Basic Electrical Circuits
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Lecture - 50
Super node for nodal analysis with independent voltage sources

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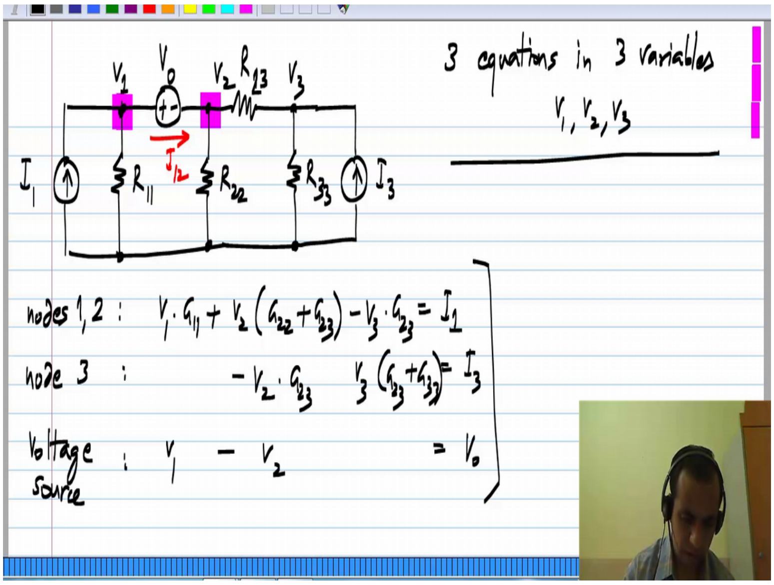

We saw how to get three equations and three variables even when we have independent voltage sources in our circuit. The three variables are V 1, V 2 and V 3; and I can solve for these three variables. Now this is fine, we took the equation for node 1 with the unknown current I 1 2, and we took the equation for node 2 also has the same unknown current I 1 2, but with opposite sign. We combine these two and eliminated that variable. What we would still like is a straight forward way of ah writing down all these equations without going through this intermediate extra variable. If you remember that is what we did for nodal analysis also they started out with the Kirchhoff's law and then identifying each of the currents with the voltages across them and the resistances in those branches, but finally, when we wrote down the structure of the equations, we could simply look at the circuit and write out the conductance matrix. So, that is what we want here also, and that is possible as well, because if you look at what is happening when we sum the equations for nodes 1 and 2.

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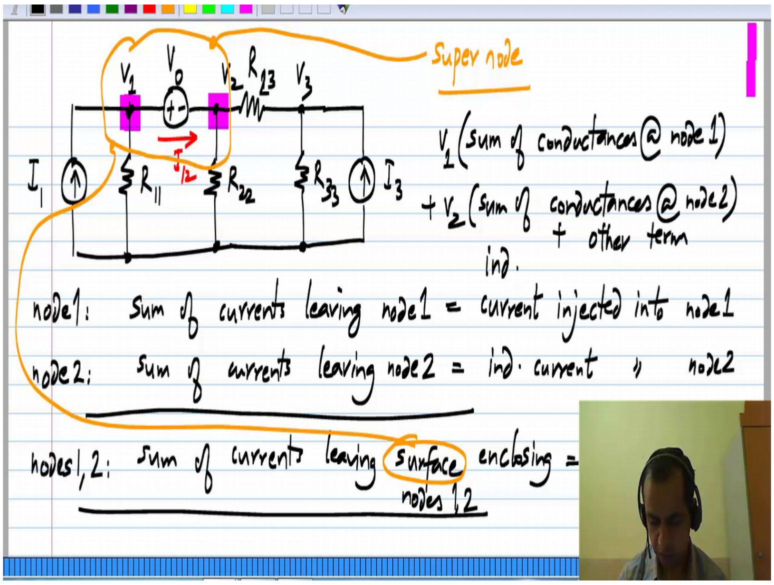

Let me copy over this circuit. The equation for node 1, it is says that on the left hand side, we will have sum of currents leaving node 1 being equal to the independent current injected into node 1. And similarly at node 2, we have sum of currents leaving node 2 being equal to independent current injected into node 2. When we sum these, what happens, we will have the sum of currents leaving nodes 1 and 2 that closed surface enclosing nodes 1 and 2.

