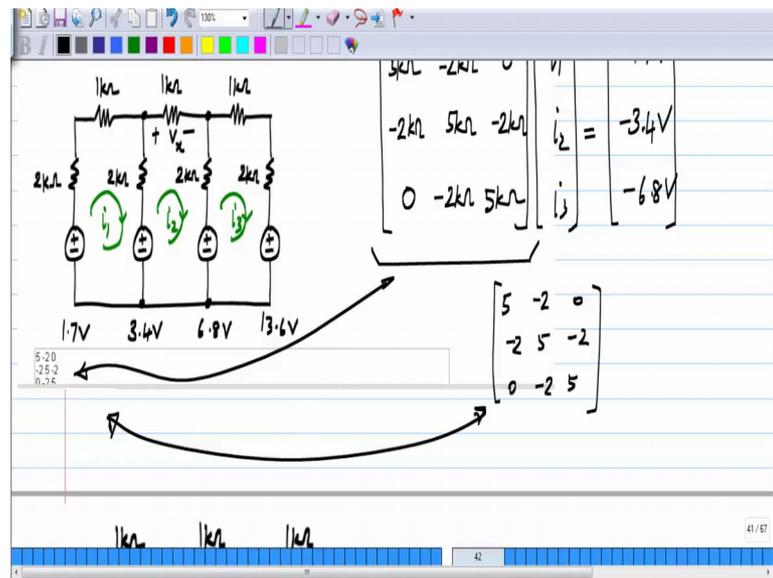**Basic Electrical Circuits**
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Lecture - 71

Now, we will discuss solution to the assignment on units 9 and 10 on mesh analysis.

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So, the first question is to set up mesh analysis equations for this particular circuit. And, so in this case we have only resistors and voltage sources, so it is the easiest case for mesh analysis. We will have three meshes, which are already given to you I 1, I 2, I 3 the mesh currents are already given and this first coefficient here is the sum of all resistance in this mesh, which is 2 kilo ohms plus 1 kilo ohms plus 2 kilo ohm, which is 5 kilo ohms.

This entry is negative of the resistance common to mesh 1 and mesh 2, which is 2 kilo ohms to have minus 2 kilo ohms. And finally, this entry is the resistance common to meshes 1 and 3 and there is no such resistor, so this is 0. And the second mesh, the total resistance is 5 kilo ohms that appears here along the diagonal, then this entry is negative of resistance common to mesh 1 and 2, which is this 2 kilo ohms, so we have minus 2 kilo ohms and this entry is negative of resistance common to this and that mesh, which is again minus 2 kilo ohms.

Finally, for the third mesh the total resistance is 5 kilo ohms, which appears here along the diagonal, then we have 2 kilo ohms, which is common to meshes 2 and 3 and nothing common to meshes 1 and 3. Anyway by symmetric, we have already filled these, so we can fill these and on the right side, we will have the total voltage rise in each mesh and that is in the first mesh if you look at this direction, we have a voltage rise of 1.7, voltage drop of 3.4, so we will have minus 1.7 volts, second mesh voltage rise of 3.4, voltage fall of 6.8.

So, the net voltage fall of 3.4 volts and the third one voltage rise of 6.8, voltage fall of 13.6, so a total voltage rise of minus 6.8 volts. In any case, what you are asked in this is resistance matrix which is given here, you should omit the units and enter only the numerical values in kilo ohms, which is 5, minus 2, 0, minus 2, 5, minus 2 and 0 minus 2 5, so that is the answer.

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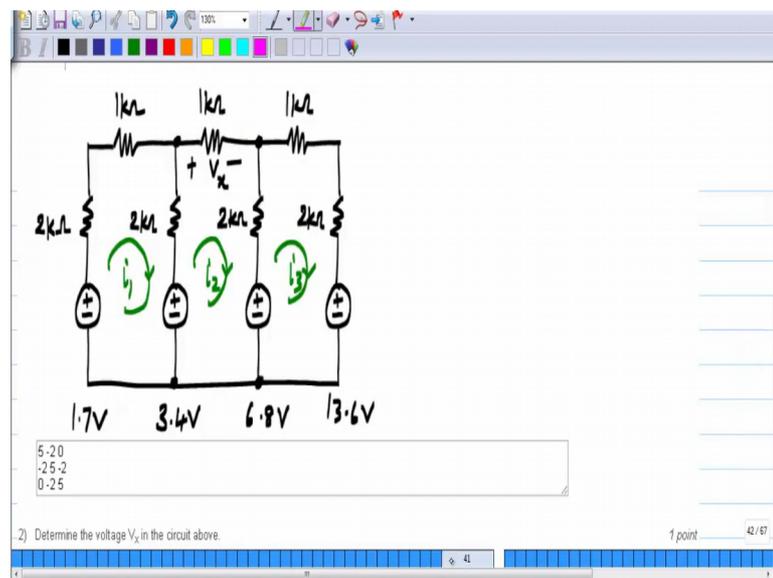
Now, in the next question it is the same circuit and you are asked to determine the voltage V x. Voltage V x is this one, which means that basically whatever we set up earlier we have to solve. So, in this we have to find V x, so we have to solve this.

