Circuit Analysis for Analog Designers
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Lecture - 24
Active-RC biquads and Impedance scaling

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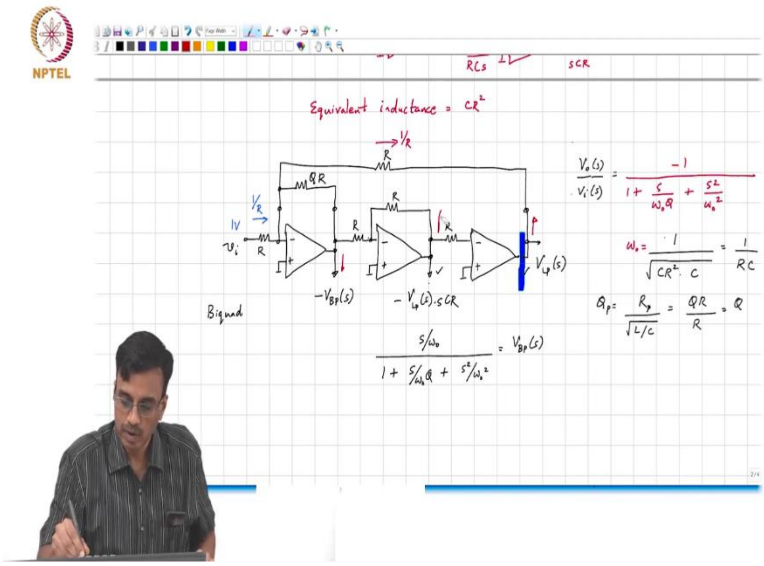

So, and so therefore, this is a low pass transfer function what comment can we make about this transfer function here? What comment can we make about this transfer function?

So, this is a this is V LP of s right. What is this transfer function? Yeah. We know V LP of s. So, what should this be? What is the transfer function of this from here to here what is the transfer function? Minus 1 by SCR. So, this transfer function must therefore, be?

VLP of s times Minus SCR right. So, that is basically S by omega naught divided by 1 plus S by omega naught Q plus S square by omega naught square. What kind of transfer function is this?

$$\frac{\frac{s}{\omega_0}}{1+\frac{s}{\omega_0}Q+\frac{s^2}{\omega_0^2}} = V_{BP}(s)$$

A DC, it is evidently 0, at infinite frequency it is? As this tends to infinity what happens? It goes to? 0 right. And somewhere in the middle it, it basically peaks. So, this is a band pass transfer function. Is it inverting band pass non-inverting band pass? Non-inverting band pass right.

So, this is said $V_{BP}(s)$ and what therefore, must be the transfer function here. So, we have seen what this is, we have seen what this is, what is that transfer function?

Well, this is nothing but an inverting amplifier. So, this is basically minus $V_{BP}(s)$ right. So, looking at the voltages at different points in the circuit, it is possible to you see that we get both low pass and band pass responses right. In our example, we are only interested in the low pass response.

So, our final output will be the V LP of s right and recall as you can see in the picture there that is an inverting low pass response right and. So, now, as I said this is what is called a Biquad ok. And which is you know the slang for bi-quadratic section. Now if you want to build a high order filter therefore, what do you do now?

There is cascade about bunch of these things right and can we make any comment on the parasitic sensitivity or insensitivity of this structure. Let us say every capacitor is associated with some parasitic to ground right. What comment can we make?

Well, all parasitic are either at virtual ground or the output of an op-amp; and therefore, just like in the first order section right the whole structure is insensitive to parasitic. And, this is extremely important practice because remember that the idea is to have filters you know on a you know you have a big SOC for instance where you have filters, the output of the filter goes into an A to D converter.

And then you have a DSP and the DSP is basically a lot of digital circuitries correct and they are all sitting on the same silicon substrate right. So, what happens? Therefore, is that the digitals you know the digital circuitry basis basically can cause you know a lot of what is called ground bonds right simply because you know that a CMOS inverter basically draws a big spike of current from the supply to ground at every clock edge right whenever the input data changes ok.

