Circuit Analysis for Analog Designers
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Lecture - 02
Kirchhoff's Current and Voltage Laws, and the Incidence Matrix

Good morning and welcome. This is Advanced Electrical Networks lecture 2. So, let us get in the last class we looked at the motivation behind the various topics that we cover in this course and looked at why they are important in practice and today let us get started in earnest.

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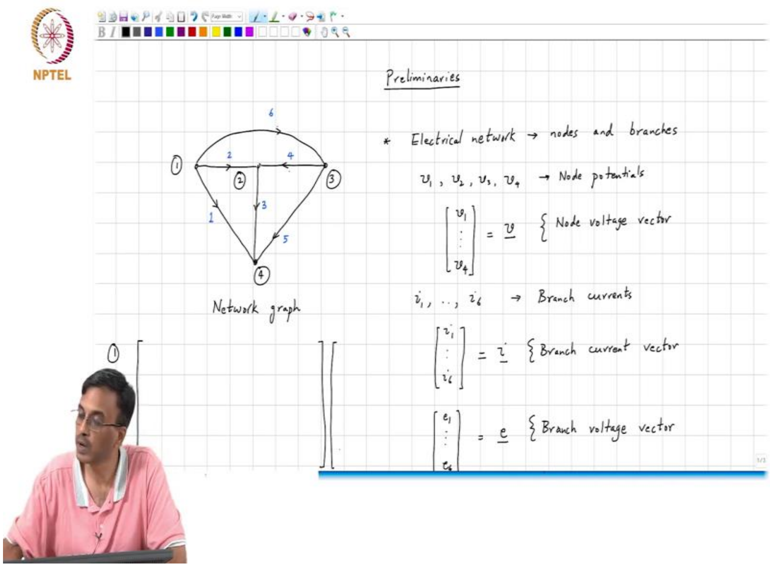

So, we will start off with some preliminaries. I am sure most of you know most of these topics and you have seen them at different levels in your earlier classes, but this will serve as a good starting point for all of us. So, and as I said in the last lecture, this is a course about; this is a course about electrical networks and so, we are going to be dealing with networks with nodes and branches. So, let us number the nodes.

It is going to quite arbitrarily choose the directions of, I am going to number the branches. So, an electrical network consists of nodes and branches. In this example, there are 4 nodes and 6 branches and every node is associated with the node potential. And we will call the node potential v_1, v_2, v_3 and v_4 are the node potentials and we will call the column vector $\begin{bmatrix} v_1 \\ \vdots \\ v_4 \end{bmatrix} = \underline{v}$.

We will term this the node voltage vector, right and every branch is associated with a node current. So, i_1 through i_6 are the branch currents and likewise $\begin{bmatrix} i_1 \\ \vdots \\ i_6 \end{bmatrix} = \underline{i}$, we call this the branch current vector and this picture is simply as you all know the skeleton on which the real network is formed.

So, each branch could perhaps represent you know an element, the elements could be linear, could be non-linear, could be time invariant, could be time varying and so on. So, this is the network graph and every branch is not only associated with the branch current, but also with the branch voltage. And we are going to use $\begin{bmatrix} e_1 \\ \vdots \\ e_6 \end{bmatrix} = \underline{e}$, as the branch voltage vector. And what is the job of you know network analysis? We have seen this in the past well the assumption is that you know the branch iv characteristics.

So, you know how the current through a branch is related to the voltage across a branch and what you would like to do is to solve the network namely. Given the branch relationships when you connect the branches in this fashion, what are the actual branch currents and branch voltages that result right and what are the cornerstones with which you can solve the network?

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The well-known Kirchhoff's current and voltage laws right.

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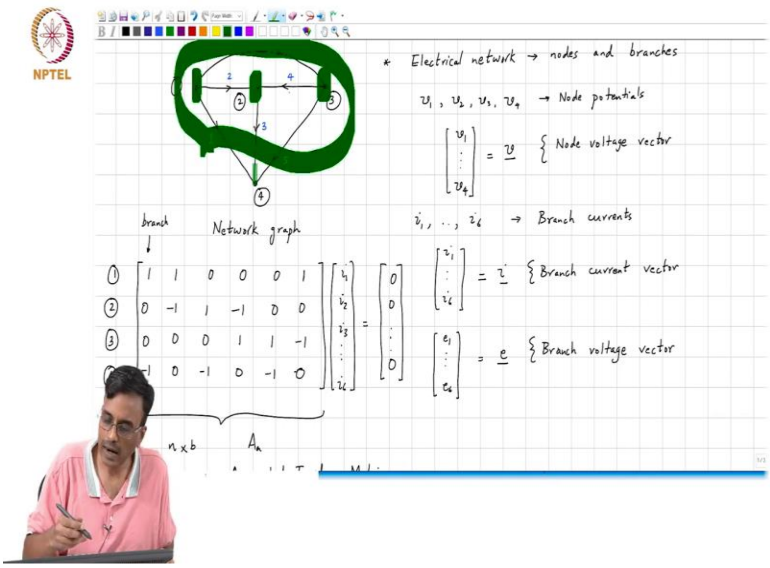

