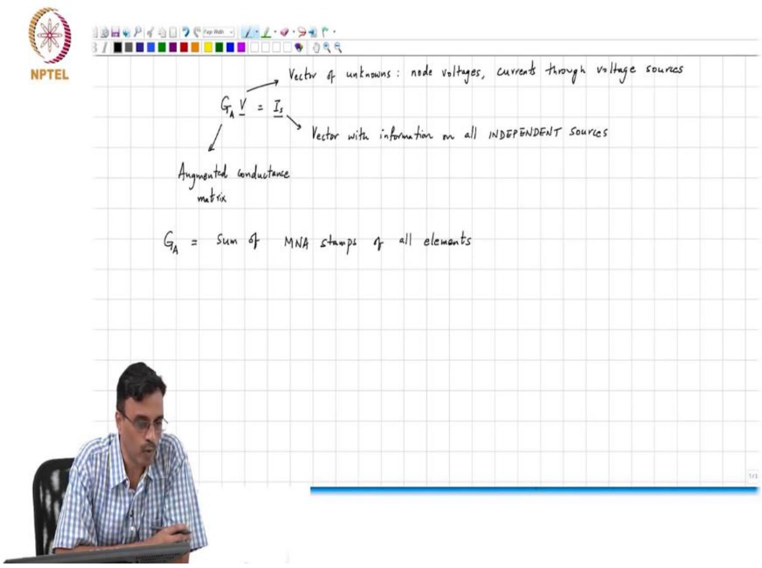
Circuit Analysis for Analog Designers

Prof. Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 12
MNA stamps of controlled sources - the CCCS and CCVS

(Refer Slide Time: 00:31)

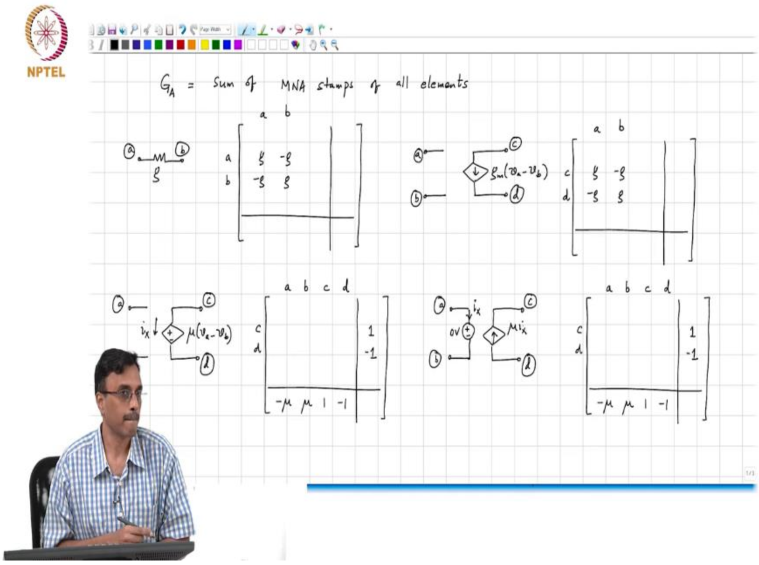


In the last class we were looking at how to write the equations of a network in a systematic way. And we concluded that in general you will get equations of the form $G_A \, \underline{v} = \underline{I_S}$, right where this, (G_A) is the augmented conductance matrix and this, $(\underline{v})$ is the vector of unknowns and what are the unknowns? What are all the unknown?

All the node voltages and currents through the voltage sources and likewise this I_S is basically a vector with information on all the independent sources. Does make sense people? Right and even though this is called the conductance matrix as we saw yesterday all the entries there need not be have dimensions of conductance. It is only that core part on the top left corner which will have all the conductance that are there in the network, if you had voltage sources and so on some of those entries may actually be dimensionless, right. Now, and how do we form go about forming the conductance matrix? This is simply nothing but the sum of. Sum of what?

Very good there is simply the sum of the MNA stamps of all the elements, correct? And, yesterday we saw what MNA stamp of current source would be, what a MNA stamp of a voltage source would be, what the MNA stamp of a conductance would be.

(Refer Slide Time: 03:16)



So, let us quickly write down that stuff. So, this is node a and this is node b and this is a conductance g and what we have? We have this is the, this is the conductance matrix or the augmented conductance matrix, we allow for potential auxiliary equations and unknown, but the only thing we need to be aware of is a, b and a, b and what you have here?

$$\left(\begin{array}{cc|c} g & -g & \\ -g & g & \\ \hline & & \end{array}\right)$$

G, minus g, minus g and g, ok. So, then we saw what a voltage controlled current source would behave like. Let us call this $g_m(v_a - v_b)$ alright, ok and how does this stamp of this look like? What we do?

