Circuit Analysis for Analog Designers
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Lecture - 14
The Adjoint Network

Welcome to advanced electrical networks this is lecture 7. So, in the last class we saw another perspective of reciprocity and the basic idea was the following. So, we have when we write the modified nodal equation analysis MNA equations.

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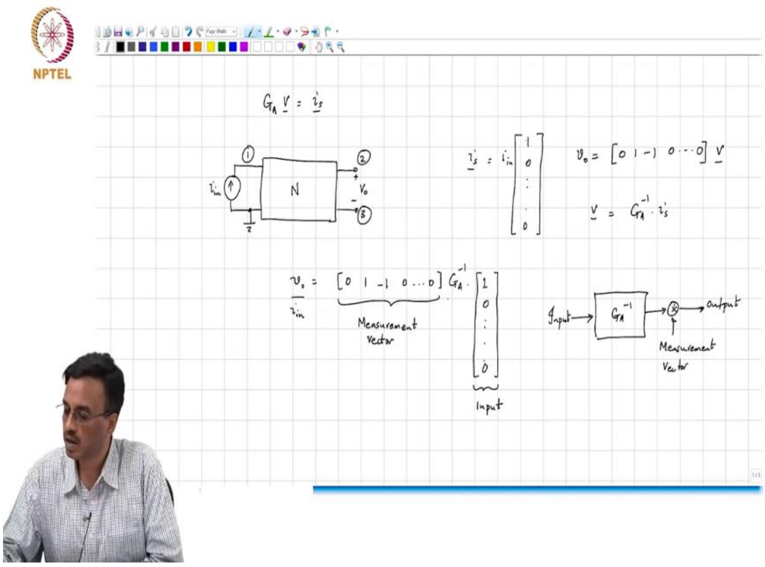

We basically have an equation of this system of equations of the form G sub-A times V equals i sub s, where G A is augmented conductance matrix, V is the vector of unknowns which will consist of all the node voltages as well as the currents flowing through the through any of the voltage sources inside the network.

$$G_A \underline{V} = \underline{i_s}$$

And the vector on the right-hand side is simply the source vector which consists of all the independent sources namely current sources and voltage sources.

And we saw that well when we try to work with reciprocity one way of looking at it is to basically, we saw that for the case of a current excitation and a voltage output I believe.

And what we do is simply recognize that when we write these equations what we are doing is injecting a current into node 1. So, this i s simply becomes i in times 1 followed by all 0s and v o is simply a row vector which is 0 1 minus 1 all 0s multiplied by V right and V is nothing but G A inverse times i s right.

$$\underline{i_s} = i_{in} \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$v_0 = \begin{bmatrix} 0 & 1 & -1 & 0 & \ldots & 0 \end{bmatrix} \underline{V}$$

$$\underline{v} = G_A^{-1}\, i_s$$

So, v o therefore, can be written as 0, 1, minus 1 all the way time 0 times G A inverse times i s which is or v o by i in is nothing but 1, 0 etcetera alright.

$$\frac{v_0}{i_{in}} = \begin{bmatrix} 0 & 1 & -1 & 0 & \ldots & 0 \end{bmatrix} G_A^{-1} \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

So, this is the corresponds to the input vector this is the circuit matrix of course, and this is the what I will loosely call the measurement vector or the measurement matrix if you like ok.

So, this you can think of this as a you know as a mathematical operation right. So, you have the input ok, it gets multiplied by a matrix G A inverse ok and produces an output which is as an input times G A inverse and then what is happening?

You have measure, you multiplying this by the measurement vector. So, this is and this is the output ok.

