Basic Electrical Circuits
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Lecture - 13
Linearity of Elements

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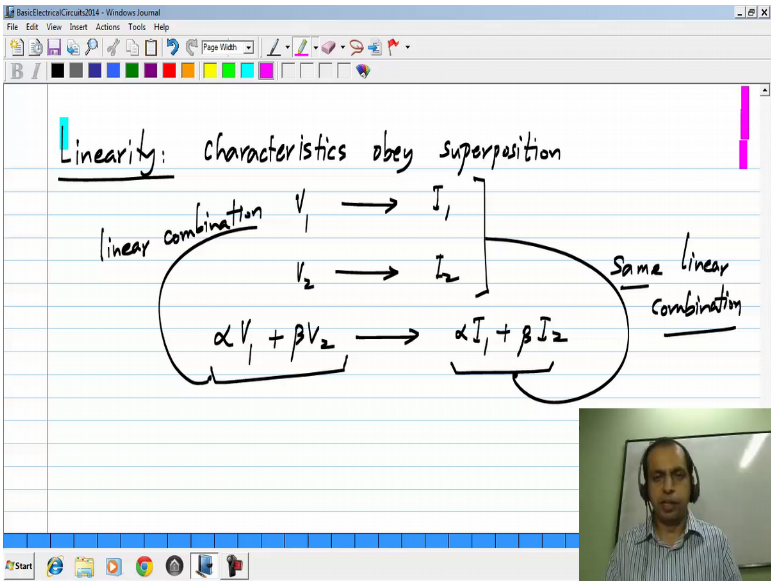

Now, I will discuss an important concept, which will revisit later in the context of circuits it is linearity. Right now, we only discussed basic elements, so we will only discuss linearity of elements, later we will expand it to linearity of entire circuits. So, the question is which of the elements is linear among the elements we have discussed so far and why. Now what does linearity means this basically means that its characteristics, characteristics of an element which is linear obey superposition. And what does this mean, so if let say you have a volt as V 1 which results in a current I 1 and voltage V 2, which result in a current I 2. What does super position mean if you apply voltage alpha V 1 plus beta V 2, which is a linear combination of these two voltages. This result in a current which is alpha I 1 plus beta I 2 that is it is the same linear combination of the effects; alpha and beta are the same.

So, this obviously means that if you double the value of V 1, you will get double the current, if you make the value V 1 half of what it was before, it gives you half the original current and so on. Now of course, I have taken voltages and current here, but this is the general property that can be applied to systems. If you have certain excitation certain responses, if the response is proportional to the excitation it is linear or in other words if excitation one gives you response one, excitation two gives you, response two then linear combination of these two excitation should give the same linear combination of the two corresponding responses. So, if this is not clear, it will be clear immediately after we discuss examples.

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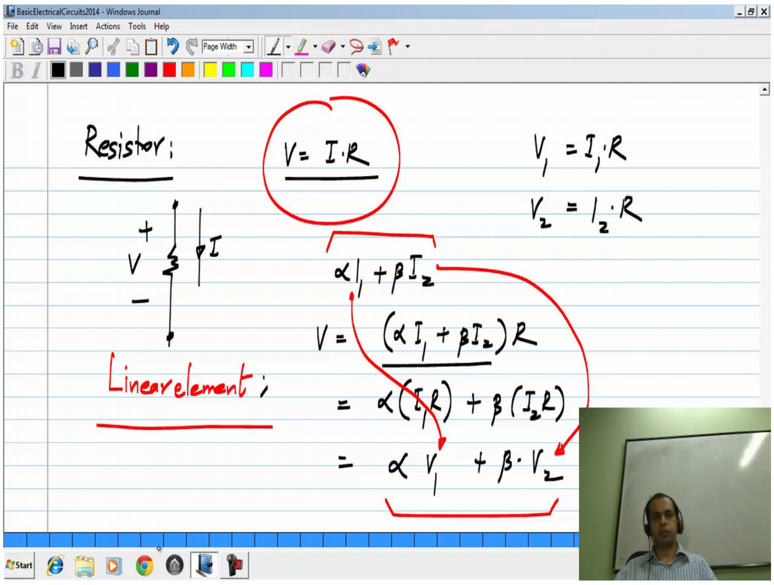

So, let us take resistor the voltage V and current I R related by a proportionality relationship. This immediately tells you, it is linear. For those of you familiar with linearity, but we will go through it in any case. So what it means is let say when the current was I 1, the voltage was V 1 obviously the resistor obeys ohms law and V 1 equals I 1 times R; and the current is I 2, the voltage is V 2 and V 2 equals I 2 times R. Now what happens if the current is alpha I 1 plus beta I 2 the voltage will be alpha I 1 plus beta I 2 times R, basically it is a current times a resistance. which of course, can be written as alpha times I 1 r plus beta times I 2 R which of course is alpha times V 1 plus beta times V 2 that is if we applied a current I 1 we would could have voltage V 1, we applied I 2, we would got V 2.