Because in the first equation, any current flowing between node 1 and node 2 will appear with the particular sign; and in the second equation the same current will appear with an opposite sign. So, any current flowing between nodes 1 and 2 will get canceled when you carry out the summation, and what is left is basically sum of currents leaving the surface enclosing nodes 1 and 2, basically this surface, this is the surface we are talking about here. And what will be have on the right hand side, we will have the total of independent currents injected into nodes 1and 2, so that is all that there to it

Instead of looking at a single node, you will look at this surface enclosing the two nodes and such a surface enclosing multiple nodes is known as a super node. So, the bottom line is that if a voltage source is connected between two particular nodes, you form a super node with those two nodes and write the equations for the super node. You take the sum of all the currents leaving the super node, and equate that to the total current being injected into the super node. How do you do this, you still have the same variables V 1, V 2; we have the super node, the super node consists of multiple nodes each with its own voltage.

So, the point is you do not have to worry about the currents going from node 1 to node 2. So, you still have these terms corresponding to V 1 and V 2. So, V 1 will get multiplied by sum of conductance at node 1 and V 2 will also get multiplied by the sum of conductance at node 2. And there will be all the other terms. If you have conductance between node 1 and some other node, you will have other terms corresponding to that. So, this will be obvious from the example.

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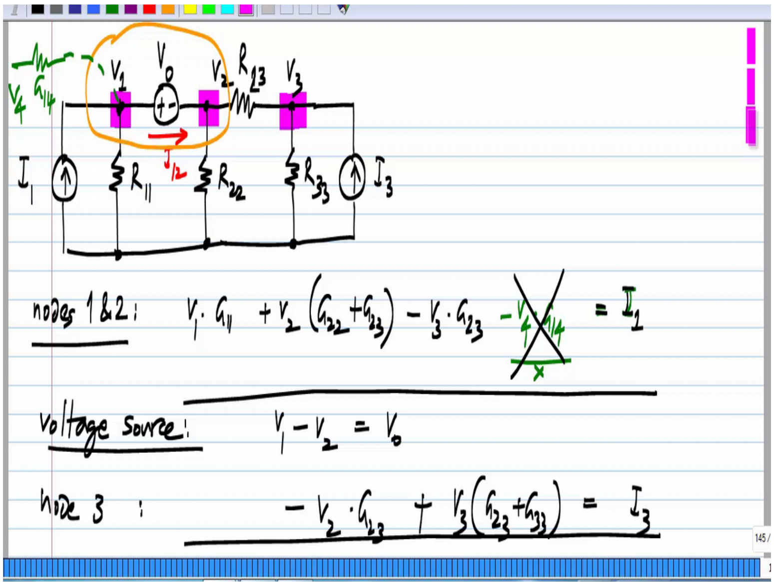

This is the super node. So, the combined equation for nodes 1 and 2 is nothing but the sum of currents leaving the super node through resistors which corresponds to V 1 times G 1 1, because at node 1, we have only single conductance; but at node 2, we have these 2 conductance G 2 2 and G 2 3, so we have V 2 times G 2 2 plus G 2 3. And you can have conductance going from any 1 of these nodes to other nodes outside the super node and those will appear with a negative sign as usual minus V 3 times G 2 3. And if we happened to have some other node with 3 four and G 1 four we would also get minus V four times G 1 4 and so on. This is not in our circuit right now, but if it was there that would also appear.

We have to look at total current from independent sources being injected into this super node. So, you have I 1 and going into the super node and nothing connected here. So, the sum simply equals I 1. So, this is not there, I just took that as an example. So, this equation, which combines the equations for node 1 and 2 can also be written down by inspection, what we have to do is, make a super node that combines the nodes between which the voltage source is connected. And on the left hand side, you write all the currents leaving the super node through the resistors; on the right hand side, you have all of the independent sources injecting currents into the super node. Of course, you always use appropriate polarities so that is how you can write this by inspection.

