Circuit Analysis for Analog Designers
Prof. Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 18
Magnitude approximation principles

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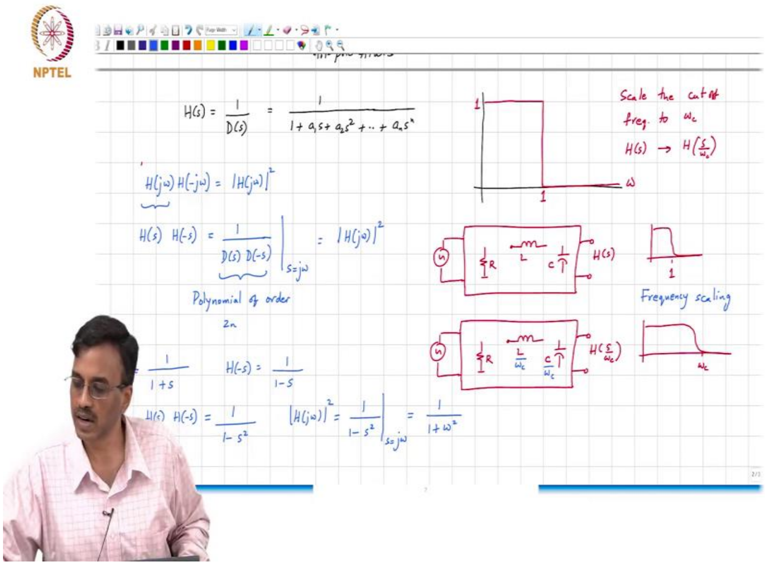

So, and the next thing. So, what we are going to do. Therefore, our problem is now within quote simplified to saying H of s is 1 over D of s ok.

$$H(s) = \frac{1}{D(s)}$$

And D of s is basically an nth order polynomial and if we want the DC gain to be 1; what comment can we make about the polynomial. This must be 1 plus some a 1 s plus a 2 s square all the way up to a n s power n ok.

$$= \frac{1}{1 + a_1 s + a_2 s^2 + \cdots + a_n s^n}$$

So, mathematically therefore, our problem boils down to how do we determine a 1 through a n so that, the magnitude response of this transfer function satisfies some speck right. Traditionally, it turned out that people in the beginning like were attempting to I mean this application to a to d converters came much later right. Long before there were A to D converters, people still needed to filter; for example, you know you were listening to some music and you have an amplifier and you know it was also picking up some noise and you wanted to remove high frequency noise. So, in that context, this concept of alias band does not exist. You just need to remove everything beyond a certain frequency.

So, a lot of filter design actually started out in this context, where people said, ok let me try my best to approximate the what is called the brick wall; very well knowing that you cannot get a perfect brick wall right and perhaps you could approach the ideal brick wall response by going on increasing the order of the filter that is seems reasonable right.

If you work harder, you know perhaps you can do better. So, and the ideal brick wall they were trying to approximate was that the magnitude response is 1 up to a cut off frequency some ω_c and they arbitrarily chose the ω_c also to be 1 radian per second ok.

And is that a problem or you know there is no issue with that? I mean it is not very likely that you designing filters whose cut off frequency is 1 radian per second right; 1 radian per second is a very very low frequency correct.

So, now suddenly if you want to cut off frequencies beyond 20 kilohertz, you know would you have to kind of throw all this over and start all over again or if you have done the work for 1 radian per second, you can do it I mean straight forward to extend it to any frequency, Scale.

What you do so, when. Let us say we found some $H(s)$ with a cut off frequency of 1 radian per second. If you wanted cut off you want to scale the cut off frequency to ω_c radians per second, what would you do? Very good. So, you had $H(s)$. All that you need to do is replace s with s by omega c right ok. This is the polynomial.

$$H(s) \rightarrow H\left(\frac{s}{\omega_c}\right)$$

What would you what would happen with the components; if you had. Let us say somebody give you a network with inductors, capacitors, and resistors and told you that this these components will result in a cut off frequency of 1 hertz or 1 radian per second; what should I do to change the cut off frequency to ω_c radians per second?

Do you understand the question right? So, in other words, let us say I had a big network with you know say r, l, c and so on some box it is a high order filter and you know that let us say r, l and c and you know that this is $H(s)$ and the cut off frequency is I mean this is a low pass filter with some behaviour like that and let us say this cut off frequency is 1. Now, the question is what do I do to get. What do I do to get a cut off frequency of ω_c radians per second? ω_c. What should I do?