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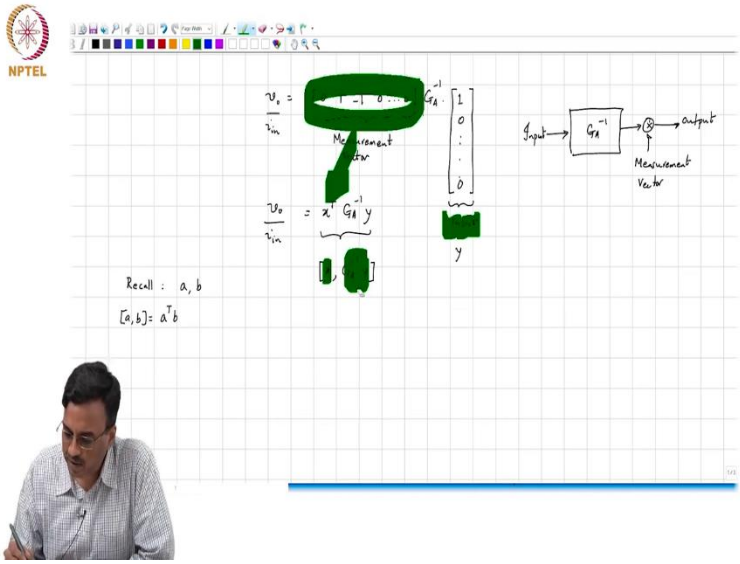

And if you think of this as x transpose right. So, this I am going to call x transpose simply because it is a row vector right. So, and x is a column vector and x transpose is that is the measurement matrix this times G A inverse times the input which I am going to call the y.

So, the scalar v o by i in is of the form g transpose I mean x transpose times G inverse times y

$$\frac{v_0}{i_{in}} = x^T G_A^{-1} y$$

and you know and remember that if you have two vectors if you have two column vectors with the same length let us say I am going to use a and b, what is the dot product of the two vectors? The dot product is nothing, but this a dot b is nothing but a transpose b correct and you look at this here and this is the; this is the I mean this is the notation for dot product.

$$[a, b] = a^T b$$

So, how can you write this as this is nothing but x comma G A inverse times y alright, does that make sense people?

Alright and so you can it, so you can think of it as the following you have a vector y right on which the matrix G A inverse is operating and the result is another vector ok. And you are finding the dot product of that vector G A inverse times i with x right x corresponds to the measurement y corresponds to the input alright.

$$\frac{v_0}{i_{in}} = [x, G_A^{-1} y]$$

Now, in mathematics it turns out that you know I mean a legitimate question you can ask is the following. I am taking a vector y operating on it and taking the dot product of the result with the vector x right to get a scalar correct. A legitimate question I can ask is if I wanted to get the same scalar, but I wanted to operate on x instead rather than y what should I have done to x to get the same scalar does make sense.

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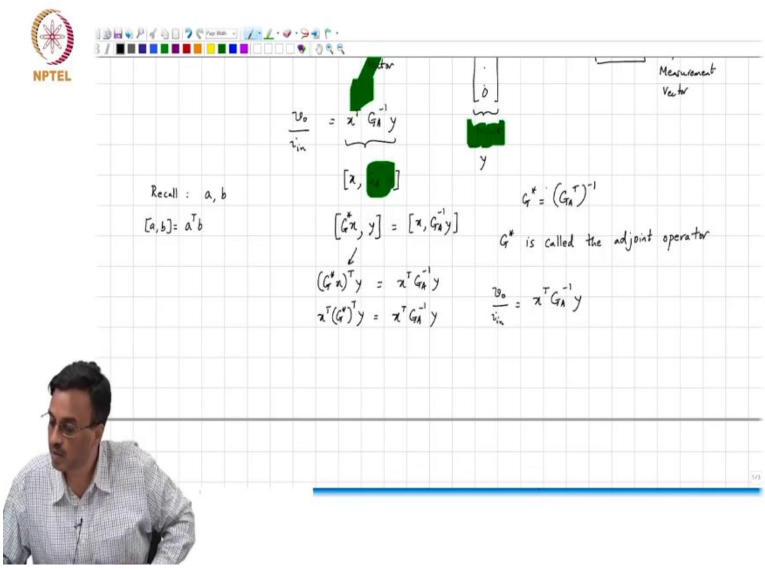

In other words, what should I do to x ok before doting it with y to get, the same answer as I would get if I; is this clear people right?

$$[?\,x, y] = [x, G_A^{-1} y]$$

I mean this is simply a mathematical equation there is nothing I mean why are we doing it no this is what the math guys love to do right and it seems like a legitimate question right ok.

So, you are messing with y first in the in this computation you are messing with you are messing with y right and then taking the result and finding the dot product with x right. Reasonable question to ask is well if I messed with x instead ok and then computes its dot product with y, I mean what should I have done to x to get the same dot product ok. So, with what matrix should I have multiplied it to get the same dot product. So, how do you solve this problem and what do you do and how can we figure this out.