Of course, the voltage source equation can be easily written down by inspection. So, V 1 minus V 2 equals V naught. And for node 3, the same thing that we always used to applies, we will have V 3 time of sum of conductance which is V 2 3 plus G 3 3 minus V 2 times G 2 3, and the whole thing will be equal to I 3. So, we have our three equations and three variables and we can solve for the node voltages. These equations can also be put into matrix form, but the matrix form will not have any symmetric structure like we did earlier. In fact, the matrix form, I will continue to call at conductance matrix but not all elements will have dimensions of conductance.

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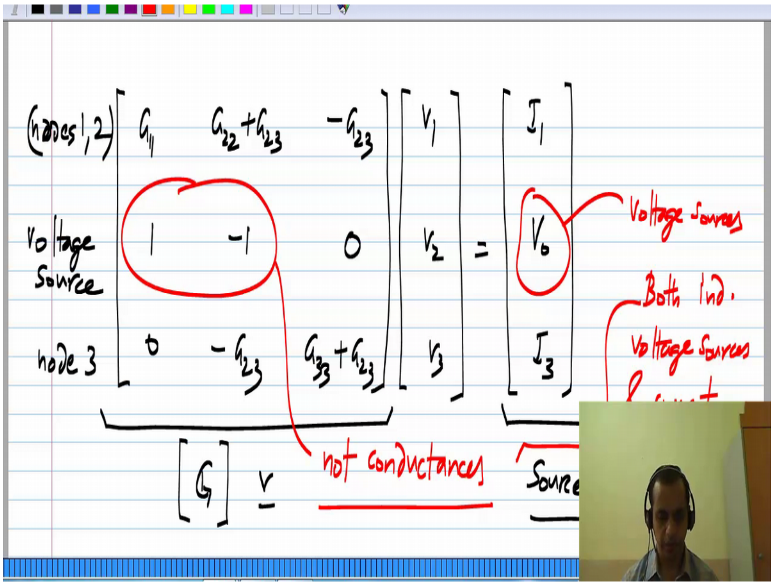

Let me show it for this example. So, let say the first row, I write it for the super node which will consists of G 1 1 G 2 2 plus G 2 3 minus G 2 3 V 1 V 2 V 3 and this will be equal to I 1. And the second equation is for the voltage source its says that V 1 minus V 2, remember this is matrix multiplication this number multiplies that this 1 multiplies that V 1 minus V 2 equals V 0 - the voltage of the voltage source. And finally we have the equation for node 3 which says that G 3 3 plus G 2 3 and minus G 2 3 will be equal to I 3. So, this is the G matrix, I will continue to call it conductance matrix, but these are not conductance, they are dimensionless quantities. And similarly the right hand side here is the source vector. So, it consists of all independent sources so that way this structure is still nice; we have some matrix times the unknown vector to be equal to the source vector; I still denoted by I, but this source vector now also has voltage sources.

So, the source vector will have both independent voltage sources and current sources is this fine? So, when you have voltage sources in this circuit, you can form super nodes consisting of the nodes between which the voltage source is connected and then go ahead with nodal analysis. So, in summary, when you have voltage sources in circuit you form a super node consisting of nodes to which the voltage source is connected, write a single equation for the super node, and you add the constraint imposed by the voltage source between particular nodes.

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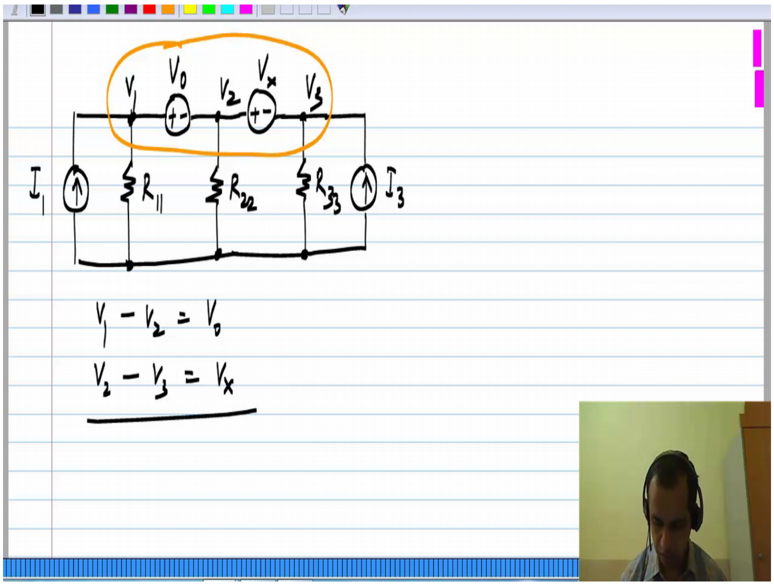

