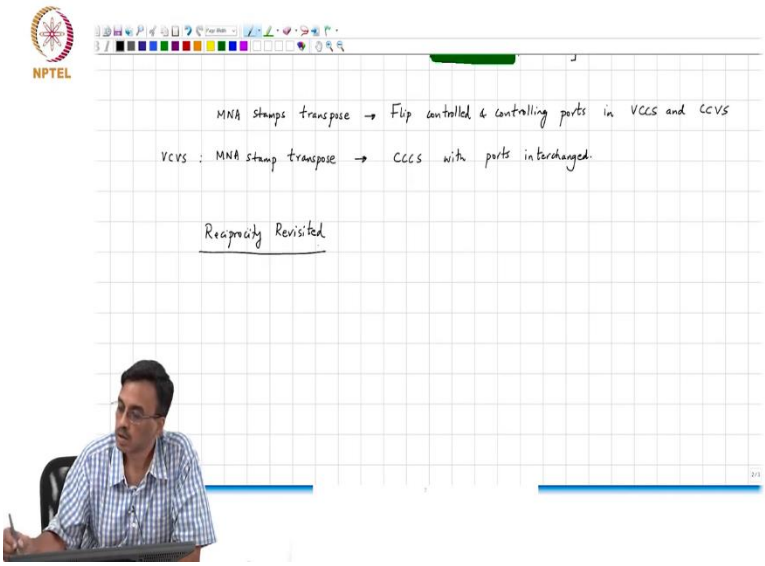
Circuit Analysis for Analog Designers
Prof. Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 13
Inter-reciprocity in linear networks - using the MNA stamp approach

(Refer Slide Time: 00:16)

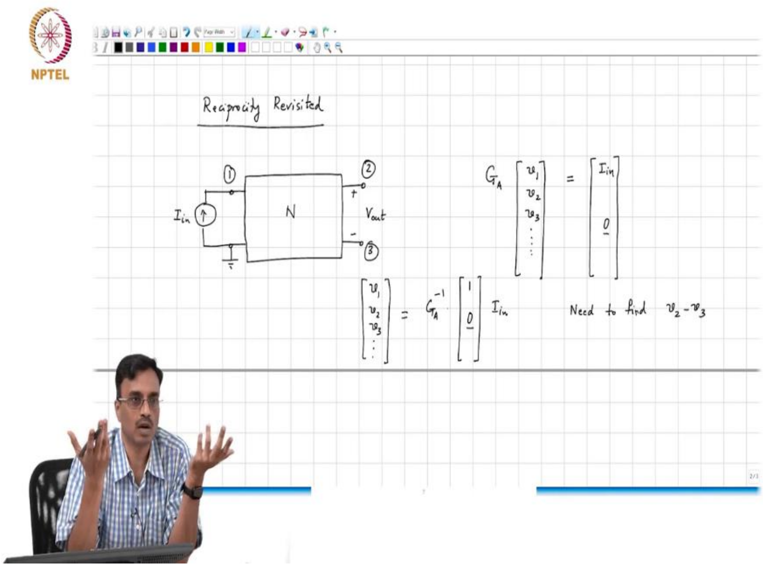


With this as I said with this background, let us take a step back and look at reciprocity again. So, the last time around what did we use to prove reciprocity?

Reciprocity and inter reciprocity how did we prove them? We use Tellegen's theorem right, I do not know about you guys but, I mean the whole thing seems like pulling a rabbit out of a hat, right. Because, it is a I mean you know Tellegen's theorem to begin with is not particularly intuitive, especially when applied to other networks. It almost seems like we knew the answer and we have getting it, right, ok.

A fortunately it is possible to actually derive it from first principles and then you know that is what we are going to do next. So for example, let me take. So, let us say we have a network here right and I am you know I am going to do simply a current input voltage output system, right.

(Refer Slide Time: 01:18)



You can go ahead and do the same math for you know voltage input, current output and any of those other combinations that we often encounter. So, this is said I_{in}, this is the network N and this is V_{out}, correct.

And you know this of course network has whole bunch of nodes and this is a free country. So, I am free to choose any node as the reference, I am going to choose that node as a reference. I will call this node number 1 right, I will call this the node number 2 and this I will call the node number 3, correct.

And how will I go about solving the network? I mean, let us say I know the internals of the network. What will I do? Basically, you just go and write the MNA equations for the entire network and you will basically find that you will get an equation of the form, some augmented conductance matrix, right where the augmented conductance matrix will contain entries corresponding to the conductance’s and you know the controlled sources and all that stuff which are potentially inside this box, right and the unknowns will be all the node voltages and the currents through all the you know the controlled sources or the zero voltage you know the zero voltage sources inside the box, correct. So, the unknowns therefore will be of this form.

