Circuit Analysis for Analog Designers

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Lecture - 21
Connection between magnitude response and pole locations in an all-pole filter

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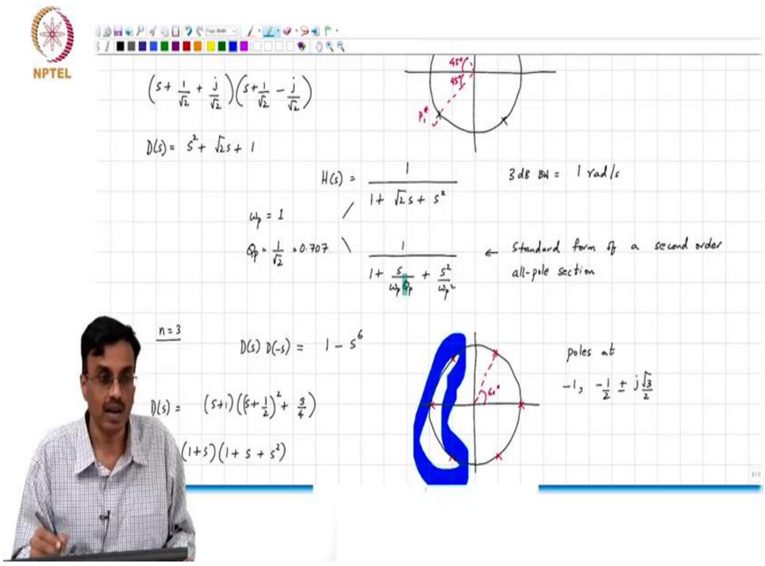

So, this peak is necessary to I mean is necessary for two things right, the peak or rather the peak make sense because if it has to be flat if the product has to be flat in the signal band and we have a first order section then the second order section must peak and remember also that the overall third order filters response falls off much, much faster, right.

So, there must be something in here which must be responsible for that quick change in magnitude with frequency, right. So, that is only possible remember as you keep increasing the quality factor of. So, if you have a standard second order section and if you keep increasing the quality factor, what comment can we make about the magnitude response?

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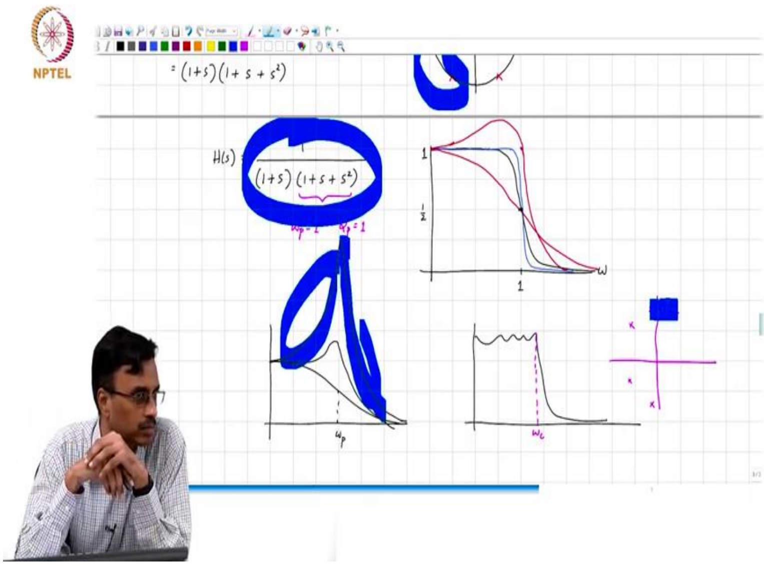

So, in other words the question I am asking is, we have a standard second order transfer function and you know for small q it does something like this as Q keeps let us say this is ω_0, ω_p as Q keeps increasing while we keep ω_p the same, what comment can we make about the magnitude response it becomes.

First it will peak like this then if you if Q becomes you know if Q tends to infinity, then this thing will do something like that right. And therefore, as you can see as Q becomes higher and higher the change the rate of change of magnitude with respect to frequency becomes higher right.

And the overall response remembers a high order filter will have a magnitude response that falls very rapidly with frequency around the band edge correct. So, that sharp fall in the magnitude response is must be coming from a, from a pole whose frequency is, I mean you know from a pole whose quality factor is very high right, but if you only had a pole with the high-quality factor and nothing else you will get a rapid fall, but in the pass band it will go up also very rapidly.

