Circuit Analysis for Analog Designers

Prof. Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 17
Introduction to Analog Active Filters

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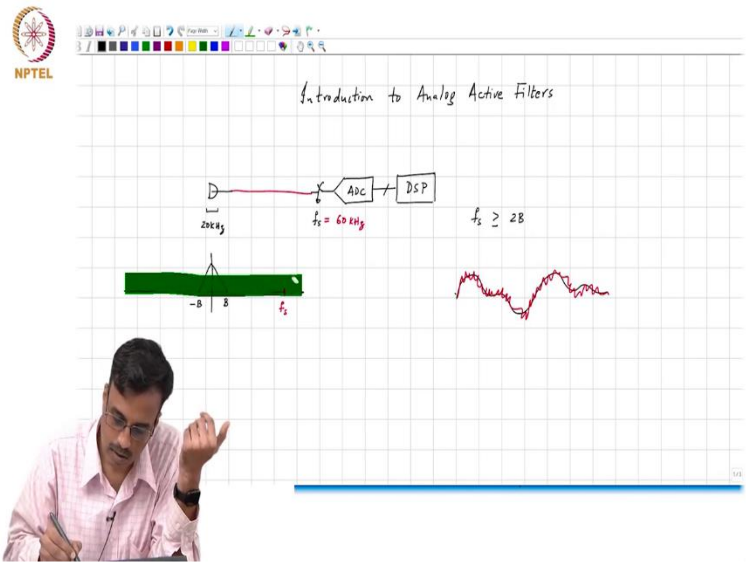

Today we will start a new topic and that is an Introduction to Analog Filtering; most specifically Analog Active Filters. Before we get into the details, I would like to motivate the topic and we kind of brushed upon this in the very introductory class. Remember that today all processing is largely done digitally and therefore, you have an A to D converter that goes and drives a DSP which then stores processors does also of open stuff.

And an A to D converter remember is a combination of sampling and quantization and it is not appropriate to take the output of your sensor I mean could be the microphone, could be an antenna whatever right and then even though they desired signal is has a small bandwidth right.

You know as per Nyquist let us recall that all that we need to do is basically say f_s must be greater than or equal to twice the signal bandwidth for us to be able to perfectly recover the signal from its samples right.

But that does not mean that you simply take your sensor for example, let us say your you have voice right and we know that voice is for all practical purposes limited to 20 kilohertz. So, in principle right it should be enough to simply if you just believe Nyquist and then basically said Well here is my mike and then you know I have some amplified inside the mike which gives me an electrical signal and then I directly connect this to the input of my A to D converter.

What is the problem with this approach? I mean the signal bandwidth is 20 kilohertz let us say f_s is I do not know I mean 60 kilohertz for argument's sake. So, as per Nyquist this is good enough right. So, are we doing or is there a problem? Yeah, remember that it while it is true that the desired signal has a bandwidth of 20 kilohertz or B in this case right the often times the sensor output is not only corrupted by, I mean it is got not only the desired signal, but often times is corrupted by what you call noise in the simplistic case I am just assuming that there is some broad band noise.

The noise does not know that you want to satisfy Nyquist it does not know the sampling rate it is just noise and what you call its frequency components can be anywhere right. Now if the there is nothing you can do about the components of noise that lie within the signal band plus minus B right. Because it is impossible to distinguish whether a component of the noise if that corresponds to the signal or if it is noise, if it is in the if it is inside the signal band correct.

However, noise signals that are outside the signal band are easy to distinguish right and here is a visual example. So, let us say your digitizing voice and you know here is an example wave form that you expect to see right, but suddenly you know you start seeing stuff like this right there is a whole lot of very high frequency junk riding on top of your desired signal correct. So, if you simply sampled the noisy wave form at the Nyquist rate what will happen in what will happen?

Basically, you can see that the samples of the black wave form and the red wave form are simply not the same right ok. And how can you do better? Again, it is a you know common sense thing what will you do if you look at the red wave form you know that that cannot be the true wave form of voice because no one can speak with you know no one's voice output will have such rapid variations right. So, what do you do what is a common sense tell you? Well before samplingyou must smooth that the red wave form right and smoothing the wave form is basically the technical term for that is filtering correct ok. In the frequency domain it simply means that high frequency noise right can potentially alias into the signal band after sampling if you do not do anything correct.