So, there is basically a lot of junk on the ground and therefore, if ground is not and if you have a structure which is sensitive to parasitic capacitance, then through that ground which we thought was ground is not really ground and therefore, can you know can inject you know noise into the into the filter and so on.

So, parasitic insensitivity is actually a very useful and important attribute in facts. Of course, there is also an issue of robustness in the sense that when you have parasitic capacitances, you basically are not in a position to exactly predict the omega naught and Q.

Simply because you know if you have some state capacitances which modify the integrated transfer function then you have you know you do not know what omega naught you are getting and what Q you are getting and therefore, there is again a problem with respect to robustness.

Now, a couple of points that I would like to draw your attention to right we would like to make sure that there is negative feedback. Remember, the difference between the input terminals of the op-amp is 0, only if there is DC negative feedback around the op-amp right and you know the question is you know is there a DC negative feedback around each of these op-amps right.

So, for example, let us take a look at the last op-amp for DC the capacitors are open. So, I am going to remove that and I break the loop alright. So, if I yank this side up sorry yeah if I yank this side up, what happens to this voltage?

If I mean well we assume that the other op-amps are negative feedback. So, if that goes up what happens? This goes down. If this goes down, what happens to this voltage? Goes up. If that goes up and we have assumed these signs. So, what happens to the output of this op what happens to this output? Goes Down right. So, is there negative feedback around this op amp?

Yes right. So, likewise you can go and do this for all the op amps and you know when you come up with the new circuit you know the first thing you should check and make sure is that you choose the signs of the op-amps properly. So, that your assumption of the virtual ground remains within ports you know is actually valid, does make sense alright.

So, that is the that is one thing that you would be aware of the next thing that I would like to draw your attention to is the following.

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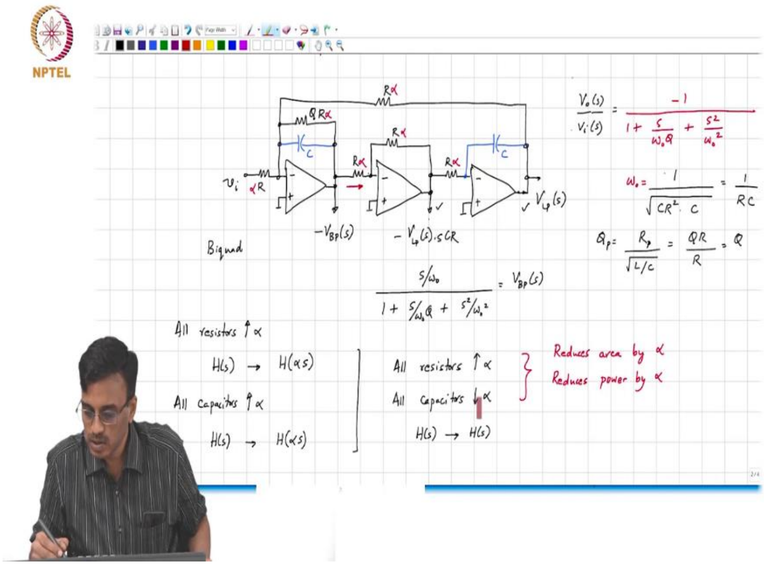

So, let us say all these resistors are scaled up by the same factor. In other words, I multiply all resistances by alpha. Now, we simply missing the capacitors alright; all resistors scaled up by alpha, what comment can we make about the transfer function?

Pardon, all resistors go up by alpha then earlier let us say you had a transfer function H of s; now what happens to H of s? What happens to Q?

Well, Q is remains the same because it is a ratio of like quantities right it is dimensionless. So, it does not change. What about omega naught?

Omega naught is 1 by RC, all resistors will change by a factor of alpha. So, the omega naught goes down by a factor of alpha. So, what happens to H of s?

This is H of alpha times s alright; now all.

$$H(s) \rightarrow H(\alpha s)$$

So, if all resistors remain the same, but all capacitors go up by a factor of alpha, what comment can we make? What happens to Q again? What happens to the quality factor?