Now, let us take a couple of other examples of having voltage sources in different ways then I showed. So, let say the circuit is like this that is there are two voltage sources and they have a common node. For instance here, this voltage source is connected between node 1 and node 2; another 1 is connected between node 2 and node 3. So, what you will do in this case, clearly you have to make a super node of all these three nodes because you do not know the current through this voltage source or that one and those have to be eliminated from our equations. So, you form single super node out of three nodes. In general, it could have a multiple nodes, because if you have voltage source connected in such a way between nodes then you have to form a super node with more than two nodes and that is ok.

So, you still write KCL equations for the entire super node; on the left hand side, you will have currents through resistors coming out of the super node; and on the right hand side, you will have the total current injected into this super node I 1 plus I 3. And then you have 2 more equations. So, if this is V 1, V 2 and V 3 you know that V 1 minus V 2 is V 0, and V 2 minus V 3 is V x. So, if you combine more than two nodes, if you combine three nodes into super node you will lose two equations, but of course, you will have two constraints from voltage sources and that will take care of it.

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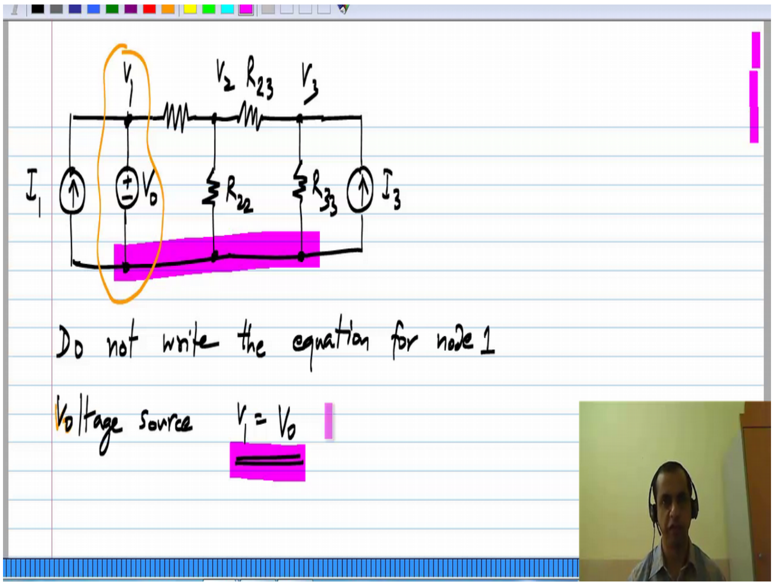

Another possibility is in circuit like this where a voltage source is connected between particular node and the reference node. If you see V 0 here this is the reference node it is connected between this and reference node. So, what you do in this case, now you simply do not write the equation for node 1. Another way to think about it is that you still form a super node and this super node consists of node 1 and the reference node. And you are not going to write an equation for the reference node any way. So, you do not write any equation for this super node, and the voltage source constraint directly gives you this node voltage; it tells you that the voltage source constraint V 1 equals V naught.

So, essentially this node voltage is already given to you; it is equal to V naught, and you do not have to solve for it. So, you do not write the equation for node 1. And you just write equations for nodes 2 and 3, so that is all that there do it. Clearly in this when you have a voltage source like this, the voltage at node 1 will be equal to V 0. So, you do not need to solve for this one, you can take it has a given and solve for the rest of it. Pr if you still want to write three equations and three variables, you would have written it for V 2 and V 3, and you have this equation V 1 equals V zero either way it is perfectly fine.