So, whenever you have. So, for example, let us say you have a sampling rate which is f_s right if you had a tone if the noise wave form had some component at f_s, how would that appear after sampling? A tone at f_s after sampling at f_s would appear like DC and therefore, you if you are simply looking at the samples you would get confused and think that the input signal consists of some DC right. So, in a similar fashion what comment can you make about what all signals can alias into the signal band, what all noise components can alias into the signal band?

In other words which all input frequency components after sampling will lead you to mistake that the input signal has a lies within a bandwidth plus minus B. So, you have that is correct. So, basically these are all.

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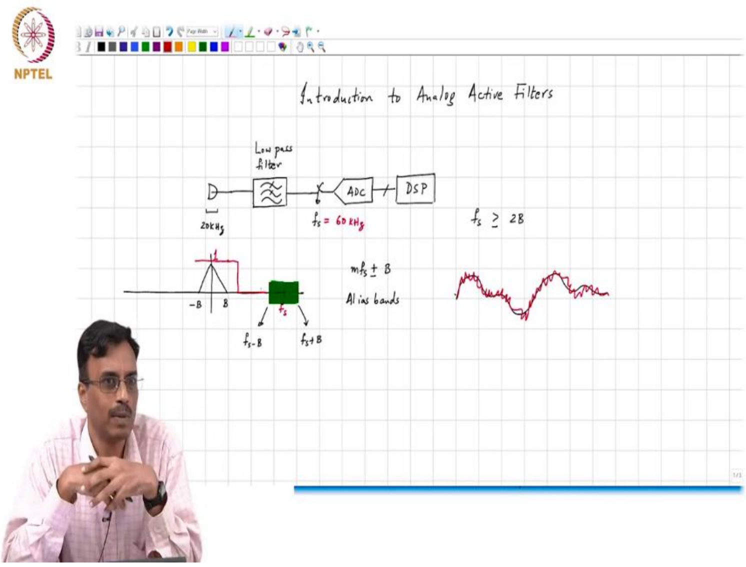

So, this is $f_s - B$ and this is $f_s + B$. So, all frequencies of the form m f s plus minus in the range of m s, $mf_s \pm B$ where m is an integer can potentially alias into the signal band after sampling if you did not know if you did not do anything else correct. So, if you want to prevent this from happening what would you do? What are common sense tell us with respect to the red wave form that you see here?

You have to smooth the wave form in other words you need to put a filter between the sensor and before the A to D converter right and in this particular example what kind of the I mean a filter basically all that it means is that it selects something and removes something else what frequencies does it select?

It selects low frequencies and eliminates high frequencies. So, this is an example of a low pass filter. So, the symbol for the low pass filter is basically you know there is you knock off the mid and high frequency component and that is the symbol for a low pass filter right. So, and what is the job of the low pass filter how will you specify the low pass filter pardon?

And what else I mean. What should be the I mean basically you want to specify the frequency response of the low pass filter how will you specify the frequency response of the low pass filter? Let us say you go to filter designer right and say you know give me a filter which satisfies these specifications what exactly you would to tell him?

So, the what is the pass band? 0 to B you want the filter gain to be 1 in the region 0 to b correct alright, what next? Right; so, basically it is not enough to say I mean. So, one suggestion that I got was to say the gain must be 1 between minus B and B and after that it must you know cut off like this for instance right. And it sounds like you know it sounds like a plane right, but it is; obviously, something wrong with this kind of specification what is it what does it mean?

I mean remember that we live in a real world where you know nothing has a derivative which goes abruptly to infinity right. So, you can see the discontinuity in the magnitude specification in the magnitude response right and therefore, it means that it is not possible to realize in practice ok.

So, you have to be a little more reasonable I mean if you are adamant and say well, I want again to be abruptly 0 beyond a certain frequency right then the filter designer is going to tell you to go and take a hike simply because it is not physically possible right. Now, you say well let me be a little more reasonable and what should I what would you then do?

I mean do you really care about the response between B and $f_s - B$. Ok, so in other words how will you specify the response? We come back to the question. So, it is clear that you guy just cannot say I want everything beyond a certain frequency to get cut off correct alright. So, how do you specify the response I mean what do you really care about. You care about the fact that the gain in the bandwidth up to bandwidth B is 1 what else do you care about? In the alias bands you want the gain to be not low you ideally wanted to be 0 right. So, you the way you specify the filter responses well here you say it is see you wanted to be 0 alright.

