Circuit Analysis for Analog Designers
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Lecture - 23
Op-amp-RC biquadratic sections

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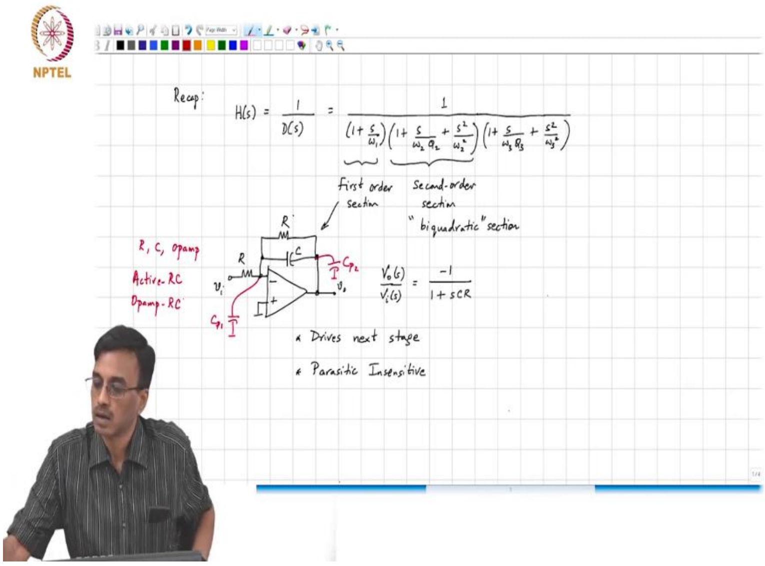

In the last class we were discussing the realization of a high order filter transfer function. As this could be an arbitrary filter, but we have looked at so far, we have looked at all pole filters which are and we approximated the ideal brick ball by using the Butterworth polynomial. And we then realize that we can break up a high order polynomial into a product of you know first and second order terms and I mean if it is an odd order polynomial then you will have, I mean you can definitely have a first order term.

And each one of these can be realized using individual building blocks and the high order transfer function can be realized using simply a cascade of these sections alright. And the first thing that we discussed yesterday was how do we go ahead and realize a first order section, whose transfer function is $\frac{1}{\left(1+\frac{s}{\omega_1}\right)}$, ω_1 corresponds to the 3 dB bandwidth of the first order section.

And it is not merely enough to have an R and C and say ok, that also gives as a first order transfer function. One must realize that when we cascade sections together loading does not affect the frequency response of the previous stage right. And as and as we again we were discussing yesterday we use an op-amp in fear and with negative feedback around it, to realize a first order low pass filter.

And the op-amp serves two purposes; one is that it facilitates cascading of stages easily because the output impedance of the open stage is 0. So, it can drive any load at least in principle without its transfer function getting modified right the second aspect that we saw yesterday is a practical one. Mainly that every capacitance, in fact, even every resistance for that matter there is always going to be a parasitic capacitance between those terminals and ground ok.

And it so happens that the transfer function in this case is not modified by parasitic capacitances on both plates of the capacitor or both terminals of the resistor. Because in all cases the parasitic capacitance is either sitting at virtual ground right where the swing of the I mean the voltage swing across this capacitor is C_{p_1} is 0 simply because one terminal is real ground the other terminal is virtual ground.

Virtual ground and as long as the op-amp is the ideal you know C_{p_1} has no role to play on the transfer function. Likewise, C_{p_2} there is a voltage swing across C_{p_2}, but realize that C_{p_2} is in parallel with appears in parallel with the voltage output of the op-amp and therefore, all the current that goes into C_{p_2} is being supplied by the Op-amp.

Op-amp, the voltage across the feedback branches the RC circuit in parallel is not modified and therefore, the output voltage remains the same, the transfer function remains the same. So, this is an important practical aspect that is associated with such a structure. And as you can see this filter structure, I mean it is very simple no doubt, consists of RC and an active element, which is the op-amp. So, these are often called active RC filters or op-amp RC filters alright. And, so that was what we were doing yesterday.