So, let us write KCL for each node. So, rather than write, we can write this in shorthand notation or in matrix notation as follows and again I am sure you have seen this in the past, but let me just do that anyway. So, this is i 1, i 2, i 3 blah blah blah i 6. So, we arbitrarily say if a branch is leaving a node the current is positive. So, I one for example, if you write KCL at node 1, what is the branch equation that you will get?

You see that you will get i 1 plus i 2 plus plus 0 times i 3 plus 0 times i 4 plus 0 times i 5 plus 1 times i 6 is 0, we do this at the second node and we will see that the matrix is minus 1 0 3 is plus 1 4 is minus 1 5 and 6 are 0. And similarly at node 3 we see that 4 is plus 1, 5 is plus 1 and 6 is minus 1 and at node 4 we have minus 1, 0, minus 1, 0, minus 1 and minus 1, 1, 3 and 5.

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ \vdots \\ i_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

So, sorry yeah ok alright. So, this; what comment can we make about the size of this matrix? So, for each branch you have every branch is represented by a branch current and therefore, you have a column for each branch and you have a row for each node. So, the size of this matrix is.

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The number of nodes is the number of rows cross the number of branches is the number of columns. So, this is an $n \times b$ matrix and this is often what is called the augmented incidence matrix. We will see why this name makes sense. Well incidence matrix is a matrix, well so, you better call it a matrix incidence, why do you think it makes sense to call this, incidence? why does that word make sense?

Well it is telling you know on which nodes a particular branch is incident right with the direction plus or minus 1. So, it makes sense to call it an incidence matrix. Why does it make sense to call it an augmented incidence matrix? Augmentation basically means that adding something extra right and actually if you stare carefully at this matrix, you can see that it is not necessary physically speaking, it is not necessary to write KCL at all nodes. If we wrote KCL at all, but one node then KCL at the other node is implied right.

How? Because if the current flowing through all these nodes, net nodes is 0. You can think of that as a super node and therefore, writing KCL at node 4 is actually redundant right. And this also reflected in the matrix and if you observe carefully what comment can you make about the sum of these of these rows.

So, basically you can see that if you add all the rows up the sum of all the rows turns out to be, turns out to be 0 which is telling you that the rows are not independent. And therefore, this also agrees well with our notation that if you have a network with n nodes you do not really need to find the potential of all the nodes right, you assume one of them to be the reference and the potential of all the other nodes with respect to the reference is what you need to find right.

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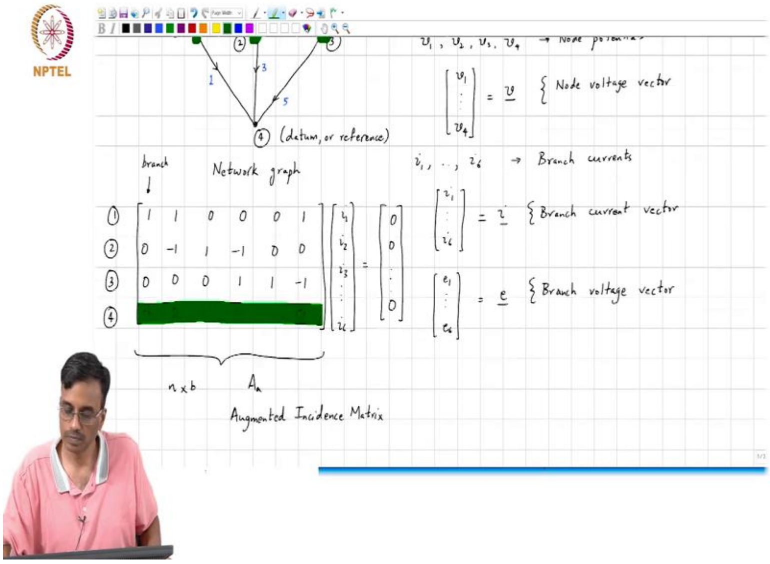

So, in other words this row for example, if we choose this as the datum or reference, then this row for instance can be neglected right, it offers no new information correct. So, this matrix which remains; which I am going to copy and paste. And by the way what is the size of this matrix?

