they remain in the same place? I am not interested in finding out where the poles are first tell me whether you think the biquads omega naught and q will change or they will remain the same, they will.

They will change correct. In fact, if you go and do the math you know it turns out that the unity gain frequency which was 1 over RC earlier now becomes 3 over RC because this is increased by 9 times, but that is not the point that the point the key point is to understand that the by changing one of those resistors there for instance I can I mean the poles of the close of the biquad and the quality factor have both changed alright.

Now, the question is what do I do to bring it back to omega naught? What do you think I could do? Of course, one thing you would say would be to do not change the 9 in the first place right, but let us say I want to do it anyway. What could you do to bring the omega naught and q back to what they were originally?

Pardon. What is the simplest thing if your DS change one element what do you think you could do? One thing you could do is increase the capacitance by a factor 9 right; alternatively I mean the many ways of doing this an alternative way would be to simply increase this resistor also by a factor of 9 right. And what is the intuition behind that observation? So, this voltage has gone up by a factor of 9 right, but if you want the transfer function here to remain the same.

This current should be divided by 9 that is all correct. So, in other words multiplying this by 9 and dividing this resistor by 9 will result in the same transfer function for the biquad correct. And therefore, the same transfer function from the input of the filter to the output of the filter right.

Now, the question is what has you know if I mean what else is happened because the resistors increased by a factor of 9 right. The input output transfer function; obviously, is the same correct what comment can we make about the transfer function from V i to v 1.

Student: Remains the same.

It remains the same. What comment can you make about the transfer function from V 1 V i to V 2? Is it the same? Why? Correct right the input output transfer function we know is not changed. So, well as far as V 2 is concerned I mean the same current comes to that branch. So, the transfer function at V 2 has also not changed right. So, that is this is nothing, but $\frac{V_2}{V_i}$ What comment can we make about V 3?

What was it earlier? If that 9 was not there what was the V 3 by V i earlier? Same as V 2 by V i, but now what has happened. Well, right. So, this is gone up by a factor actually not a particularly good artist, but right I think you know what I mean right.

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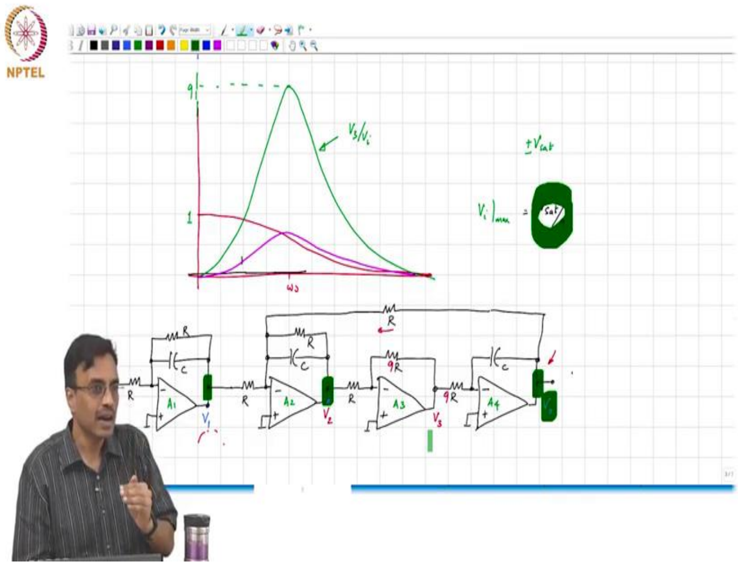

So, let me kind of the draw a cartoon. So, in other words if this was the original transfer function of V 1 was something like this V 2 was something like V 3 was something like that and V 2 was the same thing. Now, what happens? Well, we have 1 this is say 9 ok. And what is this transfer function this is nothing, but $\frac{V_3}{V_i}$ and just in case alright ok.

So, now the question is hey you know is this good or is this bad or it does not matter or what. Pardon, so let me call this A 1, amp 2, op amp 3 and op amp 4 so, if you did this A 3 will go into saturation.

What is the I mean the question I would like to ask therefore, is let us assume that the op amp saturates if its output voltages go higher than you know plus V sat or lower than minus V sat. And this V sat as a function of the power supply you already taken analog design classes you know that you know at some point in time when the voltage at the output goes very large then some device goes from goes either into the triode region or gets cut off or one of those things.