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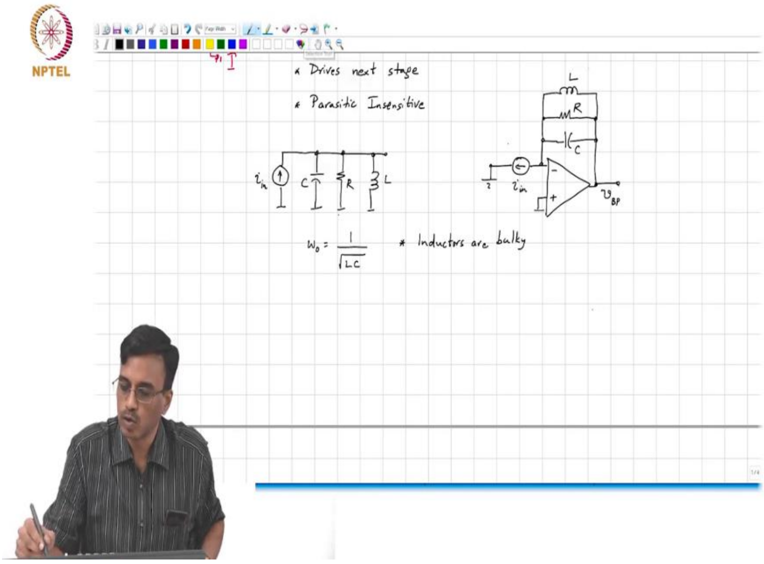

Now, the next job is to basically realize figure out, how to realize the second order section and remember and as we said yesterday, well we can draw our inspiration from any number of second order filters that we already know. For example, this is a simple structure that we have seen in our earlier classes right, the voltage output as we all discussed yesterday is a band pass output. If you want a low pass output, what would you do?

The inductor current has a low pass response. Now, the question is you know again we would like to make sure that this section right, basically does not load, I mean does not is not affected by a loading of the next stage. So, we would like to have 0 output impedance and you know a straightforward way of doing that is to avoid loading, is to say ok well. R and this are L right and we must somehow figure out the way of tapping of the current through the L alright. The what comment can we make about. So, this is i_{in} let us say, what comment can we make about this voltage transfer function? This is?

When remember, all that in the circuit on the left this current is flowing through the RLC network, the circuit on the right also the current is flowing through the RLC network. So, the voltage across the RLC network which is band pass right, that will now appear across the op-amp, except now that the op-amp is able to drive.

You know any load at least in principle. So, this is the band pass output, but there is another problem I mean and you know we hope that we will be able to kind of somehow tap off that inductor current in some way. But there is a more fundamental problem with RLC filters. And what do you think that problem might be? Or rather I would say a more you know fundamental practical problem with RLC filters and what do you think that might be?

The inductor right; and you know all of you have taken enough physics classes to know that inductors are big and bulky. And they become bigger and bulkier when the frequencies involved are small correct, because remember that the resonant frequency of this LC network is 1 over square root of LC radians per second. And therefore, if you want a low frequency right what do you call a low frequency pole pair which is what you would need, if you had a low pass filter with a low cutoff frequency, right. You know for example, you wanted to filter voice before it went into an A to D converter.

And you would need bandwidths in the kilohertz range right and the L and the C values you know either one or both would become so large so as to be completely impractical right. So, inductors therefore, the problem is that inductors are bulky and expensive right. And therefore, you know are not suitable in a whole lot of applications that target frequencies that are small. So, now the question is you know is it possible to achieve the same functionality without the use, explicit use of an inductor, alright.