Well. This whatever operation you do to x the whole I mean that is clearly let us call that I do not know I mean I am going to call that G star right. So, x transpose times, what is this guy? G transpose, G star times x transpose times y should be equal to x transpose G A inverse times y alright ok and this is nothing but x transpose G star transpose y must be equal to x transpose G A inverse times y. So, what comment can we make about G star.

$$[G^* x, y] = [x, G_A^{-1} y]$$

$$(G^* x)^T y = x^T G_A^{-1} y$$

$$x^T (G^*)^T y = x^T G_A^{-1} y$$

$$G^* = (G_A^T)^{-1}$$

G A transpose inverse the inverse transpose it does not matter ok. So, in mathematics this G star is called the adjoint operator alright and you can see that what this is telling us is that, what is this telling us? If we want the same transfer function right another way of interpreting this is to say well v o by i in we know is x transpose G A inverse times y ok. Which can be thought of as, let us call this I mean remember this in other words.

$$\frac{v_0}{i_{in}} = x^T G_A^{-1} y$$

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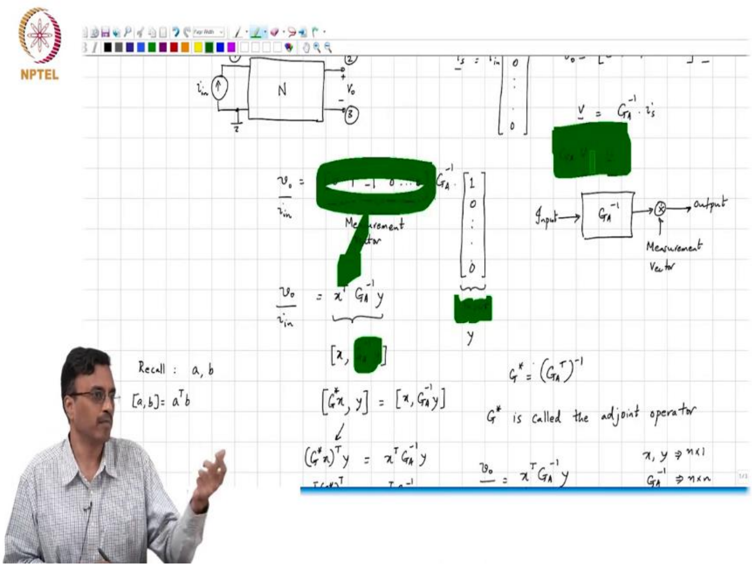

When you write an expression G A times V equals i s how do you I mean this is mathematically is the set of equations, but how do you interpret this? These are nothing but the node voltages that develop when you take a network with a with an MNA matrix G and have a source vector i s that is the interpretation of this set of equations as far as we are concerned, correct ok.

So, when we have multiple input sources as we were discussing yesterday right to do this by superposition is equivalent to finding G A inverse times i s for each source separately. So, in effect what we are doing is computing the inverse multiple times. An alternate way of doing this and a smarter way of doing this is to basically say well we know that v out by i in is x transpose times j inverse times y. Remember x is the measurement vector and y is the excitation vector or the input vector correct.

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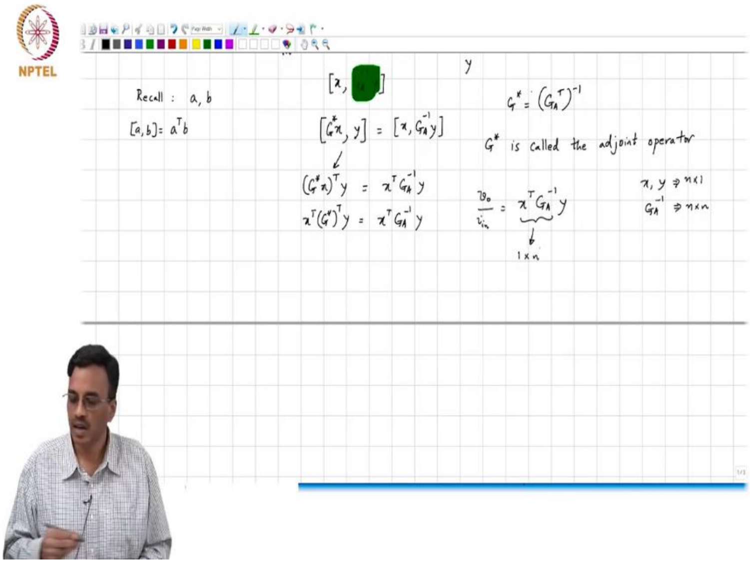

So, you can think I mean what comment can we make about the dimensions of that guy there let us say there are n unknowns let us say x, y are both n cross 1 vector because there are n number of unknowns and G A inverse, what is the size of that? n cross n, so what comment can we make about x transpose times j inverse 1 cross n. It is a row vector correct and I am going to call that v hat alright.