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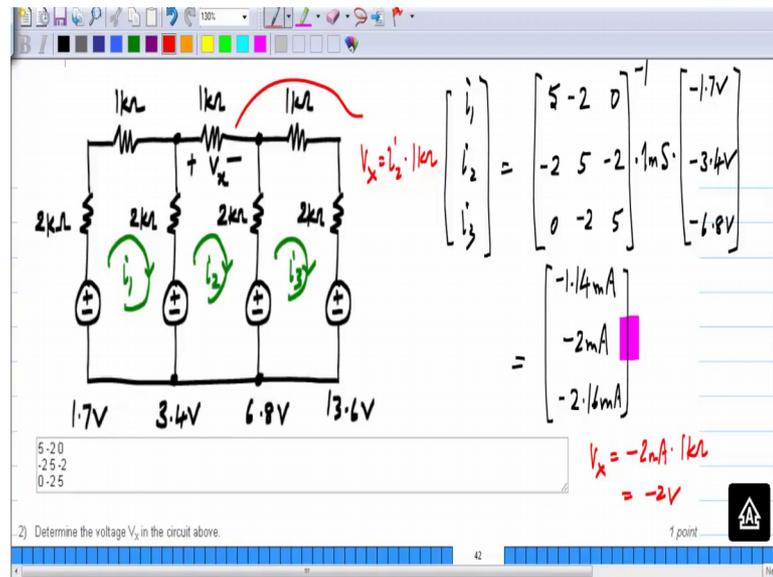
So, first define the current vector which is basically and by the way, the entries here had units of kilo ohms, so the inverse has units of Millisiemens. So, I will multiply the whole thing by 1 Millisiemen and this has to be multiplied by the source vector, which was minus 1.7 volts minus 3.4 volts and minus 6.8 volts. And if you do this, the answer comes out to be minus 1.14 milli amp minus 2 milli amp and minus 2.16 milli amp.

Now, clearly you can see that this V x is I 2 times 1 kilo ohm, I 2 is the current flowing through this resistor, which belongs only to mesh number 2 and it is resistance is 1 kilo ohm and I 2 we see is minus 2 milli amperes. So, V x is minus 2 milli amperes times 1 kilo ohms equals minus 2 volts.

And you can see that, minus 2 volts is the right answer.

Now, again there is another question, where you have to set up mesh analysis equations and the only difference from before is that, before we had only voltage sources, independent voltage sources and resistors. Now, we have independent voltage sources, a current dependent voltage source and resistors. So, now, this current here is 1 kilo ohm times I x and I x is defined in that direction. You can clearly see that I x equals i 3 minus

i 2, I x is flowing upwards in this branch and that is equal to i 3, which is defined to be flowing upwards and i 2, which is flowing downwards.

So, essentially here we have an additional voltage drop, which is 1 kilo ohm times i 3 minus i 2, so that is the voltage drop. So, now, this we have i 3 and i 2, so if we look at this mesh excluding this, we will get some resistances and if we include this, the coefficients of those currents i 3 and i 2 will be modified; that is if you write down the mesh equation here, we will get i 1 times 3 kilo ohms, which is the total resistance in this mesh and we have minus 1 kilo ohm, because of this resistance which is common to meshes 1 and 2 and there is nothing common to meshes 1 and 3.

This will be equal to the total voltage rise, which is half a volt, which is the voltage rise minus a voltage drop, which is that one 1 kilo ohms times i 3 minus i 2. Now, of course, this is also an unknown, so it will go to the other side, so what happens is that, in the first equation the right hand side source vector will be this 0.5 volts which is due to the independent source and on the left side, the coefficient of i 1 is 3 kilo ohms, the coefficient of i 2 is minus 1 kilo ohm and plus 1 kilo ohm from there, which is minus 2 kilo ohms.