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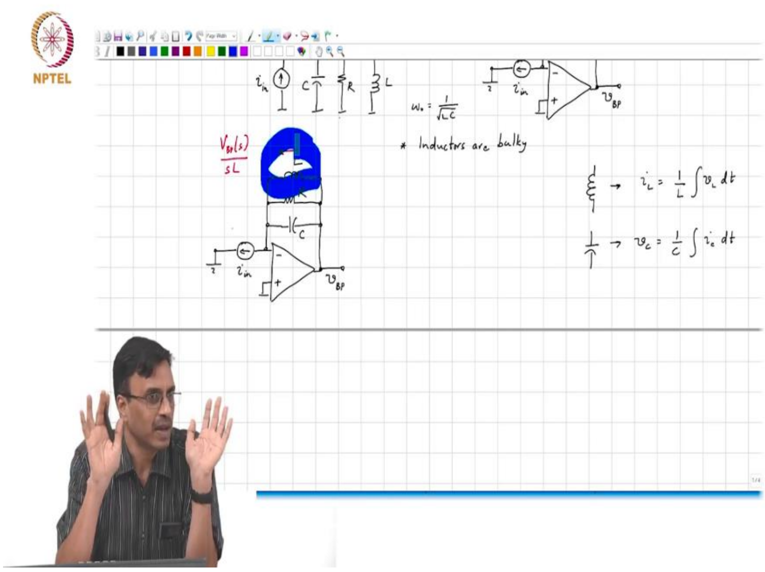

And the idea is the following. What is the current that is flowing through the inductor in terms of the, in terms of v_{BP}, what is the current that is flowing through the inductor?

$$\frac{V_{BP}(s)}{sL}$$

So, in the time domain what does this correspond to? Integration. So, I would like to kind of draw your attention to the following right. In an inductor the current is; how is the current related to the voltage across the inductor? How is i_L related to v_L?

$$i_L = \frac{1}{L}\int v_L \ dt$$

What about a capacitor?

$$v_C = \frac{1}{C}\int i_C \ dt$$

So, you can see that whether it is the inductor or the capacitor, both of them are performing the mathematical operation of integration, correct. The inductor however integrates voltage and generates a current right. In the capacitor integrates current and generates a voltage ok.

So, basically, we want to replace an inductor, an inductor is an integrator right. So, whatever you do, they must be some integration in that box. So, let us say you put a magic box here right, whose job is to do the same thing as a physical inductor without using one right. What mathematical operation must be going on in that box?

There must be integration inside that box right. We only know two types of I mean two elements, which are integrate as in some way. One of them is the inductor, the other is the?

The other is the capacitor right. We already saying that we do not want to use the inductor right. So, whatever you do inside they must be a capacitor. Does it make sense right? So, in other words if we are supposed to get rid of this inductor right with basically a black, box a magic box let us say.

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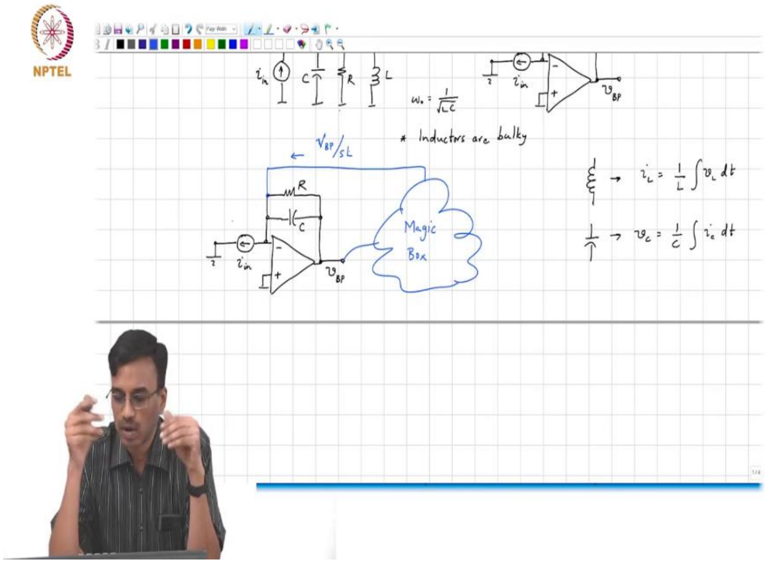

Well, its job is to look at v_{BP} correct. And what should it do? It should generate a current which is? It should generate a current which is $\frac{V_{BP}}{sL}$ ok and it should do it without using an inductor. I mean I cannot say the magic box in that inside I have another inductor, that does not make sense correct. And we need only one integration. So, in principle what do you think must be there inside that magic box?