So, if we apply a linear combination alpha I 1 plus beta I 2, we will get the same linear combination responses alpha V 1 plus beta V 2. So, a resistor is very much a linear element. So, if you have a relationship that shows proportionality like this it is a linear element. And an essential feature of these is that if I 0 V is 0 that is excitation is zero the response would be zero. Now let us try this test on other elements; particularly voltage source and the current source, because they can be confusing sometimes you can apply I would definition of linearity and say that linear but they are not.

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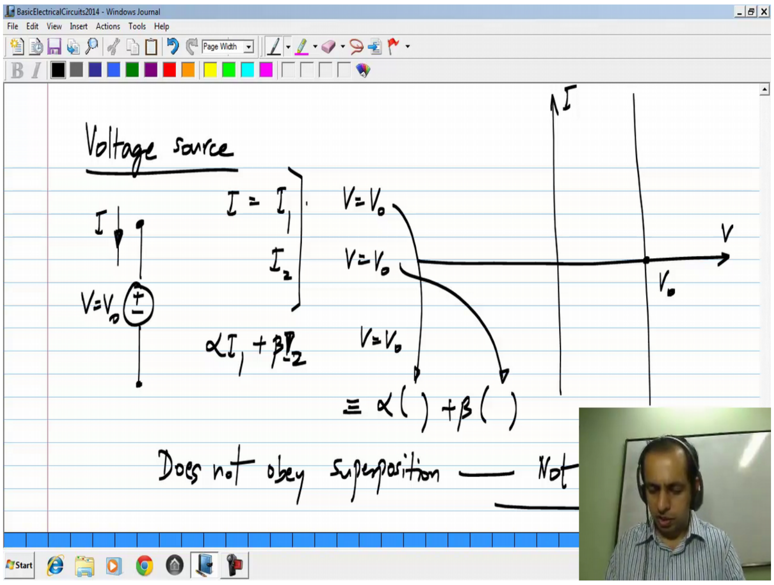

So let us look at voltage source, it maintenance a voltage V regardless of the current I that is flowing through it; and the relationship graphically is given by straight line. So, this is V naught this will be V naught. Now sometimes this confusion arises that because the characteristics consist of a straight line this is a linear element, but it is not, that we can check while trying to apply superposition. So, let say the current I is a particular value of I 1 then the voltage V will be equal to V naught because that is the property of the voltage source, it will always maintain a voltage V naught. Similarly, if the current were a different value I 2, voltage would still be V naught. And if you apply a linear combination of these alpha I 1 plus beta times I 2 the voltage would still be V naught and this is certainly not equal to the linear combination that is a alpha times this voltage plus beta times that voltage. So, it does not obey superposition which means that it is not linear. So, do not be confused by the straight line in the characteristics it is not linear, because it does not obey superposition.

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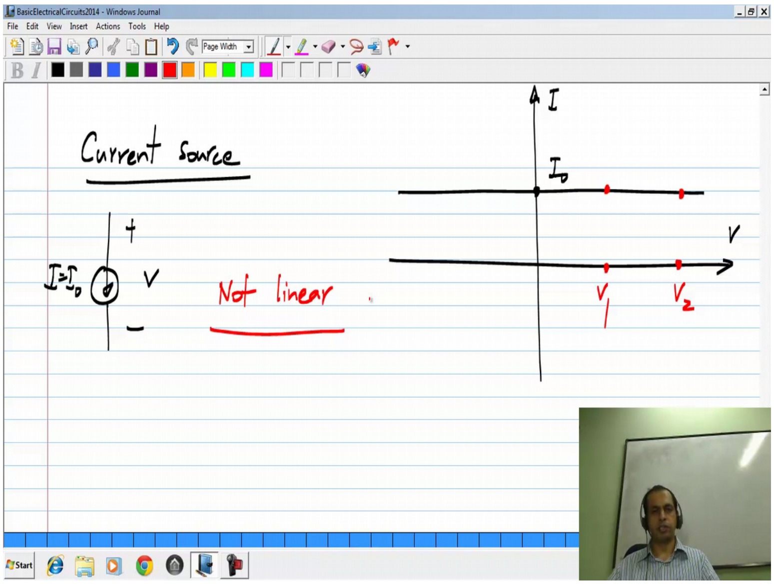

Obviously the exact same thing volts for a current source; again the I V characteristics consist of a straight line; this is I naught, but it is pretty obvious that superposition does not hold, because if you apply certain voltage V 1, you get a current I naught, you apply a certain voltage V 2 which is let say double of V 1, you will still get a current the same which is the same which is I naught, we will not get double the current. So, clearly this is also not linear. So, it is pretty obvious everybody says that resistor is a linear element and that is a correct answer, but sometimes some peoples get confused by straight line characteristics of an ideal voltage source or an ideal current source they are not linear, because they do not obey superposition. Now there will be linear only if the value of the current equals 0 that is not a very interesting case. So, we would not consider that that is if the current source is zero valued current source, or a if the voltage source is a zero valued voltage source those would be linear elements.