And therefore, there is some limit their which depends on the supply voltage, we assume those limits to be plus minus V sat. So, the question I would like to put to you is, what is the largest input amplitude of a sinusoid I can use right? Before any one of the op amps be gets saturated right. And why do we say any one of the op amps get saturated I mean the other three guys are working quite nicely why should we is it a problem if just one gets saturated.

Well, I mean once one of the op amps get saturated right the notion of transfer function does not exist anymore because this is the system has become non-linear. Right so, what is the maximum input amplitude that you can put in before any one of the op amps gets saturated it is simply V sat by 9, does this make sense alright.

In this particular example we see that even when the input is a sinusoid at omega naught for instance and is has a voltage has an amplitude which is V sat by 9 then A 3 will just get reached the edge of its swing limits and therefore, saturate and beyond that A 3 is within codes gone for it alright.

And therefore, you know the filter is basically now no longer a linear system it is non-linear and you know all sorts of weird things happened and we are not interested in that alright. So, what do you think is the design flaw therefore with this choice?

I mean evidently in the transfer function as far as the input output is concerned this is still maintained that is not changed, but it just so, turns out that this particular choice of resistor and capacitor values have led to a scenario where one of the op amps namely A 3 prematurely limits the maximum input swing that you can put in to the filter right.

And let me reiterate in an ideal network theory world where op amps are you know are can take infinite output swing there is no difference as far as the transfer function is concerned whether you know use 9R there or you use 1R right. But in practice we know that op amps must operate of a limited supply which means that they have limited swings.

Which therefore, means that if you make the input larger and larger at some point some op amp will get into the saturation region beyond which the system is no longer what you want it to be right and this particular choice of resistors and our capacitors right is a is it a good choice or can we comment on whether it is a good choice or a bad choice?

It is a bad choice because while A 1, A 2 and A 4 right have maximum gains which are, You know 1 or smaller right you have something which has a maximum gain which is several times larger than 1 right and therefore, you basically are in a situation where A 3 prematurely limits the maximum amount of input signal you can put in before the system becomes non-linear right. So, to summarize while there are an infinite number of choices of choosing the resistors and capacitors to achieve the same transfer function right many of those choices will be poor in the sense that one or the other of the op amps will prematurely limit the maximum input signal swing that you can put it does make sense?

Alright ok. So, what is the moral of the story? So, how do we fix this problem? What should we do therefore? What would you do? Let us say I you know somebody came and told you that well here is a 3rd order Butterworth filter I have done the math the transfer function is what I want alright ok.

And you know it seems like even when I put in V sat by 9 you know the filter is going berserk right help me fix the problem right. What would you do how do you go about fixing the problem?

So, what you would do therefore is plot the frequency response from the input to every to the output of every op amp right.

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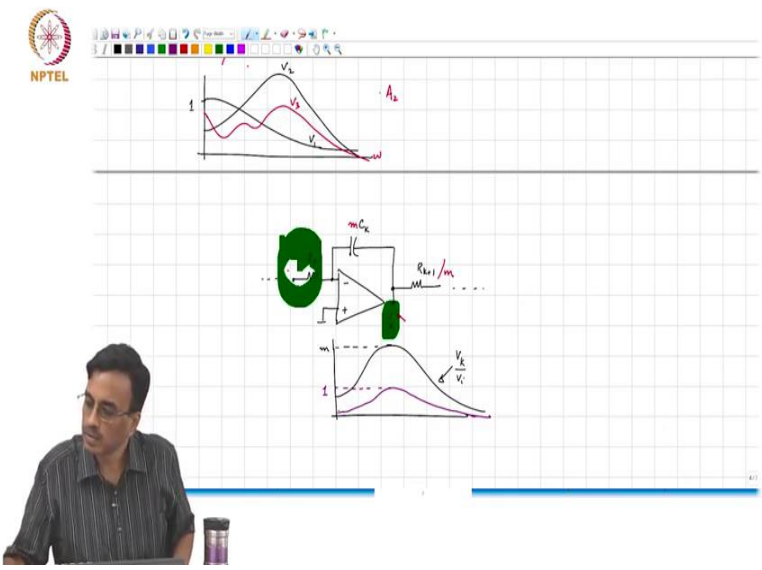

And you will get therefore, frequency responses which you know which in general you know can look this could be V 1 let us say this could be V 2. Again, this need not pertain to this filter could be something else this could be this is 1 let us say and then you have something else which is V 3 right this is all as a function of frequency alright.