There must be at least one capacitor. Does make sense? Alright.

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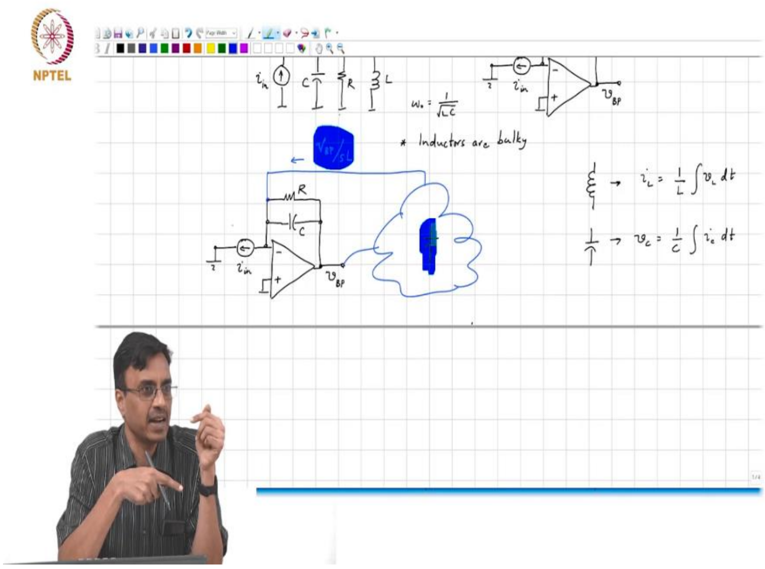

But unfortunately, this capacitor is a great integrator; however, it integrates? Current and generates a voltage. But what do we want?

We want to integrate, remember that this quantity here is an integral of, it is a current which is an integral of the voltage right, but our man inside can only integrate. We need to integrate a voltage and generate a current, which is the integral of the voltage, our man inside can only Integrate a current and generate a voltage right. So, what do you think we should do? What do you think we can do?

Convert voltage into current right, ok. Integrate that current because the is the capacitor our front inside can only integrate a current. So, you get a voltage, which is now an integral of the current right. And then what do you do? But what you are looking for is a current. So, what, but you have a voltage what do you do?

You convert a current back into, I mean a voltage back into current that is the principle right, ok. And you know this can be done in many ways and you know you already know how to convert a voltage into a current. And what do you think that is?

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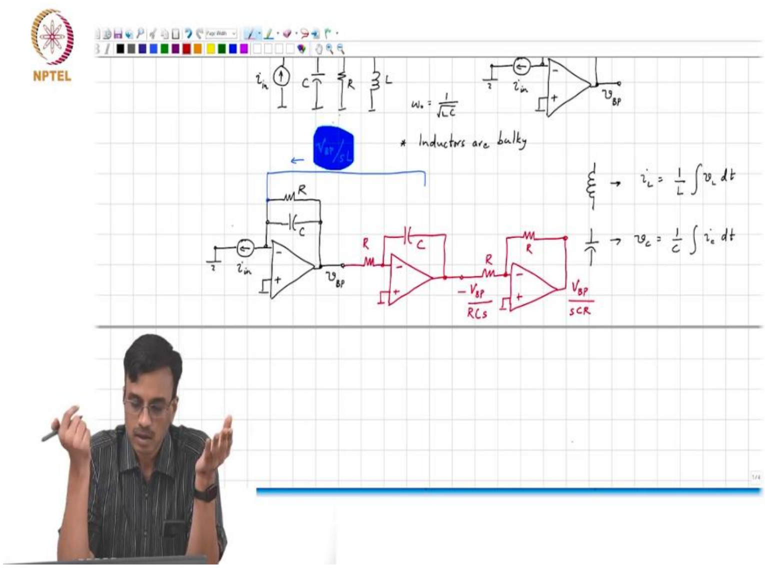