It remains the same, right. Again, the idea being that Q is a dimensionless quantity if all capacitors are increased by the same factor, then you will get. So, what happens to; what happens to omega naught? Comes down, right. So, what happens to H of s? H of alpha s right ok.

$$H(s) \to H(\alpha s)$$

So, now, I am going to ask you third question I, all I increase all resistors by a factor alpha, decrease go down by a factor of alpha right, what comment can you make about the transfer function?

Well, we can do this in two steps right. All resistors go up by alpha, you basically get H of alpha s; all capacitors go up by a factor of 1 by alpha. So, you know H of alpha s times 1 by alpha. So, you basically nothing changes correct.

$$H(s) \to H(s)$$

So, now, the question is what are we achieved by doing this? Right, it seems like if you increase all resistors by a factor alpha and all capacitors are reduced you have the same transfer function right.

So, what do we gain by doing this? Is there any practical advantage to increasing all resistors by a factor of alpha and reducing all capacitors by factor of alpha?

Well, capacitors all become small. So, and you know as you know on IC capacitors are parallel plates that are you know that are planar and therefore, if you make the capacitors all smaller, evidently the filter itself will become very small, right. There is a further advantage in the sense that if all resistors increase by a factor of alpha, what comment can you make about the currents through the resistors?

Well, if all resistors increase by factor 10, all currents through them go down by a factor of 10. Now, if all currents go down the who is actually supplying these current? The op-amps are basically supplying these currents correct. So, if all currents in the resistors go down by a factor of alpha then the op-amps job becomes much easier because it has to supply a less, you know much smaller current. If the op-amp needs to supply a much smaller current; remember, that an op-amp inside is made with transistors; transistors have to be biased and you know the current that is coming out of the op-amp is you know within quotes the incremental current right. So, if you need a much smaller incremental current that the op-amp has to supply, what comment can we make about the bias current itself?

Remember, that the increment is supposed to be a small fraction of the quiescent current. If the incremental current that you are supplying you know needs to be much smaller, then you can basically the bias currents in the active devices inside the op-amp can also be correspondingly small right. So, increasing resistance and reducing capacitance has you know two benefits. One is of course, the area occupied by the entire filter which is predominantly that occupied by the capacitors becomes fundamentally smaller, right. And second, since the op-amps now have to drive you know or supply currents which are much smaller it means that the quiescent current that they can dissipate right also becomes much smaller.

For instance, if the incremental current became smaller by a factor of 10 let us say we increase all the resistors by a factor of 10, then the incremental currents or the currents that the op-amps need to supply go down by a factor of 10, right which in principle means that the quiescent currents can also all go down by a factor of, by factor of what? By a factor of 10, correct. So, increasing I mean doing this basically reduces area by a factor alpha, it also reduces what else?

Quiescent current and quiescent current is related to the power dissipated because that is the total part dissipated is simply the sum total of all the quiescent currents in the op-amp multiplied by the supply voltage. So, this also reduces power dissipation by a factor of alpha correct. So, this seems to be a great idea alright. So, what would you do you know.

So, what is the obvious thing to do therefore? So, I choose alpha to be as large as possible which is equivalent to saying make the resistors larger and larger and larger and larger and the capacitors become smaller and smaller and smaller right. So, in the limit we will have a filter that occupies no area and dissipates no power right because alpha tends to infinity evidently the currents in the resistors tend to 0, right.

And therefore, the power dissipate in the op-amp can also in principle tend to 0 and the area occupied by the capacitors; obviously, tends to 0, because I am decreasing the capacitors right. The transfer function evidently remains the same because that is what we discussed right ok. So, it seems like you can get away by having the you know 0 power, 0 area and you know the same function right.

So, there is you know I remember one thing something that sounds too good to be true right, is often something that sounds true to be true is often you know really too good to be true right. So, there must be a catch we must be missing, we must be missing something right. I mean this sounds exactly like you know you know give me your money and I will double it in you know you know in 2 minutes right.

I mean I hope none of you is naive enough to believe. Yeah. Jishnu had a point yeah.