They will be $\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \end{bmatrix}$. And G_A is the augmented conductance matrix which we can we are now in a position to determine by simply adding up the MNA stamps of the individual elements, right. And this must be equal to what must be there on the right?

All the independent sources, what are the independent sources here? There is only one independent source namely I_{in}. So, basically that just comes that goes into node 1, the rest of this the rest of this vector is basically 0, alright, ok.

$$G_A \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \end{bmatrix} = \begin{bmatrix} I_{in} \\ \\ \underline{0} \\ \\ \end{bmatrix}$$

And we are interested in, what are we interested in finding? We are interested in finding the transfer function from Not V out to I in, I in to V out. Correct, alright.

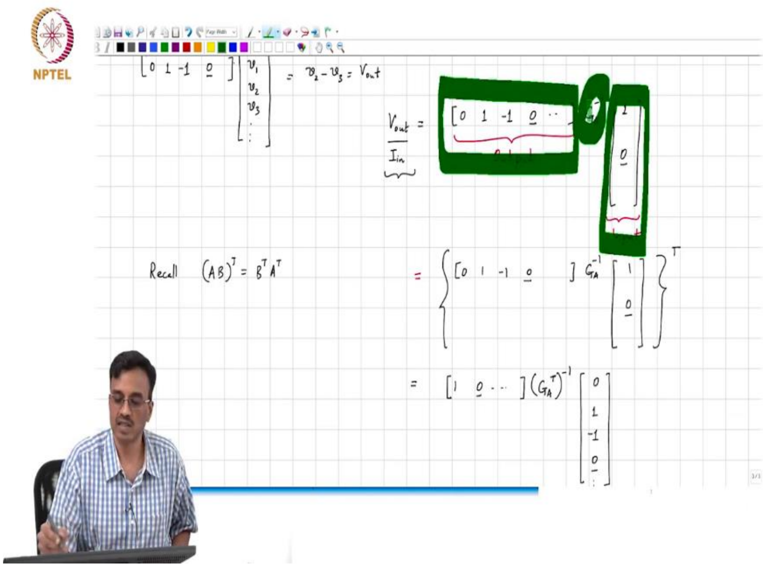
So, what will you do about what will we do? How do we go about doing this? I know it is easy but, please tell me. What will we do? We will find, we will first find v 1 v 2 v 3 I mean, I am not asking you to invert the matrix at this point, but all I am saying is this is what we will do? Right. This is G A inverse times I in followed by all 0's, which is equivalent to saying 0 I mean 1 followed by a string of zeros times I in, right I in is a scalar, correct.

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \end{bmatrix} = G_A^{-1} \begin{bmatrix} 1 \\ \\ \underline{0} \\ \\ \end{bmatrix} I_{in}$$

And this will give me the output will give me. You know the node voltage all the node voltages and all the currents that are flowing through the controlled sources and you know the zero voltage sources needed in any of the controlled source. Does make sense? Ok, but what am I actually interested in? I am actually only interested in. I need to find v 2 minus v 3, correct. $(v_2 - v_3)$

So, how will I find v 2 minus v 3? We have this common vector v 1, v 2, v 3 and so on. We want v 2 minus v 3 what you think. You multiply by a row vector.

(Refer Slide Time: 06:29)



So, v 1, v 2, v 3 blah blah blah times 0, 1, minus 1 and followed by all 0’s ok, must be v 2 minus v 3 which is V out, right.

$$[0 \quad 1 \quad -1 \quad \underline{0}]\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \end{bmatrix} = v_2 - v_3 = V_{out}$$

Does make sense people? So, what is V out? Putting these two equations together it is simply nothing but 0, 1, minus 1, 0, alright. Time’s v 1, v 2, v 3 blab, blah, blah and what is that v 1, v 2, v 3 blah, blah, blah. It is G A inverse times1 followed by all zeros, correct times I in. So, if I bring I in out here. The transfer function from I in to V out is basically of this form, correct.

$$\frac{V_{out}}{I_{in}} = [0 \quad 1 \quad -1 \quad \underline{0} \quad \ldots] G_A^{-1} \begin{bmatrix} 1 \\ \underline{0} \end{bmatrix}$$

Now, is this at is this a scalar or a vector. This is a scalar, right and what does this quantify? What information does that? Vector what does that? What information does that incorporate? I understand. So, this basically quantifies where the input is applied, right. The input is applied and you know this the circuit generates you know this vector of node voltages, right. But we are not interested in all the node voltages right, this you can think of as the we only interested in measuring the difference between selected nodes.

