

**Circuit Analysis for Analog Designers**  
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**Lecture - 22**  
**Cascade-of-biquads, realization of stray-insensitive first-order section**

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$$H(s) = \frac{1}{D(s)} = \frac{1}{(1+s) \underbrace{\left(1 + \frac{s}{Q_1} + s^2\right)}_{\omega_p=1} \underbrace{\left(1 + \frac{s}{Q_2} + s^2\right)}_{\omega_p=1} \dots}$$

Now, the question is you know how do you realize the next question is how do you realize the low pass filter and in general therefore, the high order Butterworth filter or high order any filter for that all pole filter for that matter will basically be can always be realized as a product of First and second order terms.

So, it is 1 plus s by 1 I mean I am again going to stick to if I assume that Butterworth it is 1 by 1 plus s. So, if for an odd order Butterworth there will be a pole at minus 1, for an even order Butterworth you know there will be only complex conjugate poles and you basically have you know s by Q 1 plus s square times 1 plus s by Q 2 plus s square and so, on right.

$$H(s) = \frac{1}{D(s)} = \frac{1}{(1+s) \left(1 + \frac{s}{Q_1} + s^2\right) \left(1 + \frac{s}{Q_2} + s^2\right) \dots}$$

So, the  $\omega_p$  corresponds if you write this in standard second order form what does  $\omega_p$  correspond to? 1 and what is the quality if I mean and Q 1 corresponds to the quality factor

what comment can we make about  $\omega_p$  of this second order section? Also why does it make sense that all the  $\omega_p$ 's are 1 or does it make sense.

And that is ok, but that I mean this could what I am asking you is, why is how is I mean is it a coincidence that the  $\omega_p$  of this is 1 and likewise  $\omega_p$  of this section is also 1. Very good right.

So, it is indeed a coincidence in general that need not be true, but in the special case of Butterworth filter realize that all the radius of the magnitude of all these poles is 1 right and therefore, it must follow that the product of the roots must also be 1 and you basically see that you have 1 plus. So, it is not a general thing it only happens in the case of a Butterworth ok.

Now, the question is ok. So, we have decomposed our filter I mean our filter transfer function into you know product of first order and second order sections and therefore, you know we should be able to we should be able to implement. How would we implement this what I mean what do we need to learn to do?

If we know how to implement a first order section and if we know how to implement a second order section, we can implement we can just cascade these sections and get any order filter without any problem right and therefore, that is basically what is done in practice right.

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The slide contains the following content:

- NPTEL Logo** (top left)
- Transfer Function:** 
$$H(s) = \frac{1}{(1+s)(1+\frac{s}{\omega_p} + s^2)(1+\frac{s}{\omega_c} + s^2) \dots}$$
- First-Order Section:** 
$$\frac{1}{1+\frac{s}{\omega_c}}$$
- Parasitic Sensitivity:** A diagram showing a feedback loop with a resistor  $R$  and a capacitor  $C$  in parallel, with a gain  $M$ . The transfer function is  $\frac{V_o}{V_i} = \frac{-1}{1+sCR}$  and  $\omega_c = \frac{1}{RC}$ .
- Second-Order Section:** A diagram showing a series resistor  $R$  and inductor  $L$  followed by a parallel capacitor  $C$ . The transfer function is  $\frac{1}{1+\frac{s}{\omega_p} + \frac{s^2}{\omega_p^2}}$ .
- Another Second-Order Section:** A diagram showing a parallel combination of a resistor  $R$ , capacitor  $C$ , and inductor  $L$  in series with a resistor  $R$ . The transfer function is  $\frac{V_{sp}}{i_{in}}$ .

First order section. So, general first order all pole section what is the transfer function? So,  $\frac{1}{1+\frac{s}{\omega_0}}$  right ok and how do we realize this?

Yeah, just an RC problem with an RC is that ok alright. So, one approach is to simply say ok you have R and C and  $\frac{1}{RC} = \omega_0$  alright. What comment can we make about second order section?  $\frac{1}{1+\frac{s}{\omega_p Q_p} + \frac{s^2}{\omega_p^2}}$ , What do you think we can do?

Well one suggestion is basically says ok RL and C this is one option, another option if you have an input current is basically RC and L alright. So, what comment can you make about the voltage across the inductor here? What kind of transfer function is that? Input current to output voltage here is what kind of transfer function? What is the value of the transfer function for DC 0, for infinite frequencies it is 0. So, this is a band pass transfer function right. So, if you want a low pass transfer function what are we supposed to look for? Yes. Current through inductor very good. So, basically this is  $i_{LP}$  right likewise in this; this is nothing, but  $v_{LP}$  that is a low pass voltage output right and across the inductor you have high pass filter transfer function and so, on alright.

So, let us say oh we like this we like this and we like this. So, how do we realize the third order transfer function? Very good ok.

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The whiteboard content includes:

- NPTEL logo in the top left corner.
- A handwritten transfer function:  $\frac{1}{1 + \frac{s}{\omega_p Q_p} + \frac{s^2}{\omega_p^2}}$
- A circuit diagram showing a current source  $i_{in}$  in series with a resistor  $R$ , an inductor  $L$ , and a capacitor  $C$ . The output voltage  $v_o$  is measured across the capacitor.
- Another circuit diagram showing a series combination of an inductor and a capacitor.