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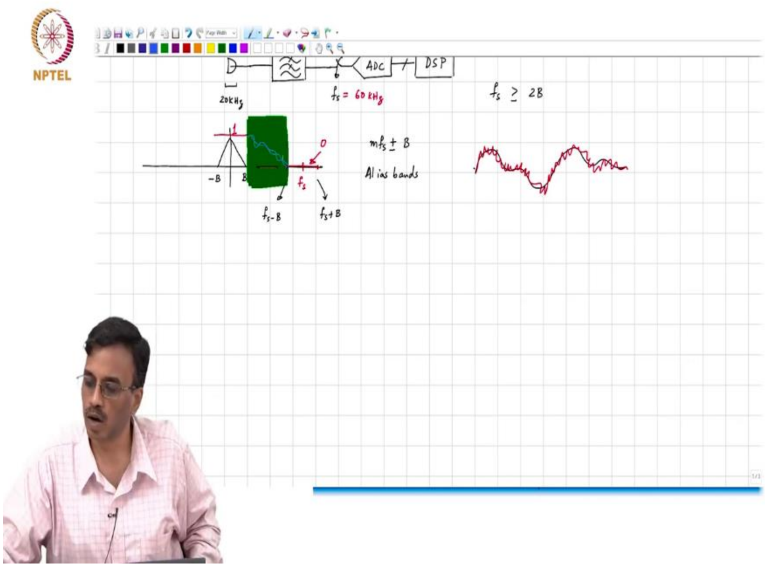

And you know in this kind of region here you are not very particular right ok, you probably as long as the attenuation in the alias band is sufficiently large you probably do not care about whether it does this or whether it does this you know you probably do not care about alright ok. So, that is how you specify the response of the filter in practice ok. So, this is what is called a brick wall specification right.

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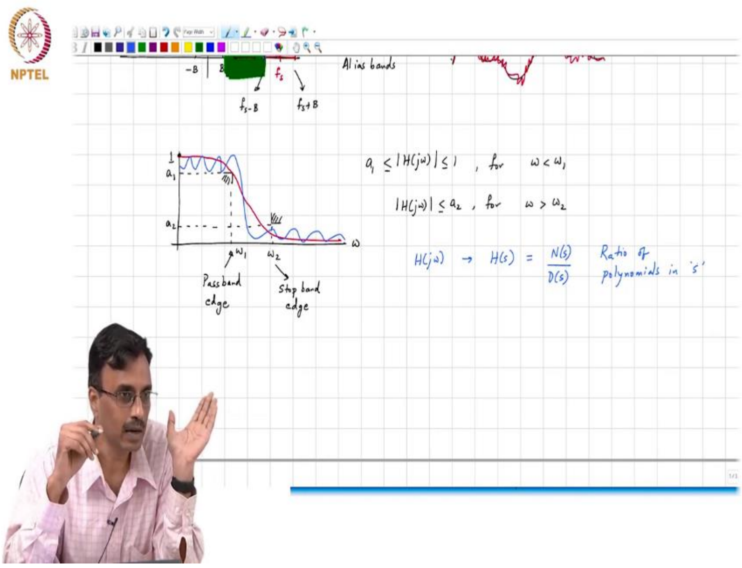

So, let me reiterate that ok. So, the question is well when the filter designer comes back and tells you sorry you know while it is a just like how it is not possible for me to give you a filter whose gain abruptly changes from 1 to 0 at a certain cut off frequency it is also not possible for me to give you a frequency response that is absolutely constant in the range 0 to B, does make sense right. And therefore, then you say you back off and then say oh well right.

This is what I will call my pass band edge this is omega right you know we can always say the DC gain has to be 1 right if it is not 1 you can always you know put an amplifier so that the gain is 1. And so, how do you specify the pass band how do you specify the filter? Filters pass band right. So, the gain of the filter or the magnitude response of the filter which I call H of j omega, $|H(j\omega)|$ right must be? Yeah, can you mathematically tell me what I should we should put down there, mod H of j omega must be greater than say basically must be greater than equal to a 1 and less than equal to 1 for omega less than omega 1

$$a_1 \leq |H(j\omega)| \leq 1, for \ \ \omega < \omega_1$$

and what else and what else could you say that is 1 specification what else would you say. Yes Karthi what do you think? Is that good enough for what? Is that a good enough pass band specification and we can move to the stop band or you want to add something else?