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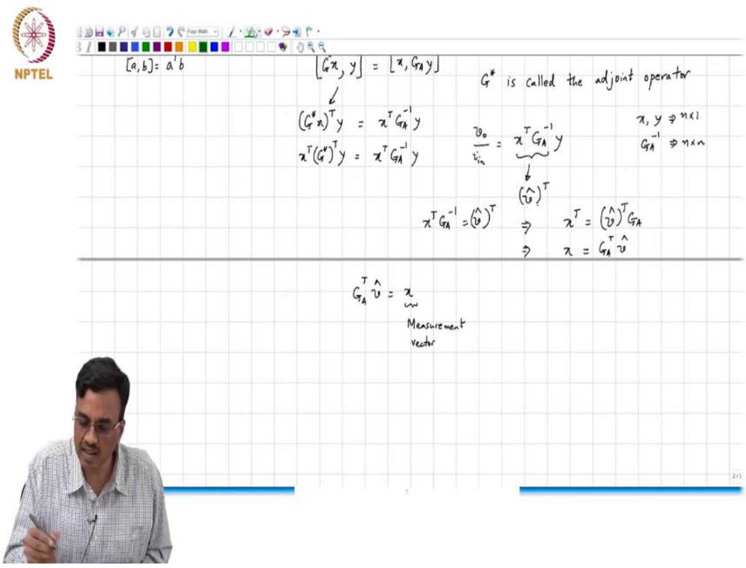

I mean it is just notation there is nothing holy about that ok. So, basically this is saying x transpose G A inverse equal to v hat I mean let me call that v hat transpose because v hat is the I would like to call all my vectors to be column vectors.

So, x transpose G A inverse is a row vector ok and if v hat is a column vector, then it must follow that I can write x transpose G A inverse as v hat transpose right.

$$x^T G_A^{-1} = (\hat{v})^T$$

Which is equivalent to saying, what does this mean? x transpose is G A times V hat transpose which is equivalent to saying x equals, what? Pardon. Hold on hold on I think I am, x transpose G A inverse equals v hat transpose v hat is basically column vector. So, what do you. Pardon. Sorry yeah sorry thank you. So, is v hat transpose times G A must go on the right-hand side and therefore, x must be equal to G A transpose times v hat correct or in other words G A transpose times v hat equals x

$$x^T = (\hat{v})^T G_A$$

$$x = G_A^T \hat{v}$$

$$G_A^T \hat{v} = x$$

And remember what is x? What does this denote, what did denote in r? This is the.

This is the measurement vector that was the name we gave to x or and so but how do you interpret these equations.

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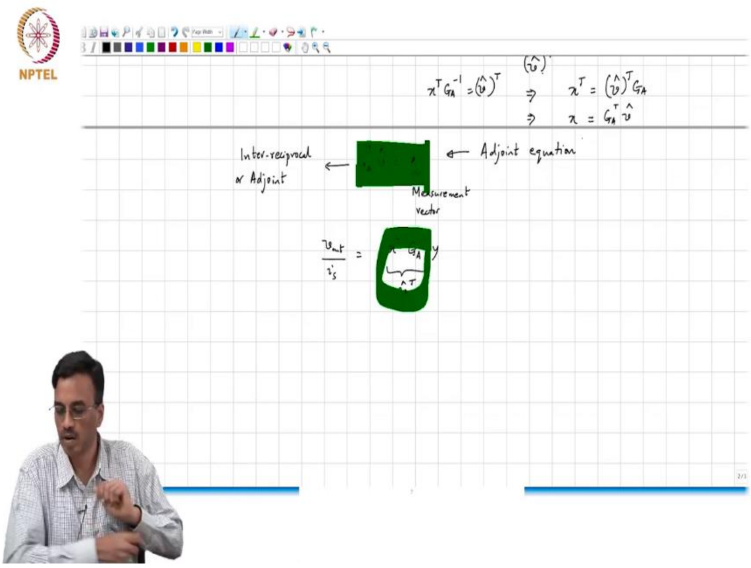

This when we said G A times V equals i s, how did we how did we interpret that? What is the interpretation of this system of equations? In English how do you describe that?