Well, yeah I mean you know he is his he says we as you keep increasing resistors right, the resistors do I mean can principle tend to get bigger, but it is not a fundamental problem, its a you know it is basically I mean well I can all to get a larger and larger resistance, what can I do? Will make it thinner, make the wire thinner and thinner and thinner.

I mean at some point of course, there will be a physical limit to how thin you can make this, but it is not a fundamental right. In principle, I can make something you know an angstrom thick and then you know I get infinite resistance right ok, alright. So, as you can see therefore, there is something that we are missing right something fundamental that we are missing.

Because you know this whole premise sounds, sounds too good to be true right and you know what we are missing is it turns out that the resistors you know also add noise of their own right. And therefore, the output of the filter consists of you know two parts; one part is the input signal which has been filtered the other part is each one of these resistors is adding noise right.

And you know that kind of from the flow from each resistor to the output there is some transfer function right. And therefore, we have to the output will not only consist of the filtered input, but also noise from the filter itself right. And if and it turns out that as the resistor becomes larger and larger it adds more and more noise and the output noise of the filter becomes larger and larger right.

So, therefore, indeed there is a catch and if you go on making you know the resistors larger and the capacitors smaller and hope that you can do the same job with fewer and fewer resources you know you are going to be terribly mistaken. So, we will. So, that will I mean that is you know motivation to go and study noise which will do you know a few classes down the line right.

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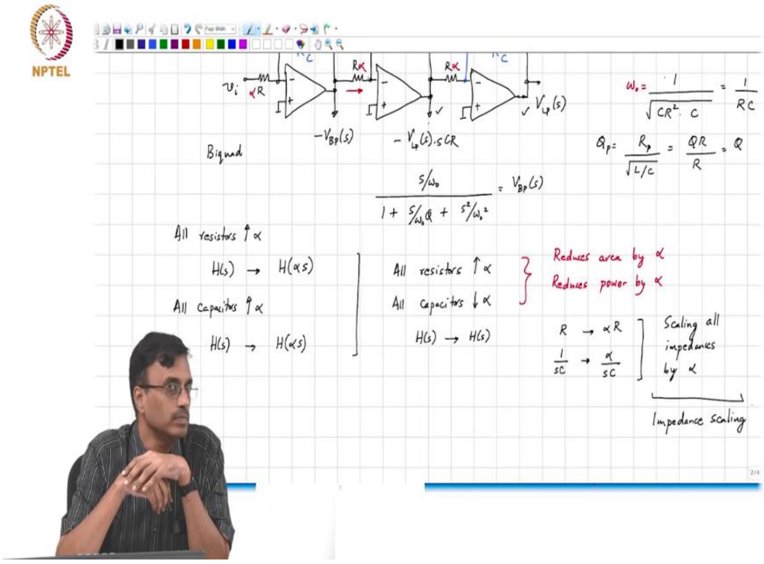

But that is something that you need to bear in mind. And this process where at all the resistors have increased by a factor alpha and all capacitors are decreased by a factor of alpha. What is the I mean decreasing the capacitor of alpha, what does it do to the impedance?

Remember, that the impedance of the capacitor is 1 over SC and it is become if you reduce the capacitor by a factor of alpha it now has become alpha over SC.

$$R \rightarrow \alpha R$$

$$\frac{1}{sC} \rightarrow \frac{\alpha}{sC}$$

And therefore, as you can see doing this is equivalent to scaling all impedances by the same factor alpha. So, if you take an electrical network and multiply all impedances by the same factor, what comment can we make about the transfer function? Remember, a voltage transfer function basically is a dimensionless quantity and must be therefore, a ratio of, a function of ratios of like elements right and therefore, the transform I mean the transfer function does not change.

And so, this is what is called impedance scaling and that does not change the transfer function alright, what does it change therefore? I mean if it does not change the transfer function, what does it change? We just discussed this you know a few minutes back.

It I mean it changes it turns out that it changes the noise produced by the filter itself ok. which will have a opportunity to study in great detail you know a little down the line right, but you should go away with the impression of you know saying you know let me go and increase all resistors and reduce all capacitors you know by a factor two million and then therefore, you know I have a tiny filter and yeah you know it does the job as well, so.

Thank you, and.,