The only thing that changes is this becomes c and d, is that clear?

Then we were discussing the voltage-controlled voltage source $\mu(v_a - v_b)$ and how does the MNA stamp of this look like? We need an auxiliary variable now, right. The auxiliary unknown is basically the current flowing through the voltage source which is I x and that is there in the unknown vector.

And, what comment can we make about MNA stamp? They are in the rows? c and d and what are the entries there? 1 and minus 1, ok and in the last row in the a b c and d, what will you get? So, you will get minus mu, mu 1 and Minus 1, alright. So, this is where we stopped yesterday, I believe ok.

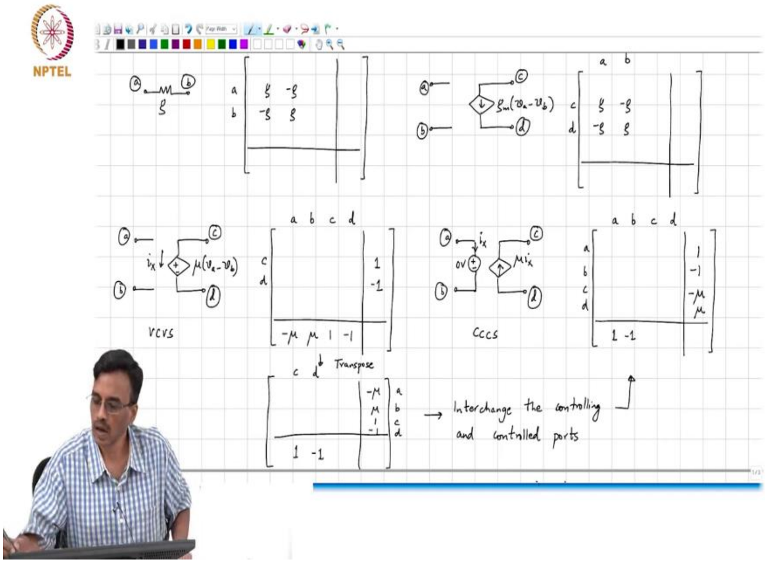
$$\left(\begin{array}{cccc|c} & G & & & \begin{matrix}1\\-1\end{matrix} \\ \hline -\mu & \mu & 1 & -1 & \end{array}\right)$$

The next control source is the current controlled current source and I am going to, remember a current controlled current source always measures the current through voltage a 0-voltage source. So, this is 0 volts and so, this is i_x ok and this is $\mu \times i_x$, alright. And, what comment can we make about the MNA stamp of this character? How many extra variables do we need now?

How many extra variables do we need now? You? Do we need an auxiliary unknown or is there any extra unknown or do we? Do not need an unknown. Why? What are all the unknowns that are there in the network? So, all node voltages remember please remember that all node voltages and currents that are flowing through voltage sources right are or all unknown. So, now, the question I am asking you is do we have an extra unknown now, when you have a current controlled current source is there are unknown extra unknown that is introduced that needs to be introduced. Yes and what is that?

That i x is unknown is flowing through the 0-voltage source and therefore, it also needs an extra unknown. So, you will need an extra row and an extra column for that, and after saying that what comment can we make about the MNA stamp of the current controlled current source? So, this is voltage-controlled voltage source, this is the current controlled current source.

(Refer Slide Time: 09:27)



What comment can we make about the stamp? Yeah, what is this stamp? Do not jump to conclusions. So, in which rows will the last column have entries? c and d and what will be the entry on c, in the c-th row? Minus mu and d-th row? Plus mu, ok and are there other rows on I mean where all in what all rows will. You will have entries in all 4 a b c and d because there is basically i x flowing out of node a and into node b and mu times I x flowing into node c and out of node d. So, in nodes a and b you will have?

You will have? 1 and minus 1 and node c? Minus mu and? Plus mu, alright and what comment can you make about the last row?