If you have a high order filter at the output of every op amp you will be able to plot a frequency response from the input to the output of every op amp correct. And in general, there is no reason that the peak gains of all the op amps are there is no reason for the peak gains to be identical right ok. So, one op amps you know peak gain is bound to be higher than all the others right after all it is a maximum of n numbers right ok.

So, what comment let us say you know A 2 has the highest peak gain when you plot the frequency response curves as shown in this picture. What comment therefore, what conclusion can you draw? If A 2 has a peak gain and that gain turns out to be I do not know 22, what comment can we make?

What comment can we make? No that is the how do we fix the problem, but what do we what do we do with how do we so?

So, the conclusion is that A 2 limits the maximum input you can put in and it limits it to You know V sat by 22 right. And ideally what is the maximum input you would like?

V sat correct. So, what so, what would you do therefore, how do we how would we fix the problem? We would like we would like to. So, therefore, if you look at, I am going to illustrate it when we have a capacitor here right. So, let us say this is the kth op amp alright and let us say the peak gain at V k when we plot and then this goes on and this goes on alright. And let us say we have a peak gain $\frac{V_k}{V_i}$ and that peak gain is saying m alright. And what comment can we make about what should we do now?

We would like to reduce the peak gain of the frequency response from V i to V k, but without change in the transfer function that is the key point right. So, if you want the many ways of doing this you know a simple foolproof way that works is the following remember that the voltage V k is basically this current times is that current times the impedance correct. So, if you want to reduce the voltage by a factor of m and what you do you want to reduce it to.

The peak gain is m what do you want to make the peak gain? You want to make the peak gain 1 correct alright. So, a to reduce the peak gain by a factor of m what should you do? The current that is flowing in let us say you do not want to mess with the voltage is current times the impedance that is across the op amp, so, what would you do?

Reduce the impedance. So, what should I do to this capacitor? I should reduce the impedance so; I should basically increase the capacitance by a factor of m alright. But if I just did this and did nothing else what would happen to the transfer function.

The transfer function will change because the transfer function is what it should be only if this is if the peak gain here is m times V k now it is m times smaller and therefore, the current flowing through R k plus 1 right is also become m times small alright. So, if you want to bring the transfer function back to what it was earlier, what do you think you should do?

You should; We should have the same current that was flowing in that resistor ok as we had before. So, what should we do? You reduce this resistance by a factor of m alright. So, this way as a result of this operation what comment can we make about the frequency response now. If I do not want to draw the frequency response on the same picture what will happen?

What will happen to the shape? Shape remains exactly the same its only the it is only the peak gain that goes down by a factor of m. So, what was m earlier is now 1 does make sense right ok. So, we figure out a way of making the peak gain at the output of the op amp of one particular op amp 1. And what should I do now?

So, if we have to do this procedure for, all the op amps there is no need to do it you know 1 op amp at a time right ok it is very straightforward to I mean you all that you need to do is plot these frequency responses that is you know that is one measurement right you know the peak gains right.

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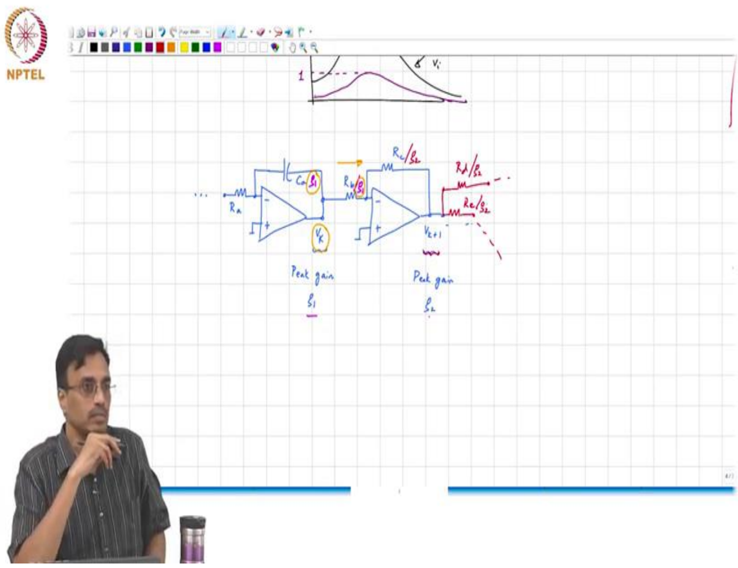

And you can do this in all this in one shot for example, again I am going to take a section of a larger section and just for arguments sake I am going to make this a resistor. Let us call this R a, C a, R b, R c this is V k this is V k plus 1. Let us say if argument's sake that the peak here of the frequency response from V i to V k let's say the peak gain was g 1 and the peak gain was g 2 we want the peak gains at both these outputs to become one.