Finally, is basically plus 1 kilo ohm times i 3 when you transfer this to the other side. And similarly, if you look at the second row I will first write the conventional terms, which basically are the total mesh resistance 3 kilo ohm, which appears here and resistance common to meshes 1 and 2, which is 1 kilo ohm, which appears there and the distance common to meshes 2 and 3, which is again 1 kilo ohm, which appears over there.

But, additionally we also have a voltage rise of 1 kilo ohm times i 3 minus i 2, on the left hand side we count voltage drops, so that becomes 1 kilo ohm for i 2 and minus 1 kilo ohm for i 3 and on the right hand side, we have the independent voltage rise, which is equal to minus 2 volts, because we have a voltage drop of 2 volts. So, now, of course, we have to combine these, this one remains as minus 1 kilo ohm, this becomes 4 kilo ohm and this becomes minus 2 kilo ohms.

Finally, the third mesh it is conventional, because we have only voltage sources and resistors. So, the total mesh resistance of 3 kilo ohms appears here and whatever is common to meshes 2 and 3 appears here with a negative sign and whatever is common to

meshes 1 and 3, which is really nothing appears there. And on the right hand side we have the total voltage raised, which is plus 2 volts minus 4 volts, which is minus 2 volts.

So, if you look at just resistance matrix this part it will be 3 minus 2 1 in the first row minus 1 4 minus 2 in the second row 0 minus 1 3 in the third row and it has units of kilho ohms and in the answer you are required to entire the numerical values in the kilo ohms. So, this is the answer.

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Again your given some V x, which is over here you can first of all see that V x is nothing, but i 1 times 1 kilo ohms. So, if you find i 1 you find this it is of course, possible to treat this is separate problem and use whatever circuit analysis method you want and find the answer. But, if you already set up the mesh equation and also the point of this assignment is learn mesh analysis, you can solve the mesh analysis equations and find out not only i 1, but all the mesh currents.

We know that i 1 i 2 i 3 is basically 3 minus 2 1 minus 1 4 minus 2 0 minus 1 3 inverse with units of Millisiemens, this matrix the resistance matrix as units of kilo ohms its inverse will have units of Millisiemens times the source vector which as we saw was half volt minus 2 volts and minus 2 volts. And if you calculate i 1 i 2 i 3 it turns out to be minus 0.2 milli amps minus 1.06 milli amps and minus 1.02 milli amps for our calculation of course, we need only i 1, so V x is I 2 times 1 kilo ohms, which is minus 0.2 volts and that is what is given here, so the correct answer for V x is minus 0.2 volts.

Now, with these solutions; that is, knowing the value of i 1 i 2 and i 3 you can calculate any variable in the circuit.

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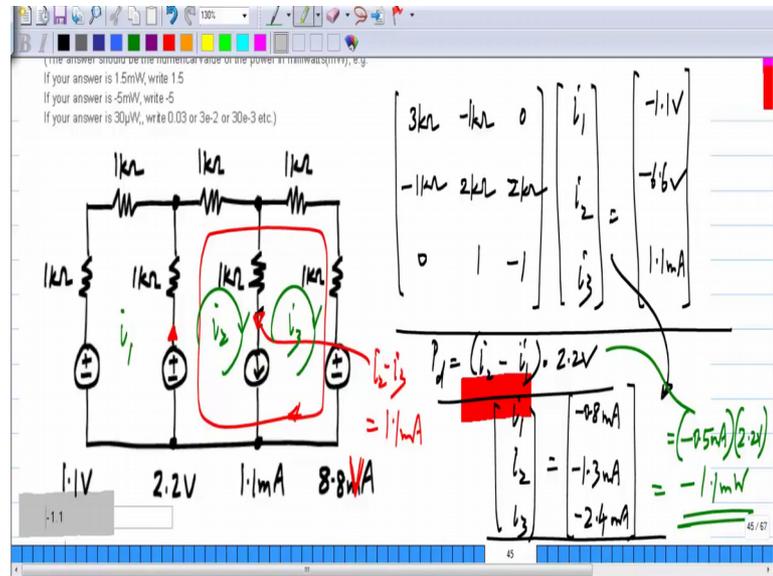
Now, coming to the next problem you have a circuit, you ask for the power delivered by the 2 point volt sources. Now, since the answer require to the power delivered you can again go and do it any way you want, what the whole point of these assignment is to learn mesh analysis, so I am going to use that. Now, in these circuits if you see we have resistors independent voltage sources and an independent current source. So, we have to use this super mesh concept.