Well, I said you know you have seen this before right. We want; you seen. So, this is our friend the capacitor inside right. So, let us call this R and there is no need for this R to be the same as that R, but I just chose that that way ok. So, what is the output voltage here? Minus V BP by.

So, how we converting V BP into a voltage, into a current? That resistor R in red is basically converting. Voltage into a current and that current is flowing into the Capacitor. So, the output volt, output voltage is therefore an integral of the input voltage by and is related through RCs ok alright. But we have a small problem now, what is that problem?

$$-\frac{V_{BP}}{RCs}$$

We want that negative sign is a problem, we want the integral of V BP not the inverted version. So, well if you want to get rid of a negative sign what do you do?

And you want to get, you want to have an inverting amplifier with a gain minus 1. So, do you know how to do that? We know how to do that, what do you suggest that we do?

Ok, well; this is R and this is R. So, this is $\frac{V_{BP}}{sCR}$ correct. So, we now have that you know all important $\frac{V_{BP}}{s}$ term correct, but we have a voltage now, we need to convert it into a current. So, what do you do? This current must be, we need a current which is $\frac{V_{BP}}{sL}$ right, we have a voltage. So, what do you suggest, how do we get from $\frac{V_{BP}}{sCR}$ to a current, which is $\frac{V_{BP}}{sL}$?

V and what is the intuition? The well, we know that this potential is what is that potential?

A 0 right. So, simply connect converting voltage into current is straight forward, we just simply put a resistor here, alright.

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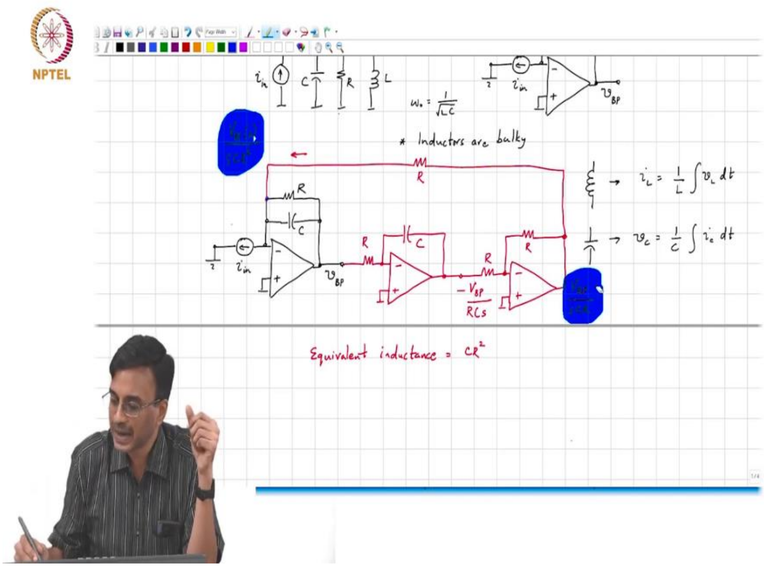

And so therefore, what is the current that flows here?

$$\frac{V_{BP}(s)}{sCR^2}$$

So, this magic box in red is therefore, doing the job of an inductor correct; and you know what is the equivalent inductance? CR square;

Alright ok. So, what is this correspond to? It is a voltage alright, but in the original circuit, I mean this how is I mean this corresponds to what does that current correspond to?

It is the current through the inductor correct ok and $\frac{V_{BP}}{sCR}$ is?

It is a voltage.