So, rather than do it 1 op amp at a time if you want to do it in one shot what do you recommend that we do, what do we do? So, we would like to push this gain down by a factor of g 1. So, what would I do?

Student: (Refer Time: 33:21)

Well, you know you just reduce this by I mean you increase this by g 1 and reduce that by g 1 ok I need to increase the reduce the peak gain of this by ok. Let us say this is going somewhere else this is going somewhere else that happens I mean that scenario is very clear for example, look at look at this guy for example.

We see that its output I mean not only goes to I mean it is the is driving 2 resistors right one is you know of course, fed back on itself. So, if there are multiple resistors connecting at the output of an op amp I mean. So, let us call this R d and R e what should I do now? What would I do?

R c must become; The peak gain at V k plus 1 is g 2 I have to reduce its gain by a factor of.

Student: g 2.

Of what?

Student: By (Refer Time: 35:02) we have already you know main R r to R v by g 1. So, now, already m become R c divided by.

No, no see that is remember changing multiplying this by g 1 and dividing this by g 1 has not changed the current that is flowing into the through that resistor correct all that it has changed is changed the peak gain there.

Right so, what do we need to do? We just need to change R c to R c by g 2 right, that will push the peak gain at V k plus 1 down to 1 and then what do we need to do?

All the other resistors that are sensing this voltage should not know that the voltage has changed I mean you know the I mean the current through them should remain as it was before right. So, what comment can we make about R d and R e?

Both of them should be divided by g right ok.

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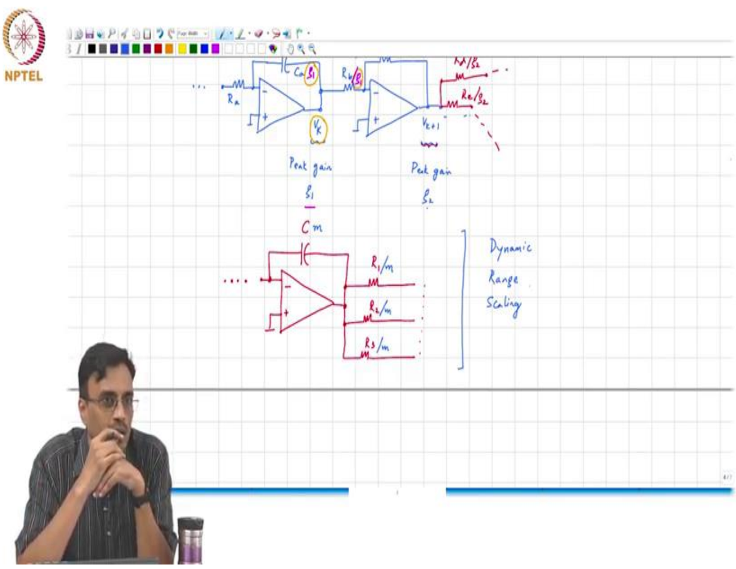

So, in general a filter basically will have potentially had this is the output of some op amp this goes to some virtual ground or the other right. If this is C this is R 1, R 2, R 3 and if you want to dynamic, I mean if you want to scale the peak by a factor of m you basically multiply this by a factor of m alright, that will pull the peak down by a factor of m and then you divide all these resistors by a factor of right.

You do this at every op amp and all this can be done in one shot ok. The end result will therefore, be I mean this process is what is called dynamic range scaling alright ok. And what is the end result after dynamic range scaling what happens what comment can we make about the peak gains at the outputs of every op amp?

They are all 1 right. So, therefore, no single op amp prematurely limits the peak swing that you can put in to the filter ok. So, all the op amps kind of you know hit saturation at the at recent right and the and you basically all the I mean the input that you can put in is the sinusoidal input that you can put in is at least in principle V sat, I mean this assumes that the op amp is all fine until you hit V sat and after that suddenly gives up right that is good enough for most purposes right.