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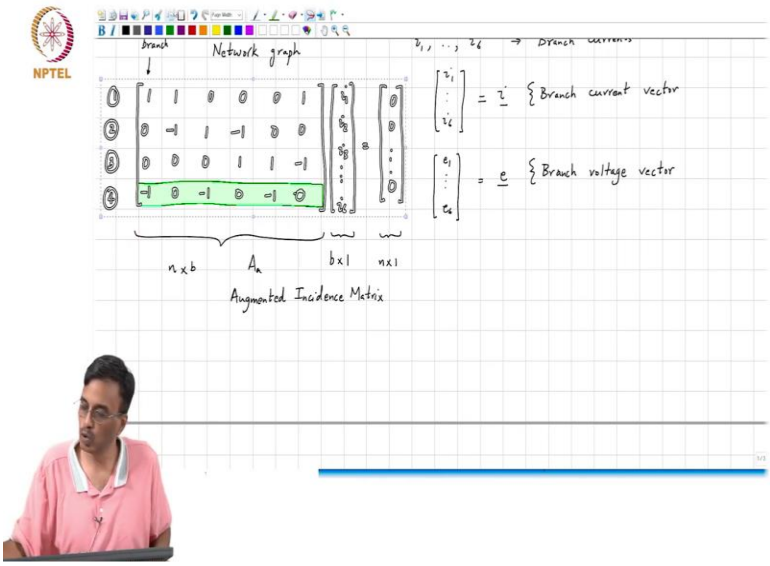

How many rows and how many columns?

b rows and 1 column and what should be the number of what are the dimensions of the vector the 0 vector on the right?

Very good is n rows cross 1 alright. Now what we realize that we do not really need all these rows.

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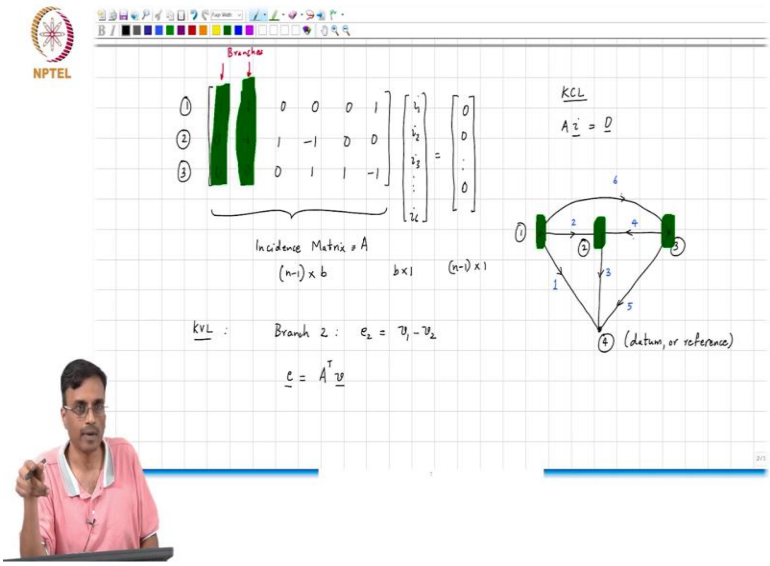

So, we can do with one fewer row. So, we eliminate, we can eliminate a 0 here ok. So, this is now if you remove the argumentation right, what do you get? What do you think you will call this matrix? Incidence matrix. Well, the augmented incidence matrix minus the augmentation basically means that this is the incidence matrix and this is an $(n-1) \times b$ matrix.

This is a $b \times 1$ matrix and this is in $(n-1) \times 1$ matrix alright.

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ \vdots \\ i_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

So, and the incidence matrix is often denoted by the symbol A there is nothing holy about that notation right. And therefore, KCL expressed in matrix form is simply A times how will you express KCL in matrix form?

$A\underline{i} = \underline{0}$, where all the matrices have appropriate dimensions alright. Now remember that we also, I am going to copy the graph over alright. So, remember we wanted to solve this network. We assume that the branch relationships are known. So, we need to use KCL and

KVL. KCL in matrix form is written in terms of the incidence matrix as follows and the next thing is to you know write KVL right and how do you express KVL in matrix form? What is Kirchhoff's voltage law?

Well, you can think of it in many ways. One way is to say is the familiar way of thinking about it as if you go around a loop then?

Well, the sum total of all the voltage drops you see you know across the branches in the in that particular loop is 0, its actually equivalent to saying that you can relate the node potential to the branch potentials by simply, for example, let us say you take branch 2. what is the potential of the branch 2? We call that e_2 and, that is simply? $v_1 - v_2$ ok.

$$e_2 = v_1 - v_2$$

So, relating the node I mean calling the branch potential as the difference between the node potentials is equivalent to saying that when you go around a loop the sum total is 0, correct, because you are starting and stopping at the same node correct. So, this avoids the hassle of having to find loops correct. So, KVL is therefore, expressed conveniently as you denote the branch potential as the difference between the node potentials.