So, this basically measures I mean what you call quantifies across which nodes the output is taken, does it make sense.

Alright. And this is a scalar so, nothing will change if I yes, what happens? I mean if it is a scalar what how can I write this as? That is the expression for the scalar. A scalar is the same as the transpose. So, I can write.

I can think of this as simply equivalently, this must also be equal to 1, 1, minus 1, 0. G A inverse times 1 followed by all 0s this transposed, correct, alright and

$$= \left\{ \begin{bmatrix} 0 & 1 & -1 & \underline{0} & \cdots \end{bmatrix} G_A^{-1} \begin{bmatrix} 1 \\ \underline{0} \end{bmatrix} \right\}^T$$

what is the transpose of this? Recall AB whole transpose is what? B transpose A transpose. So, what do you get here?

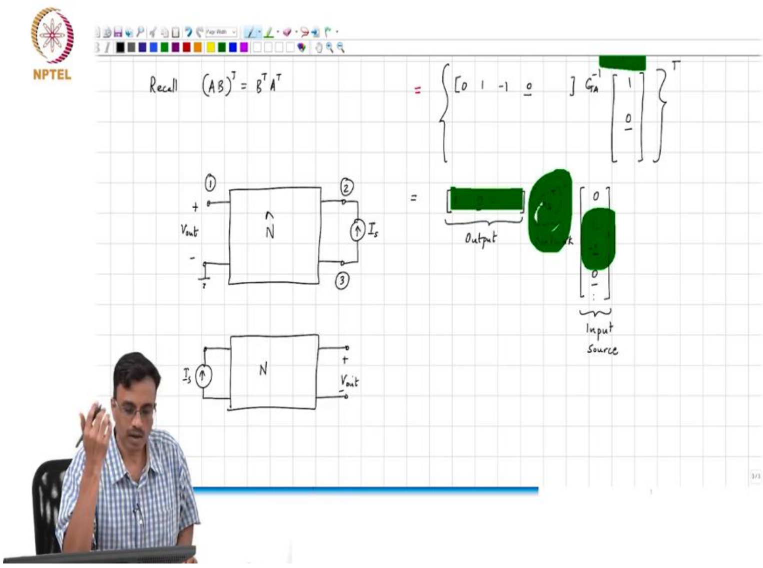
Yes come on people.

One followed by all 0s times G A transpose inverse, whether you invert first to transpose and or do it the other way round is the same thing right, times 0, 1, Minus 1 followed by all 0s. Is this clear people? Ok.

$$= \begin{bmatrix} 1 & \underline{0} & \cdots \end{bmatrix} (G_A^T)^{-1} \begin{bmatrix} 0 \\ 1 \\ -1 \\ \underline{0} \\ \vdots \end{bmatrix}$$

So, now how can I interpret this result as remember this fellow here, this column vector is the input and this is the this goes through the circuit and results in some node voltage vector whose outputs you are measuring, correct. So, likewise this can be thought of as how can you interpret this expression?

(Refer Slide Time: 12:11)



You can now think of this as the input source. Right, this as the network and this as the output the output measurement, alright. So now, a couple of comments is this network the same as this guy.

Is it the same as the original network? In general, I mean they have got different augmented conductance matrices and therefore, the networks are not the same. However, evidently the MNA matrix of this network the lower network is simply the transpose of the MNA matrix of the original network, correct. So, that is and the output is basically this. So, in other words we can interpret this result as we have another network $\hat{N}$ whose MNA matrix is simply the transpose of the MNA matrix of the original network right and what comment can we make about where we are injecting the input now? We are now injecting an input.

And what kind of input is that? There is something between 1 and minus 1 I mean, between nodes 2 and 3, right. So, remember how did we number the nodes? This was node 1, this was ground, this was node 2 and this was node 3. So, if I gave you this matrix and told you, you know where are the independent sources what kind of independent source is there and where is it occurring. What would you say?

It is leaving. So basically, so, this is you can think of it this I_S right and so, this is a network N whose MNA matrix is simply the transpose of the MNA matrix of the original network and where do we sense the output?

What is the output matrix saying? The simply taking the node voltage vectors and simply measuring the voltage at node 1 so, this is the output.

Yes, alright. Is this clear people? Right. So, if you want in other words what I mean what is the moral of the story? If you have an original network N and you inject a current I s here and measure V out here. You can get the same transfer by injecting current into the output port of a network which is not necessarily the same as N, but whose MNA matrix is simply the transpose of the MNA of the MNA matrix of the original network. Is it this clear so far?