Yeah. So, basically remember that what we are doing is; if this is $H(s)$ what we want is $H\left(\frac{s}{\omega_c}\right)$ alright and therefore, whatever you see in this network which has an s term, it must be replaced by $\frac{s}{\omega_c}$. So, the impedance of this inductor which was originally s times l must be that s must become $\frac{s}{\omega_c}$. So, what do you think you will do?

You divide this by ω_c. The capacitance had an admittance which is which was S times c. Now, that s gets replaced by $\frac{s}{\omega_c}$. So, this gets divided by minus c right. And as you can see, all time constants. What do you do with the resistor?

Well, the resistor does not have any s term in it. So, you leave it alone right. So, this is what is called frequency scaling right. And it is almost a straight forward as. Well, saying we know I mean when we wanted the transfer function to move to a critical frequency ω_c. We just scaled $H(s)$ replaced s in $H(s)$ with $\frac{s}{\omega_c}$. This is the doing the same exact same thing except that it is done with the network.

Does it make sense people? Alright. Having gotten that out of the way, you know we still have you know a couple of minor irritants that we need to take care of. The first one being, well, our specification only says that we want a magnitude response that approximates a brick wall correct. Now, a magnitude response; obviously, is a real quantity alright whereas, this is nothing if you plop an s is equal to j omega what do you get? What you will get for H of j omega; is it a real number or is it a complex number? You will get a complex number right.

So, what you need to do is basically figure out you know how to get the magnitude response from H of s right, then we recall that the magnitude response is nothing but H of j omega times H star of j omega which is nothing but replace j omega with minus j omega right. So, this is the same as mod H of j omega the whole square.

$$H(j\omega)H(-j\omega) = |H(j\omega)|^2$$

Correct. And So, H of j omega is gotten by replacing s with j omega. By the same token, H of minus j omega is gotten by replacing s with minus j omega in H of, or equivalently it is gotten by replacing finding H of minus s and replacing s with j omega. I mean that is an easier thing to do rather than say for some part of it I will replace it by j omega some other part I will replace it by minus j omega. The easier thing is to simply say I will find H of s times H of minus s right and replace I will get a big polynomial of order. What will be the denominator polynomials Order here now?

Which basically will be 1 over D of s times D of minus s and this is nothing but a polynomial of order 2 right and replace s with j omega. And I will now therefore, be able to get mod of H of j omega the whole square.

$$H(s)H(-s) = \left.\frac{1}{D(s)D(-s)}\right|_{s=j\omega} = |H(j\omega)|^2$$

Does it make sense. So, let me give a quick example. Well, let us say we have 1 plus s was. This is H of s.

$$H(s) = \frac{1}{1+s}$$

H of minus s is 1 by 1 minus s.

$$H(-s) = \frac{1}{1-s}$$

So, H of s times H of minus s is 1 by 1 minus s square and therefore, and mod H of j omega whole square is obtained by replacing s here with j omega correct and this is 1 by 1 plus omega square which you knew already alright.

$$H(s)H(-s) = \frac{1}{1-s^2}$$

$$|H(j\omega)|^2 = \left.\frac{1}{1-s^2}\right|_{s=j\omega} = \frac{1}{1+\omega^2}$$

Now, there is. So, it is very straight forward as you can see going from. H of s to mod H of j omega the whole square right. So, now, our job therefore, right is to go and find this H of s whose magnitude response satisfies that brick wall speck, which is equivalent to finding first how I mean. So, this is only specified as a function of omega. So, what we are interested in therefore, is going and finding mod H of j omega square which satisfies that specification; which approximates the brick wall in some fashion right.

Let us say we find some mod H of j omega square which actually does what we want, but we are not satisfied with mod H of j omega whole square. We want to get back from mod H of j omega whole square to H of s not H of j omega I mean H of j omega whole square to H of j omega is simply square root correct ok. We want to get back to H of s. So, what do you recommend we do; in other words, let us use the same example to go backwards. Let us say this is the mod H of j omega square. How do we get back to H of s from H of j omega whole square?

What do you recommend that we do? What pole locations you have 1 by omega 1 by 1 plus omega square. So, this is from H of s to H of j omega.