That is an equivalent statement for Kirchhoff's voltage law. Does this make sense? Alright. Now, given the incidence matrix can you think of a way of finding the branch potentials in terms of the node potentials. Well remember what does each, I mean remember the incidence matrix basically contains all the information regarding how many branches are there, how many nodes are there and how are the branches connected between the nodes, correct.

So, simply staring at the incidence matrix should be able to give us this information that we are after, mainly, how are the branch potentials related to the node potentials. And let us stare at this matrix remember that each one of these columns corresponds to, each one of these rows corresponds to? A node and each one of these columns corresponds to? To a branch right.

So, can you stare at the columns and take a guess as to how the branch voltages can be related to the node voltages? Well, what is this 1 minus 1, what is this telling us let us take this column this corresponds to which branch?

This corresponds to the second column which basically means that this corresponds to the branch number 2 correct and it says 1 for I mean that column indicates that this branch starts at which node and ends at which node? It starts at node 1 and ends at? Node 2 and therefore, what comment can you make about the potential difference what comment can you make about the branch potential in relation to the node potential?

It is simply $v_1 - v_2$. So, taking this forward what comment can you make about the branch potential vector in terms of the node potential vector.

Similarly, ok so, let me lead you to the answer what about the potential of node 1 of branch 1? $v_1 - v_4$, v_4 we know is our datum or reference node who whose potential we arbitrarily defined to be 0 and therefore, it simply v_1. So, we can see that the branch potentials in terms of the node potentials are simply given by? A^T.

So, the branch potential vector e is simply the transpose of the incidence matrix multiplied by what? The node potential right alright ok.

$$\underline{e} = A^T \, \underline{v}$$

You can see that in each column there either what you call 1 or 2 entries if there is only 1 entry it basically means that that particular branch goes to? Ends at the reference node right. If there are 2 branches, it means that that particular I mean there are 2 non zero entries, it means that that particular branch does not touch the reference node and the plus and minus basically tell you where the branch starts and where the branch ends. Does that make sense people ok? So, this is KVL ok.

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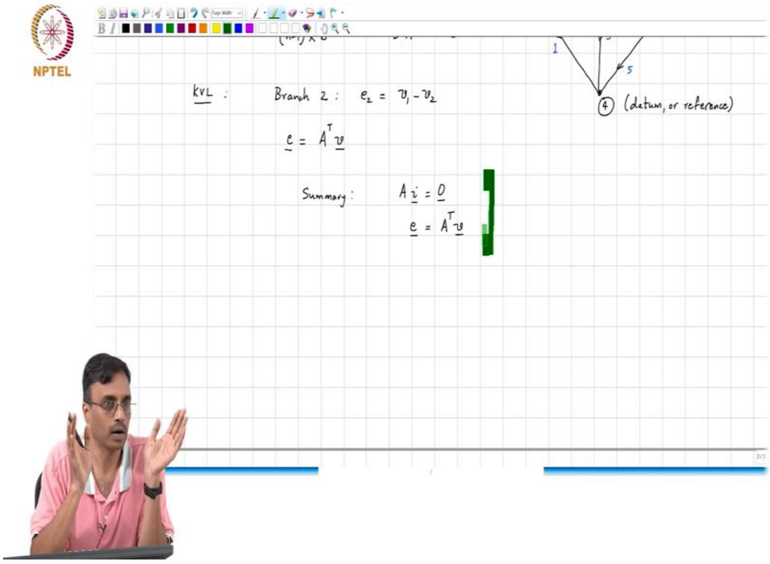

So, to summarize therefore, once the topology of or the graph of the network is known, KCL is simply written as $A\,\underline{i} = 0$ and e which is the branch voltage vector is simply written as A transpose times the node voltage vector is that clear yes.

Correct this is not; this is not enough for us, he brings up a good point, these are not enough to basically find the actual voltages and currents. We still need some more information right, what is that information?

Well, we need to know how each branch voltage is related to Branch current. To the branch current in a linear network that relationship will be linear right in a non-linear network that will not be linear yes.

Yes, I mean as so, if current is flowing this way, we always assume that current flows from a higher potential to a lower potential. So, the direction of current is always you know if current is flowing this way, then the potential is higher on the left. Is that clear? Alright now this makes these two are simply KCL and KVL and make no assumptions whatsoever about the nature of the network right.