So, all the other filter you know sections in the high order filter or the job of all the other poles pole pairs is to kind of you know prevent this rapid rise in the pass band right and in the stop band they are also falling off right.

So, they will aid the response of the high cube pole pair at in the pass band in the stop band. Does it make sense right? So, based on this intuition let us say. So, now, that I have told you the background.

So, let us say you are in the lab and you measure you are looking at the magnitude response of the filter and the magnitude response of the filter looks like this ok. Of course, you have never seen this response before, but what comment can we make about the location of one of the poles, can we make any comment at all?

Exactly right. So, they will they must be whatever else you can say or cannot say right you must you will be able to say that, there will be a there will be there must be a pole at if you look at the s plane there must be a pole corresponding to this is the cut off frequency let us call this ω_c this is $j\omega_c$ right, there must be a pole pair there.

And that pole pair will have the highest quality factor right and remember if you have a pole pair whose quality factor is very high what is the meaning of physical meaning of quality factor, I mean what is that mean is the ratio of the?

Yeah, that is fine, but what which of these pole pairs has a higher quality factor the one on the left or the one on the right yeah, so why?

Yeah, so basically the quality factor of a pole pair is basically is related to the ratio of the imaginary part to the pole to the real part of the pole correct ok. Which is why I like to say all our students are very high-quality students because you know only imaginary part is the real part is very small you know do you understand ok. So, the as the quality factor keeps going you know higher and higher what it means is that the poles tend to move towards the j omega axis ok.

So, now, another point of interest what comment can we make about the impulse response or the step response of such a filter? If you have and in a time domain what is high quality factor what does that indicate? If the quality factor of a pole pair is very large what comment can we make about the impulse response?

Remember that if you have a pole at a certain frequency P i the impulse response will have a component at frequency, I mean in the time domain it will have component which is e to the P, e to the power P i times t right and therefore, if you have a pole which has got a large imaginary part and a very small real part, what does that mean?

You will see a slowly decaying exponential right the larger the Q the more the number of cycles it takes for the that complex exponential to get attenuated. So, with that background what comment can we say can we make about the impulse response of this filter, because you go and touch the filter you just hit the filter with a step what would you expect, you would expect? Ringing at what frequency? At that band edge frequency right,

You do not need to know anything else about the details of the filter you do not need to go and find its poles you do not need to compute its routes or the quality factor right. Simply looking at the magnitude response you should be able to say that you conclude that if you excite the filter with a step or with an impulse right you will see.

You will see the longest lasting part of the impulse response will correspond to the pole pair with the highest quality factor. Because that is the one which is getting damped off the slowest right and that persistent and it will you know it will be a damped exponential and the it will be a damped sinusoid and the sinusoidal frequency will correspond to roughly ω_c does make sense alright.

So, all this is stuff that we should expect from commonsense alright and this is all done without having to go through just staring at the impulse response is good enough I mean at the if the magnitude response of the filter is good enough to allow us to conclude all these aspects. Does it make sense ok?

So, this is what happens in the third order Butterworth filter. And the I mean in a higher order Butterworth filter what do you think will happen? Let us kind of see what happened when we went from second order to third order, second order we had two pole you know poles here ok.

Third order the poles are those shown in red now what do you think we will happen if you have fourth order Butterworth basically again you know cutting the piece of pie into 8 parts right. So, you will have poles here right and then somewhere here and I think we are done right 1, 2, 3, 4; 1, 2, 3 and 4 and the poles we would be interested in are this those 4 guys there right and what comment can you make about the highest quality factor in the fourth order Butterworth polynomial versus the third order one?

We can see that the quality factor is? Higher and why does that make sense? Remember that a higher order Butterworth filter cuts off you know much more sharply then a lower order one and that sharpness in the magnitude response must be coming from very sharper it must be coming from a pole whose quality factor is much higher right.

And as you keep increasing another thing that I like to draw your attention to, is as we keep increasing the order see that the pole free the frequency corresponding to the highest Q pole pair starts to approach 1 right ok.

Alright it approaches the j omega axis where the I mean that and the this point therefore, is ω_c equal to 1. So, basically as the for a high order Butterworth filter right we as we expected if you touch it will basically ring at add that frequency 1 radian per second right ok.

Otherwise, it is close enough to 1 radian per second alright ok. So, once we have the low pass filter I mean once we have a Butterworth filter you can now scale it for arbitrary bandwidth by simply changing scaling that s with s over ω_c and that is all there is two Butterworth low pass filters.