So, let say this is i 1 this is i 2 this is i 3 these are three mesh currents, because now there is branch with the, with an the independent current source we have to form a super mesh around it and right the KVL equation around the super mesh. So, first the equation for the regular mesh total resistance in the first mesh, which is 3 kilo ohms resistance common to meshes 1 and 2, which is 1 kilo ohms. So, it appears minus 1 there and the no resistance common to meshes 1 and 3, so 0 over there.

On the right hand side we have the total voltage raise, which is plus 1.1 minus 2.2, which is minus 1.1 volts. And secondly, we have to write the super mesh equation and in the supper mesh equation i 2 is flowing through these two resistors, so the total is 2 kilo ohms and i 3 is flowing through these two resistors in the super mesh, so that is also 2 kilo ohm. And finally, i 1 is flowing through this resistor in the opposite direction to the

direction we use for traversing the super mesh, so the coefficient of i 1 is minus 1 kilo ohm.

In other words, is 1 kilo ohm is the resistance common to the super mesh and some other mesh and the total voltage rise in the super mesh is plus 2.2 minus 8.8 volts, so there is the mistake here this should be 8.8 volts. So, we have minus 6.6 volts over there plus 2.2 minus 8.8. When finally, we have the write the relationship between the currents in the super mesh we know that the current in this branch is i 2 minus i 3 in terms of the mesh currents.

So, this current here is i 2 minus i 3 and we know what its value is that is equal to this independent current source, which is 1.1 milli amperes. So, the third equation simply state this fact which is that i 2 minus i 3 equals 1.1 milli amperes, the entry in the, so called in the resistance matrix are both resistances and dimensional constant and the entry in the source vector or both voltages and currents. How do you find the power delivered by this? So, the power delivered by these 2.2 volt sources is 2.2 volts times the current flowing in that direction opposite of the passive sign convention.

The other hand if you take this V and the current flowing in that direction and multiply the two that will be the power dissipated in 2.2 volts. So, now, you can see that this current that is flowing upwards is nothing but, i 2 minus i 1 the current in this branches I 2 minus i 1 flowing upwards. So, the power dissipated is i 2 minus i 1 times the voltage of the voltage source, which is 2.2 volts. Now, to do this we can solve for i 1 i 2 i 3 from the mesh analysis set up.

So, solution vector i 1 i 2 i 3 turns out to be minus 0.8 milli amperes minus 1.3 milli amperes and minus 2.4 milli amperes. So, I 2 minus I 2 is minus 1.3 minus of minus 0.8, so this term here is minus 0.5 milli amperes, so this is equal to minus 0.5 milli amperes times 2.2 volts, which is really minus 1.1 milli watts. So, it turns out that this 2.2 volt voltage source is actually dissipating power it is dissipating 1.1 milli watt and, because we calculated the power generated it came out us minus 1.1 milli watts and that is the answer shown over there.

The next question you are given some circuits, so this m consists of only resistors it means something, which already means and there are some two sources. Now, in this context the network and consisting only of resistors means that whole network is linear with two independent sources i 1 and V 2. So, this means that any branch variable anywhere in the circuits either with in m or in these will be of the form some proportionality constant alpha 1 times i 1 plus some other proportionality constant alpha 2 times V 2 alpha 1 and alpha 2 have different dimensions, but that is not relevant the point is this can be expressed this way.

Similarly, you take some other variable, let us say I x. it will be some beta 1 times i 1 plus some other beta 2 times V 2. Because, we have only two independent sources and the rest of the circuit is linear every variable in the circuits either voltage or current is a linear combination of i 1 and V 2. Now, you are given some conditions I x is 1 milli amp i 1 is 2 milli amps and V 2 is minus 5 volts and V x is 6 volts i 1 is 8 milli amps and V 2 is minus 10 volts here first of all we have 4 variables alpha 1 alpha 2 beta 1 and beta 2, so we have to determine these things.

Now, we also know that I x equals V x divided by 2 kilo ohms that is from the definitions here or V x is I x times 2 kilo ohms. So, now, in that case we can set up two simultaneous equations that is I convert this also I x or I convert this also do V x. So, let me take the first case itself I x is 1 milli amp, which means that V x is 1 milli amp times 2 kilo ohms,

which is 2 volts. So, 2 volts equals when i 1 is 2 milli amps that is alpha 1 times 2 milli amps plus alpha 2 times V 2, which is minus 5 volts this correspond to the condition given over there, what it says ix is 1 milli amp for I know that if I x is 1 milli amps V x will be 2 volts.