You know which is proportional to, which is related to the current through the inductor by a multiplication factor R, correct. So, this voltage is a proxy for the inductor current correct, because if you know this voltage the current is simply this divided by R ok, alright. And we wanted a low pass filter. And what was the idea behind the low pass filter? The low pass filter was the inductor current alright. So, if we simply tap this voltage of, what does that correspond to?

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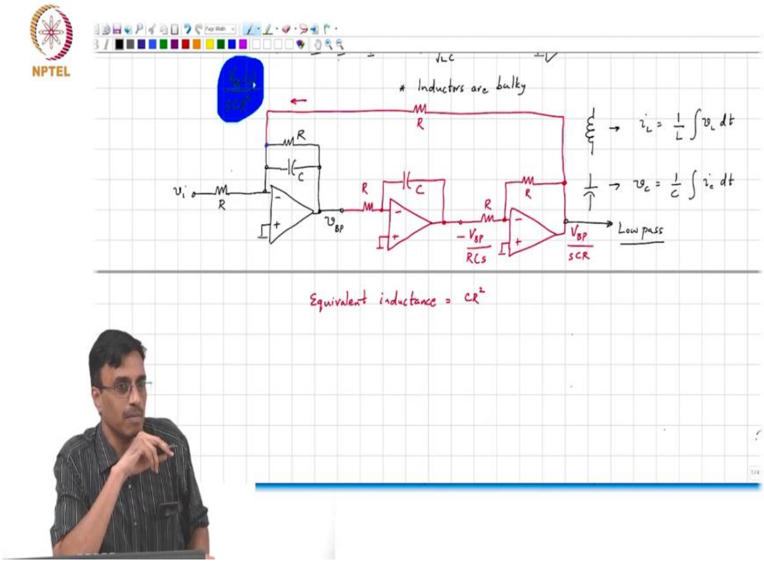

It is a voltage which is low pass; does make sense people? Alright. So, and the last thing is that we would like to have voltage input, voltage output. So, if you wanted the input to be a voltage, but not a current what do you think you can do?

Well, let us straightforward basically say, all well this is R and this is v_i ok. So, this is an example of. So, this basically represents a you know a second order low pass filter at this node ok and the way it is drawn in the textbooks; I mean you know can you comment on the order of the integration and the inversion, can it be interchanged or it can need it be the same or what?

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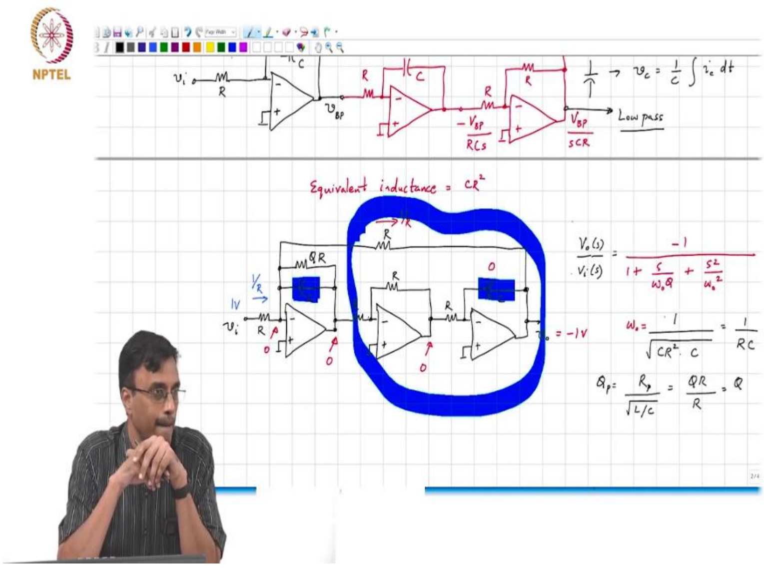

It can be interchanged and you know the way it is drawn in the textbooks is simply. So, and I just again as I said there is no reason for this capacity two capacitors to be the same, there is no reason for all the resistors to be the same right. In fact, they must at least be; I mean how many independent parameters are there in the second order low pass filter section?