The auxiliary the extra equation that we get what is that relates the voltage source? V a equals v b or v a minus v b equal 0. So, you must get? 1 and? Minus 1, does it make sense?

$$\left(\begin{array}{cccc|c} & & & & 1 \\ & & & & -1 \\ & & & & -\mu \\ & & & & \mu \\ \hline 1 & -1 & & & \end{array}\right)$$

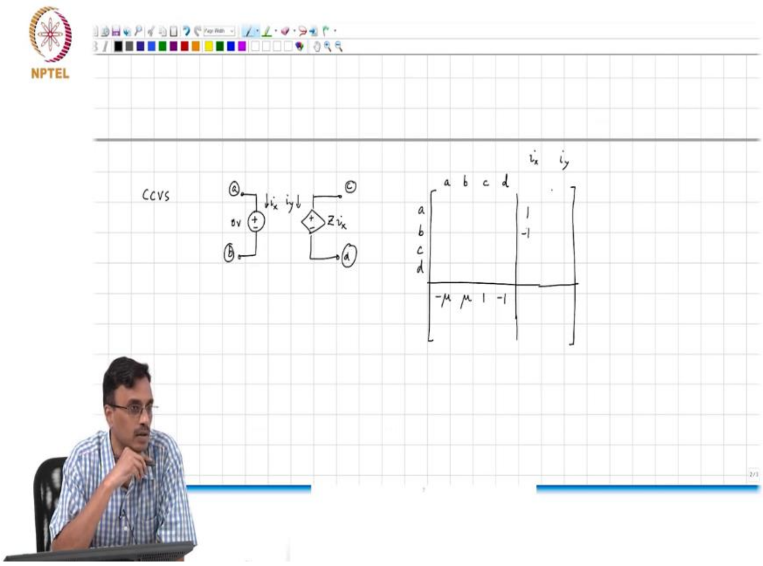
The which is I mean see you always I mean the assumption is that in a circuit simulator what you always assume is that you measure current through a voltage, 0 voltage source, ok. In it is also likely that you know you can just have an arbitrary branch current and make this mu times i x multiple of that arbitrary branch current in which case it straight forward it simply introduce an artificially introduce 0 voltage source and add an extra node there, right and the formulation is simply ones. Is this clear? Alright.

So, and I like just like to draw your attention to one fact and that is that you know do you see any you know if you stare at these 2 MNA stamps you know what do you what comment do you think you can make of the voltage controlled current source and the current controlled current I mean the voltage-controlled voltage source and the current controlled current source. You stare at the MNA stamps, I mean do you see anything at all?

Transpose plus a d a b c d and so on, transpose plus a d a b c d and so on. It is not the transpose, right. I mean it is please note that the MNA stamp is not simply the transpose.

Right you also have to swap you know swap the rows you know a with c and b with d ok and this is just something that I draw your attention to. We will use this fact later on, right. At any rate it looks very similar suspiciously similar to the MNA stamp of the voltage control voltage source ok. The next thing is basically the current controlled. We have seen the voltage controlled current source, the voltage-controlled voltage source the current controlled current source and the last man standing is the current controlled voltage source.

(Refer Slide Time: 13:31)

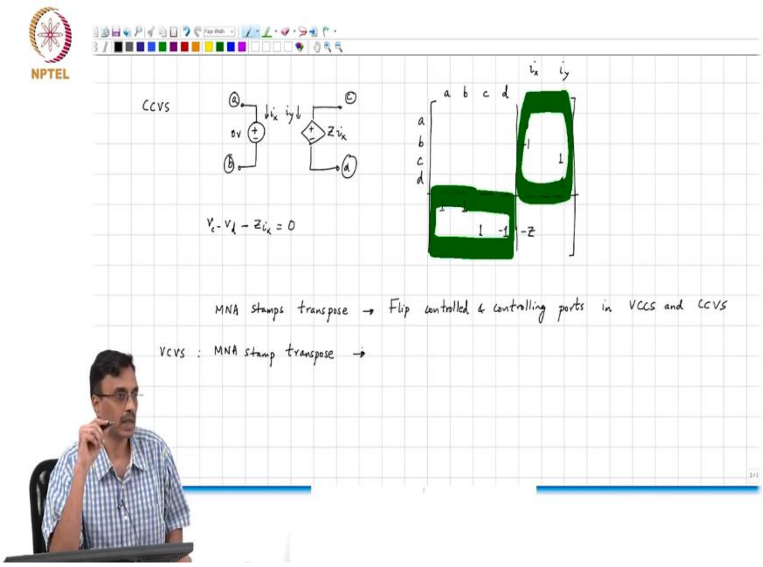


And how will this look like? So, you have a again we have a 0 voltage source and this is 0 volts, this is i x and what are the this is node c and node d and this is some z some impedance times i x, alright. So, alright. So, how many unknowns do we have now?

You have 2 unknowns now because of? And, the remember that the current flowing through any voltage source is unknown or is an unknown and therefore, we need 2 extra columns and 2 extra rows, ok. And, now you will tell me what we need to do. Let us assume that this corresponds to I x and this column corresponds to i y. What do we do?

So, between nodes which of the rows will you have entries in the last, but one column? In a and b. So, that is 1 and minus 1 alright. And what else? And what comment can you make about the last, but one row. Last, but one row. No.