Student: It is possible to specify within 1 (Refer Time: 14:48) for the frequency falls down (Refer Time: 14:52).

Yeah, but is not that automatically specified if you basically say. So, you have basically the edge of the pass band what else do we need to specify therefore? Yeah, you right. So, what?

Student: The start of the (Refer Time: 15:07).

Yeah, can you basically there must be mathematical way of saying that right. What do you think you can say?

Student: It must be less than some (Refer Time: 15:18).

Very good. So, basically, we say well there is some omega 2 which I call what would I call if I call that the pass band rate what would I call this?

Student: Stop.

Stop band edge alright and this is a_2 and basically you say well I want mod of j H of j omega to be less than or equal to a_2 for omega greater than ω_2 alright.

$$|H(j\omega)| \leq a_2, \qquad for \quad \omega > \omega_2$$

And you know as you can probably imagine there is no unique way of achieving this there are any number of ways of doing this for example, you could have a response that say does something like this does not meet the speck something that does this right you could have somebody else come up with something else which does.

Something that as that right as you can imagine there are any number of ways of achieving this magnitude response alright. So, the question so, in this course I am you know so, the way of approximating these this magnitude response right is a huge topic in itself and there are whole courses they are devoted to approximation theory.

And what are the constraints under which we operate? I mean you just cannot draw curve like that and say well this is my response I mean finally; it has to be realized and practiced right. And has to be realized using you know elements that we know namely resistors, inductors and capacitors and you know perhaps active elements to be able to replace some of the inductors because inductors can become very bulky right.

So, what we are dealing with is H of j omega therefore, is the frequency response corresponds to the frequency response of a transfer function H of s right and this H of s because it consists of only resistors capacitors and inductors must be a ratio of polynomials in say ratio of polynomials in s because of because we using lump circuits say ratio of polynomials in correct.

And because it is a ratio of polynomials in s it is there is a; obviously, a constraint on what form that H of j omega can take it cannot be an arbitrary function, it has to be a ratio of polynomials in s whose magnitude response must satisfy these specifications right.

$$H(j\omega) \rightarrow H(s) = \frac{N(s)}{D(s)}$$

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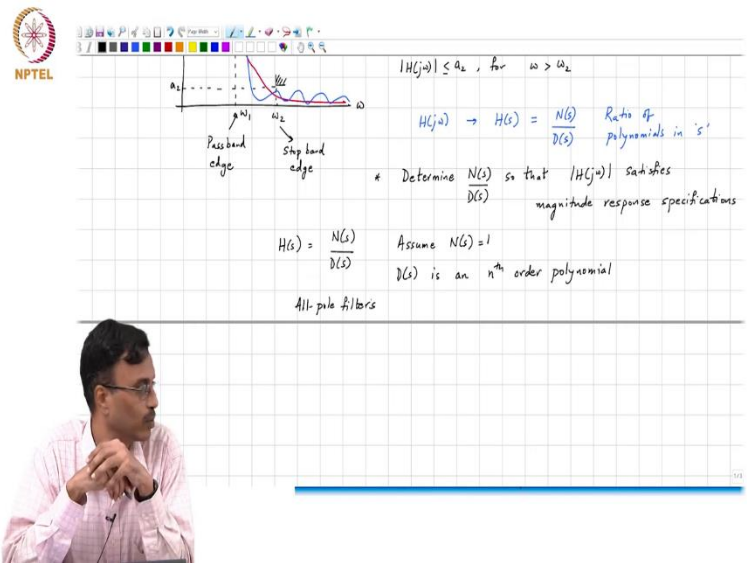

So, the name of the game therefore, is to say determine N of s by D of s. So, that right mod H of j omega satisfies magnitude response specifications alright. And to make a life simple right we basically say well let us start with simple things first.

So, H of s is N of s by D of s;

$$H(s) = \frac{N(s)}{D(s)}$$

we will assume and there is no particular reason to do this except simplicity assume N of s equals 1 alright. And so, therefore, its only D of s is a polynomial and we I mean finally, when you say you have polynomial how do you specify it do you specify by its order right.

You assume that is an nth order polynomial alright and such filters so; obviously, you can see that this transfer functions got no 0s, it is got only poles. So, these what do you think you would call these such filters these are all what are called all pole filters and the reason why they are called all pole filters is obvious.