In and so, therefore, therefore, the process of dynamic range scaling is necessary whenever you when you have when you make a filter because as we saw there are many choices of resistors in capacitors that yield the same transfer function right; however, they will not all be identical as far as the swings at the internal nodes are concerned right a wrong choice of component values can cause premature saturation of one of the op amps that are internal to the filter and therefore, they will that will premature I mean that will limit the maximum input swing that you can put into the op amp does make sense alright. And this is called as I said dynamic range scaling right.

Now the question nobody asked me was you know why should the peak gain be 1 what if it is 0.5? I mean all the peak gains are the same. So, no one particular op amp is messing you up what if the peak gains are only say you know 0.25 or 0.1 or you know 0.01?

Pardon. So, well you know one argument is that well this filter will be able to handle signals that are. If it is the peak swing is 0.1 V sat then in principle you can put in 10 times V sat at the input and the output at every op amp will only be will then hit V sat. He says that is you know that is a terrible thing to do because well you know that your supplies allow you to swing only between plus minus V sat. So, it is very likely that the input is also restricted to plus minus V sat.

Now, you unnecessarily seem to have a large range right and the outputs of every op amp if the output of every op amp is swinging very little right and it turns out that you will see why I mean on the face of it does not seem bad right. I mean what is the problem I mean you have a range which is much input range which is much larger than the signal level that the op amp can handle right. What do you think might be the problem?

Yes (Refer Time: 41:37) I mean remember whenever you are working with signals you know with circuits if you basically if all your voltage swings everywhere are very small then you basically the output becomes tends to be the fidelity of the output tends to be limited by noise right.

I mean it is like you are writing a computer program says you are doing some numerical I mean numerical solution of some you know equations and you find that all your numbers are changing only in the third decimal place right ok. Let us say you are doing some you know some attempting to solve some equation numerically on a computer and you know all the numbers somehow seem to be only changing in the you know in the third decimal place right is this good or bad.

Well, you know that every number has got finite precision correct and therefore, if all your numbers are changing only in the fourth decimal place or third decimal place it basically means that machine precision will now have a lot more effect on round off errors right in the algorithm when compared to somehow let us say before you got into this algorithm you scaled all numbers.

So, that you are using the full dynamic range of your number representation on the machine correct ok. I mean most of the time you know none of you guys bother about it because you are working with MATLAB or whatever which has you know 64 bits and you know that is its very forgiving of stupidity right.

But when you are actually implementing signal processing algorithms on a chip for instance and were having 64 bits is simply not an option because then you, I mean you would your cell phone would have to be plugged into the power socket all the time right.

Where you know the number of bits is something that you want to keep as small as possible while still doing what you want to do with adequate fidelity.

Then you want to I mean then you will end up facing the same issue because round off noise in a digital system is analogous to thermal noise in analog systems right. A resistor adds noise when you apply voltage right, I mean where you apply a voltage or not a resist you know a resistor adds some noise. If you apply a very small voltage the signal power is very small, but the resistor continues to add the same amount of noise right and therefore, the signal to noise ratio becomes op amp yes.

Yeah, I so which is why we will discuss; we will discuss why we cannot have peak swings which are much smaller than 1 right. It will become I mean at this point may seem harmless, but when we study noise, it will become very apparent why you do not want to work with very small internal levels right. You do not want to work with very large internal levels because then you stand the risk of saturation you do not want to work with very small signal levels because you stand the risk of getting drowned in noise alright you always want to work with signals which are you know not too large to cause saturation not too small to get drowned in noise right. I mean when I am speaking to you right, I mean it does not help me to kind of yell at you right because your ear is going to is going to saturate right. But then if I say well, you know these guys are very sensitive ears and you know if I whisper nobody hears anything right ok.

So, basically you know like the Buddha said its always the middle path is always the best do not either be this extreme or that extreme alright. And this so, the range of what you call good signals to use right alright which is you know a reasonable thing to define that would be you know.

What is the largest signal I can process without getting into saturation and non-linear effects what is the smallest signal that I can process without losing information because it is simply drowned in noise right? So, every system will have a minimum possible signal you can work with and a maximum possible signal you can work with right.

This ratio of minimum possible signal to maximum possible signal is called the dynamic range right. Today with the you know what you know so far you can you probably only appreciate that there is a maximum limit right, when we discuss noise, we will discuss the minimum limit right and the ratio the maximum limit to the minimum limit is the is what is called dynamic range right ok.