ω_0 and Q right so; obviously, you know there must be another factor here, which is independently variable and that is it turns out to be that resistor there. And as we will see you know simply making that QR it turns out and I leave this as homework, is results in a transfer function here which is.

So, let us say this is v_i this is v_o. So, V o by V i of s is nothing but; what is the DC gain? How do you figure out what the DC gain is? Let us say this you are looking at the circuit for the very first time, right. I am only interested in the DC gain ok, one way to find out is to go and solve the entire transfer function and put s equal to 0, right and that is kind of you know a silly way of doing it.

Because if I am only interested in the DC gain it does not make sense to find the whole transfer function put in all that effort and then put s equal to 0 and throw away most of that information that you have, correct. So, if you are only interested in the DC gain you should be able to do it simply staring at this circuit and. So, what how do you think we will do it?

At DC what happens? All capacitors are open. So, this capacitor both these capacitors are open alright. So, if I have a DC voltage say 1 volt here right. No; I have the current that is flowing here is what?

1 by R; and where must all that current flow? Pardon. It must flow as? It must flow into; nothing can flow through the capacitor correct. It can only flow through QR and R, we do not know how much flows through R and how much flows through. QR alright. But is there a way of finding out how much flows through QR? We look forward, what is the current flowing through this capacitor at DC?0; if this current is 0 on average, what comment can you make about this voltage on average, what is the DC value of that voltage? 0; if this DC voltage is 0 what comment can you make about that DC voltage? Or this is simply an inverting amplifier. So, that is 0 right. If that is 0 what and what is this voltage?0. So, if; so, what is the current flowing through that QR resistor? 0 right. So, all this 1 by R current therefore, flows through which resistor the feedback resistor on the top. So, that is 1 by R. So, if you put 1 by 1 volt here what is the output voltage at DC?

Minus 1. So, what is the DC gain? Minus 1 divided by 1 plus S by omega naught Q plus S square by omega naught square ok. And what is the expression for omega naught? Remember.

$$\frac{V_o(s)}{V_i(s)} = -\frac{1}{1+\frac{s}{\omega_o Q}+\frac{s^2}{\omega_o^2}}$$

This whole box here, what was it trying to do? It is trying to emulate the functionality of an Inductor; and what is the value of that inductance? Right value of the inductance and that is nothing but CR square. So, omega naught is the resonant frequency of that LCR network that we had earlier. And therefore, that is 1 over square root inductance times capacitance, the inductance is CR square the capacitance is C. So, this is simply nothing but 1 over RC alright ok.

$$\omega_o = \frac{1}{\sqrt{CR^2C}} = \frac{1}{RC}$$

What about Q? How do we establish the quality factor of the tank? The quality factor remember for an RLC in a parallel network is nothing but very good its R divided by square root of? L by C and a good way of remembering this is that you know quality factor basically quantifies you know the amount of loss in the LCR network right. So, when they will you lose a lot of energy when R is high or when R is low in a parallel RLC times.

$$Q_p = \frac{R}{\sqrt{L/C}}$$

Ideally when R tends to infinity you must have you have no loss and therefore, the quality must tend to infinity. So, in the parallel RLC network a good way of remembering the formula for Q is r divided by Q is dimensionless. So, it must be a ratio of like quantities if R is on the numerator the denominator must have something of dimensions of impedance or resistance and that is simply square root of L by C right, which you no doubt seen in your you know EM classes when you are studying transmission lines. So, Q is R divided by square root of L by C, in a series RLC network Q is?

Square root of L by C divided by R right in a series RLC network if R tends to 0, then you have no loss ok and the quality tends to infinity. So, here what do we see? What is that the parallel resistor here. Is simply nothing but QR divided by. What is square root of L by C? L is nothing but R square C and the C goes away. So, this is nothing but R. So, you can see that the quality factor of the tank is. Does make sense people?

$$Q_p = \frac{QR}{R} = Q$$