Because, that 1 milli amp is for 2 kilo ohm resistor and V x is measured acorss it and V x is 6 volts that is 6 equals alpha 1 times i 1 is 8 milli amps plus alpha 2, which is minus 10 volts. So, by solving for these two we can fine alpha 1 and alpha 2 and if you do that you will get you get alpha 1 to be 0.5 kilo ohms it has dimensional kilo ohms because it multiplies current to result the voltage and alpha 2 is minus 0.2 and this is dimension less, because it multiplies voltage to give your voltage.

If you substitute these for the given the values I 2 is 6 milli amps and V 2 is 15 volts you will get this V x, which is the voltage across the 2 kilo ohm resistor to the half kilo ohms times 6 milliamp minus 0.2 times 15 volts, which is exactly 0. So, it turns that 1 I 2 is 6 milliamps, so V 2 is 15 volts. This, the voltage across this 2 kilo ohms exactly 0 and; obviously, no power is dissipated and 2 kilo ohm resistor. So, that is the answer over there that is 0.

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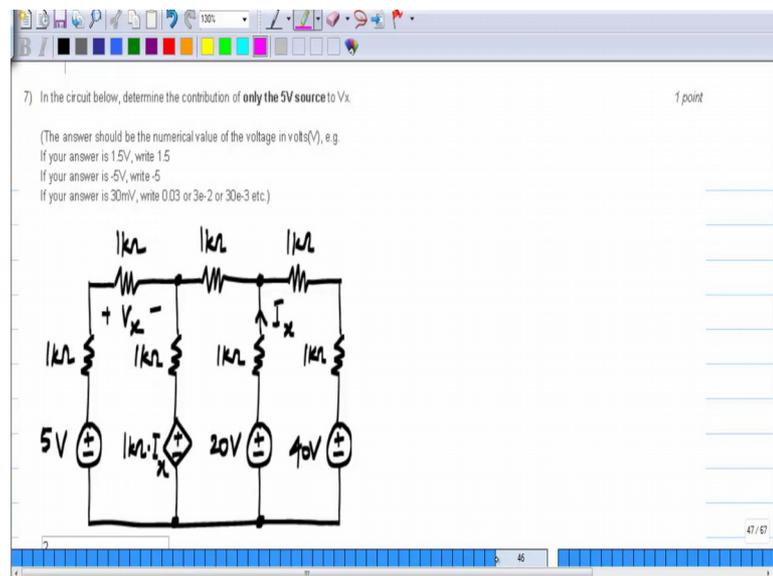
In the next question your ask to find the contribution of on the 5 volts source to be x of course, it can be done and many ways by initially setting this 20 volts and 40 volts sources is 0. Again in the spirit of learning mesh analysis I am going to continued used

mesh analysis. So, in this case we have a circuit with independent voltage sources resistors and in current control voltage source.

Now, if you compare the circuits to the one we saw earlier you see the exactly the same except that the values of these voltage sources are now different. So, this means that basically the resistance matrix will be remain exactly to the same the right hand source vector will be different.

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So, the resistance matrix it turns out is exactly the same as in the earlier problem, let me show you again, which one that is and this is problem number 3 and this is the resistance matrix. And right hand side we have the voltage rise in the first mesh, which is 5 volts the voltage rise in the second mesh, which is minus 20 volts and the voltage rise in the third mesh which is plus 20 minus 40 volts, so I have again minus 20 volts.

Now, we have to find V x and also not V x completely put only the contribution of 5 volts and we know that V x is i 1. So, by the way I dint mark the mesh currents i 1 i 2 i 3 are the three mesh currents and V x is I 2 times 1 kilo ohm. So, if we do this will get i 1 i 2 i 3 to be the inverse of this matrix, which happens to be 2 by 5 1 by 5 0 3 by 25 9 by 25 1 by 5 and 1 by 25 3 by 25 and 2 by 5 with units of Millisiemens times the source vector, which is 5 volt minus 20 volts and minus 20 volts.

We are required to find the contribution of only 5 volt source, so we can set these two 0, then you see that i 1 is simply this number here 2 by 5 times 5 volts and times 1 Millisiemens. So, i 1 due to the five volts source alone is 2 milli amps it is the product of this number and that number. So, V x which is I 1 times 1 kilo ohm is 2 milliamp is to 1 kilo ohm. So, V x due to the 5 volts source by itself is 2 volts, which is the product of this 2 milli amps and 1 kilo ohm and as you can see that is the right answer.