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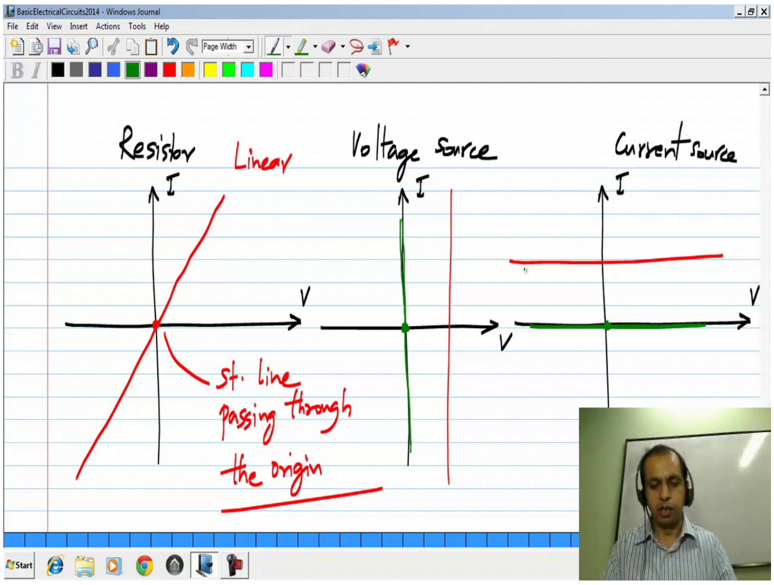

Now what distinguishes the characteristics of a resistor, a current source and voltage source, all of which as straight lines. Let me write here, resistor has a characteristic which is like this; voltage source has this characteristics parallel to the y axis; current source has characteristics parallel to the x axis. All these three are to the straight lines, but only this one is linear that you can guess from the characteristics also, it is not enough for it to be a straight line, but it has to be a straight line passing through the origin, it passes to through origin its linear characteristics; if it is not, it is not, it would not superposition, so that is why also said if the voltage source happens to a zero valued, the characteristics would be this, which passes through the origin; and similarly if the current sources zero valued characteristics would be this which passes through the origin, so otherwise they are not linear.

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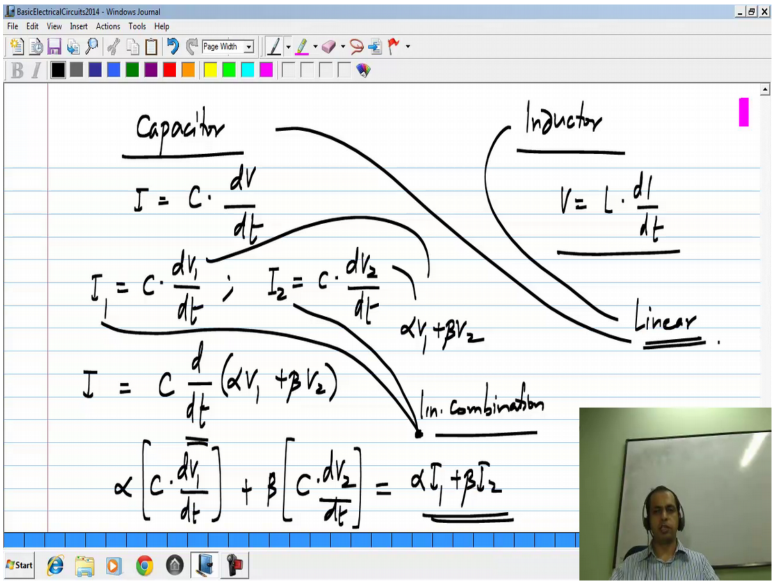

Let us quickly look at the other two elements the inductor and the capacitor. The capacitor I is C dV dt. And if you have certain current I 1, because of particular voltage V 1 which is varying with timing in some way; and another current because of another voltage way form V 2, these are all functions of time, all though, I am not showing them explicitly to be so. Then if you apply linear combination of these voltages, so let say you make linear combination of these as alpha V 1 plus beta V 2, you will get alpha V 1 plus beta V 2, this is the current which because of the linearity of the time derivative. You can pull out as C times alpha times C dV 1 dt plus beta times C dV 2 dt which of course, is alpha times I 1 plus beta times I 2. So, it is same linear combination of I 1 and I 2. So, basically the time derivative linear characteristics, so capacity is the linear element; similarly inductor which obeys relationship V is L dI by dt is also linear. I will leave it as an exercise for you it prove it, it must be again pretty obvious from the relationship. So, both of these are linear elements, we will see later, see linearity of whole circuits, and how super position applies to them, and it is not just a property; it is something which can simplify the solutions of circuits.