(Refer Slide Time: 16:07)



I mean the last, but one row will basically you know quantify what is happening with that 1 minus 1. So, v a must be equal to v b, ok and we are done with one row and one column. In the last column, what do we see? They must be 1 and minus 1. Alright, and what must you see in the last row.

You see V c what do we. So, V c minus V d equals or minus Z times i x equals. 0.

$$V_c - V_d - Zi_x = 0$$

So, what you see? What is that equation? 1 minus 1 and minus Z and 0.

$$\left(\begin{array}{cccc|cc} & & & & 1 & \\ & & & & -1 & \\ & & & & & 1 \\ & & & & & -1 \\ \hline 1 & -1 & & & & \\ & & 1 & -1 & -Z & \end{array}\right)$$

I guess the entries which are not there are all assumed to be 0. So, they should be put, alright, ok. Now, if I flip the ports if I interchange a with c and b with d what you think will happen to the MNA matrix or the MNA stamp of this guy. First of all, what you notice with respect to this matrix and this matrix?

That is simply the transpose ok. So, if I flip the controlled and controlling ports, what comment can we make about the MNA stamp? What will happen to that last entry minus Z? Which row will it enter now?

It will enter, Yeah, it will enter here and therefore, if by flipping the ports all that you do is if you flip the ports what you basically have is the transpose of the MNA stamp of this guy, correct right? So, which control which of the controlled sources does flipping the port result in the transposed MNA stamp?

This is just an observation flip the controlled and controlling ports in which controlled sources? VCCS and CCVS

So, if you transpose MNA stamp of the voltage-controlled voltage source, if you transpose it is MNA stamp what will you get? The 2 things that you need to observe here. If we transpose this guy here what do we get? We will get? Please, see this clearly what will we get?

So, you will get minus mu 1 minus 1 and which rows are there which rows are they in? a b c and? d and what will happen on the last row? 1 and minus 1 and they will be in the? c and d-th column ok, alright.

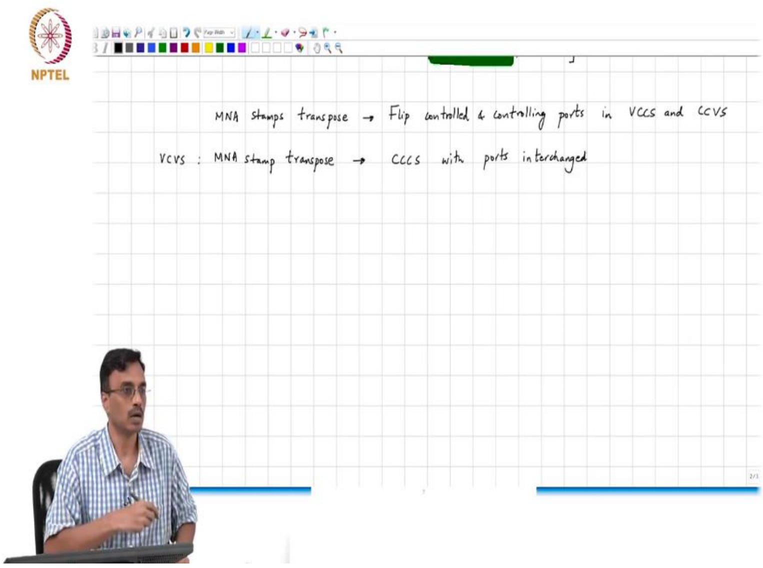
$$\left(\begin{array}{cccc|c} & & & & 1 \\ & & & & -1 \\ \hline -\mu & \mu & 1 & -1 & \end{array}\right) \xrightarrow{Transpose} \left(\begin{array}{cc|c} & & -\mu \\ & & \mu \\ & & 1 \\ & & -1 \\ \hline 1 & -1 & \end{array}\right)$$

So, that look suspiciously similar to this to this, but there is a small twist. What is that twist? I mean is this the same as that? It is not that. So, what. So, if you take the transpose of the MNA stamp of the voltage-controlled voltage source right, you get something which seems like a current controlled current source, but it is there is a twist and that is?

The controlling and controlled ports are interchange the controlling and controlled ports and then you will get the current controlled current source. Does it make sense people? Ok, at this point this is an observation. Is it clear so far? And, if you take that the transpose of a conductance if you take the transpose of the MNA stamp of a conductance what comment can we make?

Well, that is a symmetric matrix. So, if you take the transpose, it remains the same conductance, is that clear? Alright.

(Refer Slide Time: 22:26)



With this background so, this basically you get a CCCS with ports interchanged.