Right. So, v is I mean the in English what this means is that v is the vector of node voltages produced when you excite a network G A with i s, correct? Now by the same token what comment can you how can you interpret this yes, what are we exiting with x? The network G A transpose.

Yeah. So, basically now we have how do we the interpretation of this is that we are now exiting a network with whose MNA matrix is G A transpose with x which corresponds to the measurement vector right. So, earlier wherever you are measuring now you have to excite alright and the voltages that are developed on the on this network are v hat alright ok. And once we find v hat to find the individual transfer functions is simply from the multiple inputs is simply multiplying people, once you find v hat what do you do?

Yeah. So, basically remember that in the original network v out is basically what we call that x transpose times G A inverse times, G A inverse times y and what is x transpose times G A inverse? What is it? v hat transpose ok alright.

$$\frac{v_{out}}{i_s} = x^T G_A^{-1} y \; = \; \hat{v}^T$$

So, and once we have v hat transpose you know finding the various transfer functions from multiple inputs is simply a matter of multiplying v hat transpose with you know whatever input you want correct. So, in other words V hat transpose that matrix, that row vector contains information with regard to all the transfer functions I mean the transfer functions from all the inputs to a given output right and this y merely selects which of those transfer functions you want that is all, alright ok.

And so therefore, if you want multiple transfer functions to the same output, this obviously, entails a lot less labor because you are solving and you know the how do and how do we obtain v hat?

We just solve not the same network, but a related network where what is the relationship between the that network and the original network. The MNA matrix of the modified network is simply the transpose of that of the original network ok. So, we form the network whose MNA matrix is the transpose of the original network that is what is call the, what is that called? If you have a network whose MNA matrix is the transpose of the original network, what is that called?

What is what do you call that network? It is that is called either the inter reciprocal network or the Adjoint network alright, is that clear?

So, and this equation and mathematics is often called the adjoint equation and. So, the bottom line therefore, is that inter reciprocity can be simply thought of as evaluating the first two terms of this equation rather than.

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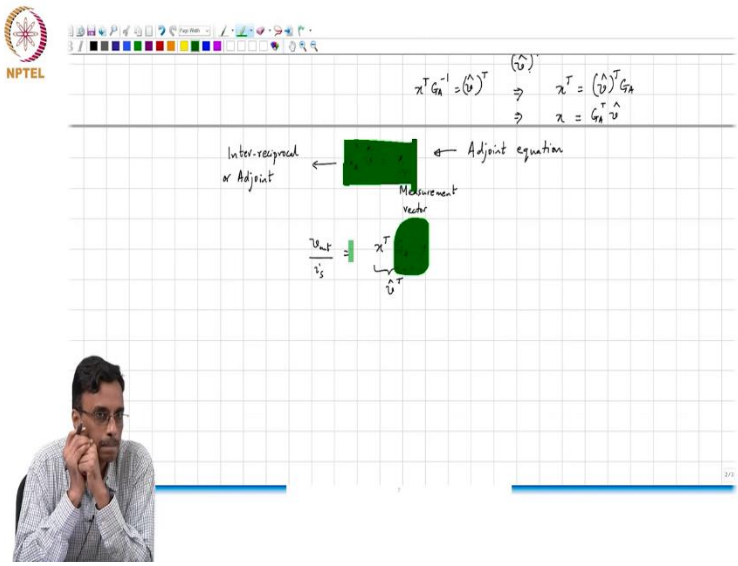

If we did superposition, we would be evaluating that product first and then dotting it with x transpose it does not make sense because that output vector is not changing, it is only the input vector that is changing.

So, it makes sense to compute the stuff that is not changing right ok, because that G inverse is difficult. So, it makes sense to compute that x transpose j inverse once ok and then as you keep changing the input sources the y will keep changing. So, the transfer finding all the transfer functions are easy. Does it make sense people?