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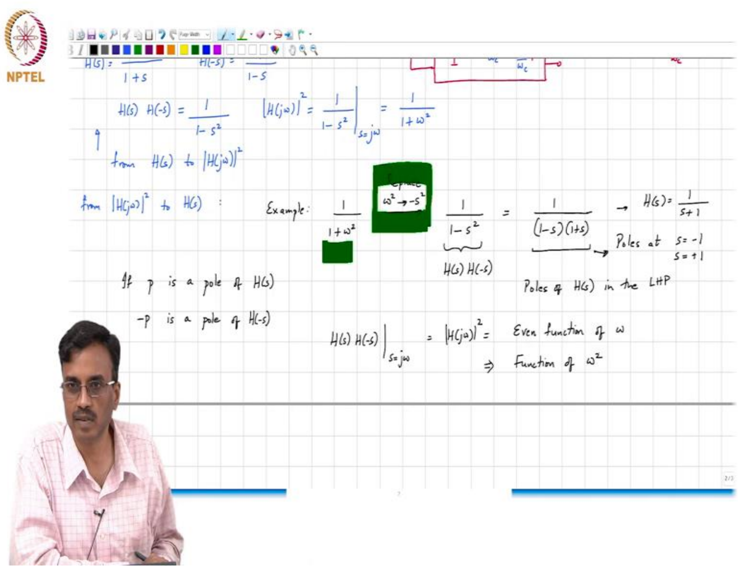

Now, how do we go back from or mod H of j omega whole square. So, from mod H of j omega whole square to H of s what would you do. Again, we will deal with this example. So, 1 by 1 plus omega square is the mod H of j omega square. What you suggest we do? How did we get 1 by 1 plus omega square? We replaced s with j omega or s square with s square with minus Omega square. So, basically replace omega square with minus s square correct ok. Now, what do you get? What do you get?

You get 1 by 1 minus s square and you know that this is. This is nothing but H of s times. H of minus s correct alright.

$$\frac{1}{1+\omega^2} \xrightarrow[\omega^2 \to -s^2]{Replace} \frac{1}{1-s^2}$$

$$\frac{1}{1-s^2} = H(s)H(-s)$$

So, how do we get H of s from this? We factorize it. So, what let me factorize this? Basically, I get 1 minus s times 1 plus s ok. So, what?

$$\frac{1}{1-s^2} = \frac{1}{(1-s)(1+s)}$$

So well, then you say well this corresponds to H of s time H of minus s. We finally know that we want to make a filter which is stable. So, therefore, the poles of H of s are in the left half plane correct. So, what comment can we make about the poles of this polynomial.

Remember, if P is a pole of H of s, then what comment can you make about pole of H of minus s, yes? Minus P is a pole of H of minus s and therefore, H of s times H of minus s will have poles which have mirror symmetry about the vertical axis right. So, what should you do? Therefore, what is a common sense telling us? So, this H of s times H of minus s is got poles at s is equal to minus 1 s is equal to plus 1. So, we know that the poles of H of s must be in the left of s plane. So, which pole will be pick? Plus 1 or minus 1 guys. Minus 1 right. So, what is the polynomial?

H of s is 1 by s plus 1 right. So, this now tells you how to go from H of s to mod H of j omega whole square. We know how to get there. We also know how to get back from. If we know mod H of j omega the whole square, we also know how to get back to H of s that make sense. Now, we said we are going to replace omega square with minus s square. What do we do with the omega terms?

I mean here we I mean I do not know. Maybe we got lucky. We just have 1 plus omega square; is it possible to get an omega term? Can we get 1 plus omega plus omega square in the magnitude square response. It is possible? I mean there are only two answers right is; either possible or not possible. It is not possible? Why? I mean he; obviously, thinks that it is possible.

Student: We have a denominators polynomial is there in denominator can be split in two factors. So, for that each factor I mean (Refer Time: 17:32) only factor (Refer Time: 17:36). So, they will form (Refer Time: 17:39).

Is there yes Alfred is you know dying to tell me the answer.

Student: (Refer Time: 17:46).

Very good right. The most obvious argument is to say. Well, the remember that a magnitude response. So, H of s times H of minus s evaluated at s is equal to j omega is mod H of j omega the whole square right, which basically is an even function of omega correct.

$$H(s)H(-s)|_{s=j\omega} = |H(j\omega)|^2$$

The magnitude response an even function of omega right; which basically means that it is a function of. It can be written as a function of omega square right.

So, there is no possibility of you ending up with 1 plus omega plus omega square for instance in the squared magnitude response, because if you change omega to minus omega, the response must remain the same and that will not happen if the denominator polynomial contains a in the mod H of j omega square polynomial you get a you have odd terms. Is this clear?