So, one suggestion I get is oh, how about taking the first order transfer function and then the second order transfer function and is this ok. Well, if you do this then you know the transfer function is all messed up it is not they say the product of the two transfer functions because the second section will load the first one and therefore, that is a problem and so, therefore, we need a buffer right some kind of controlled source so, that loading is not a problem right.

So, which is why people try to design sections where you can cascade the individual sections you know without having to worry about you know loading effects in other words the section itself must have you know either a high input impedance. So, that it does not load the next stage or a low output impedance so, that it does not load the I mean. So, it can drive the next stage.

And you know as you all know the opamp is the workhorse of Analog engineering. So, you know one thing you can say is ok let me try and see if I can use an opamp to realize the R C low pass filter right first order transfer function and why do you need the opamp? Strictly speaking if you just want the transfer function and you are not interested in loading it right or if you know if your driving sources got zero output impedance.

Then just the RC is good enough right, but often times that is not sufficient and therefore, what would you do what do we what do we need to do? I mean how can you use an opamp to get a first order R C low pass filter?

Ok very good. So, we have seen this before in earlier classes. So, this is an example of how one could do it. So, this is R, this is C and this is R this is  $V_i$  and this is  $V_o$ . And what is  $\frac{V_o}{V_i}$ ? What is the D C gain? Minus 1 and what is the denominator? Please evaluate the transfer function and get back to me. Yes.  $1$  by  $1 + SCR$  and therefore, omega naught is  $1$  over  $RC$  straightforward right fine.

$$\frac{V_o}{V_i} = -\frac{1}{1 + SCR}$$

$$\omega_o = \frac{1}{RC}$$

The only kind of minor irritant is the fact that do you have an inversion right or then you say ok maybe I will leave with it and move on.

Apart from the zero input I mean the zero-output impedance of a section like this there is another very interesting aspect that I would like to draw your attention to and that has to do with robustness right it so, turns out that whenever you have a capacitor right every capacitor turns out will have some stray capacitance at every plate right.

This is the two stray capacitances need not be the same right and they are there whether you make an integrated circuit or whether you make it on a discrete circuit there is nothing you can do about it alright. Now what is the effect of this stray capacitance?

Well, both the nodes are connected to ground. So, it does not matter what comment can you make about this stray capacitance? So, we see that; that stray capacitance is in parallel with C and therefore you thought you had an three d b bandwidth of  $1 \text{ over } R C$ , but unfortunately what you have is  $1 \text{ over bandwidth}$  which is  $1 \text{ over } 1 \text{ plus}$  which is  $1 \text{ over}$  you know  $1 \text{ over } R \text{ time } C \text{ plus } C \text{ stray}$ .

So, this is basically what you call a parasitic sensitive filter right. Now let us see what happens with stray capacitances in this structure. So, this is C stray, this is C stray what comment can we make about the effect of this stray capacitance the opamp is ideal. So, what comment can we make about the effect of the stray capacitance there on the transfer function?

Yeah ok. So, any other comments? Well, if you have a doubt simply work it out what is this voltage?

Ground right. So, what comment can we make about please sit and work out the transfer function. Yes Darwin please do it. You want to use the fact that the virtual ground potential is an ideal opamp. So, what is the potential of the virtual ground? It is 0. Yes. No he is got a descending opinion what is your, what did you conclude?

C stray will still add in parallel to C. What you think? It will not why? Which same. So, this he claims that oh this stray does not matter because.

One terminal is at real ground, the other terminal is at virtual ground anyway the potential is 0, the potential difference across the capacitor is 0. So, no current will flow through the capacitor. So, this stray is of no consequence at all right ok. So, we will do worry about it. So, therefore, where is where will all this input current flow  $\frac{V_i}{R}$  where will it flow.

It will flow through the parallel combination of R and C. So, therefore, what combination what comment can you make about that voltage?

It is minus  $V_i$  by R by you know divide by you know the same thing that we had earlier without the stray ok alright and what comment can we make about this stray capacitance at the output?

Will current flow through that or no? It will flow through that right, but does that bother us does it change the output voltage? No because the opamp is a voltage source anyway correct and basically if the opamp will do whatever it takes remember it is a negative feedback system the opamp is continuously monitoring the virtual ground voltage and does whatever it takes to keep that virtual ground voltage at 0 right and that basically means that the output voltage will be minus 1 over 1 plus SCR even in the presence of strays ok.

So, the use of the opamp is not only beneficial in the sense of being able to drive the next stage alright ok, but it also adds an element of robustness into the whole. Yes.

Yeah, I mean the stray is actually a small I mean about a few percent typically it may be 5 percent, 10 percent depending on how you and the if you have a 10 percent uncertainty in C that is a lot more problematic than I mean if you do not know that whether it is 10 percent or 5 percent right that is you know saving of a little bit of capacitance is usually not the problem.