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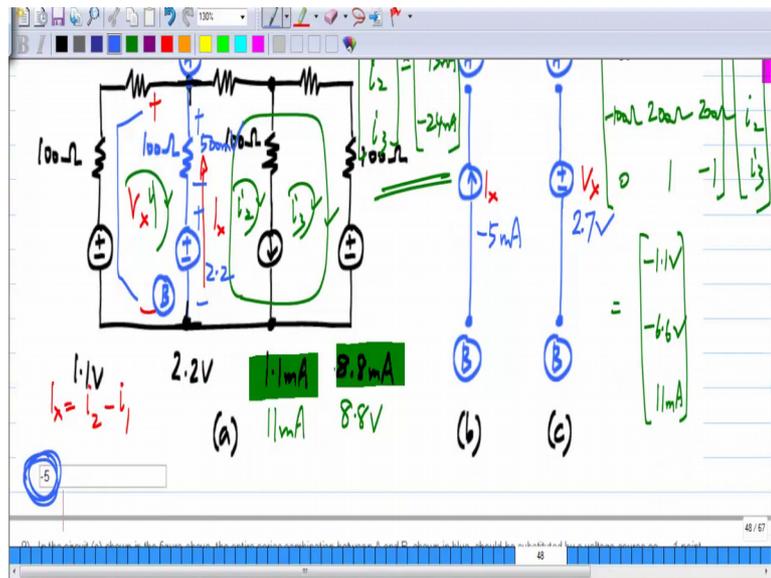
If you go to the next problem you are given some circuit and your asked to replace this entire plans between A and B consist of a resistance and a voltage source and series with current source actually it does not matter what this consists of all you have do is if you have a branch if find the branch current and you have to use the current source whose value equals that current. Now, essentially because this current source with which we have to substitute this flowing from bottom to top we have to find the current flowing that way and let me call that I x, so current source of value ix will do the job.

Similarly, when you want substitute with voltage source all you have to do it does not matter again what exactly is in this branch all you have do is find this voltage, let us called V x then if you use voltage source of value V x that way it will do the job. Now, again you can do this anywhere you want I will continued used mesh analysis. So, now, in this case we have an independent current source, so we have to use is super mesh surrounding these two meshes that way let me define i 1 i 2 and i 3.

By the way again there seems to be a mistake here this is 11 milli amperes and this is 8.8 volts these one corrected in the assignment in my copy is not corrected. So, the equation for the first mesh will have the total resistance 300 ohms here resistance common between mesh 1 and 2, which is minus 100 ohms over there and this nothing common three meshes 1 and 3. So, it will be 0 i 1 i 2 and i 3 and for the super mesh i 2 is multiplied by these two resistors.

So, that is 200 ohms i 3 is multiplied by these 2 resistors, which is also 200 ohms in total and this one basically it is the resistance common to the super mesh and mesh number 1 that is 100 ohms, so we get minus 100 ohms over there. And finally, if you look at the last equation that is for this current source the current in this branch is i 2 minus i 3 and that has to be equal to 11 milli amperes, so have i 2 minus i 3 equals 11 milli amperes.

The other two entries will be the voltage rise in the first mesh, which is 1.1 volt minus 2.2 volts minus 1.1 volt and the voltage rise in the super mesh, which is plus 2.2 volts minus 8.8 volts, which is minus 6.6 volts. And if you solve for this you will get i 1 i 2 and i 3 you invert the matrix and you solve for it and it comes out to be minus 8 milli amps minus 13 milli amps and minus 24 milli amps and it makes sense, because I 2 minus i 3, which is the current flowing down words is 11 milli amps.

Now, in our case we have to find the current I x and this I x is nothing but, i 2 minus i 1, which is minus 13 plus 8, which is minus 5 milli amps, so if you sub suiting this branch shown in blue with current source shown in this direction it has to be sub suited with minus 5 milli amps and that is the answer. Now, the very next question asked for the same branch to be sub suited by a voltage source.

So, we have to find the voltage V x across these two points we know that the voltage drop across the 100 ohm resistor in this direction is i 1 minus i 2 times 100 ohms and I 1 minus and I 2 is 5 milli amps. So, this is 500 milli volts or half volt and these voltages 2.2, so the total is 2.2 plus 0.5 volts, which is 2.7 volts. So, instead of substituting into the current source of minus 5 milli amps we can also sub suited the voltage source in this polarity of 2.7 volts and that is we have over there, so that takes care of assignment 5.