Solid State Devices
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Lecture - 17
Procedure for Device Analysis

In this lecture we will see the Procedure for Device Analysis.

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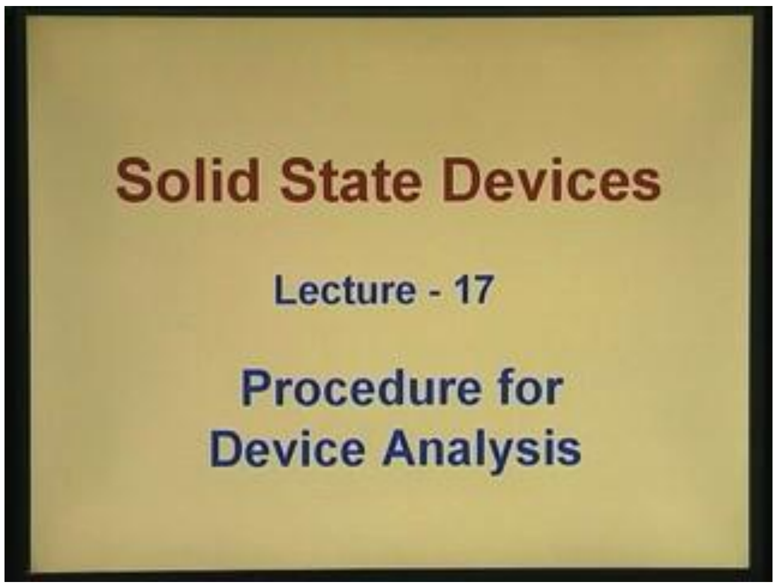

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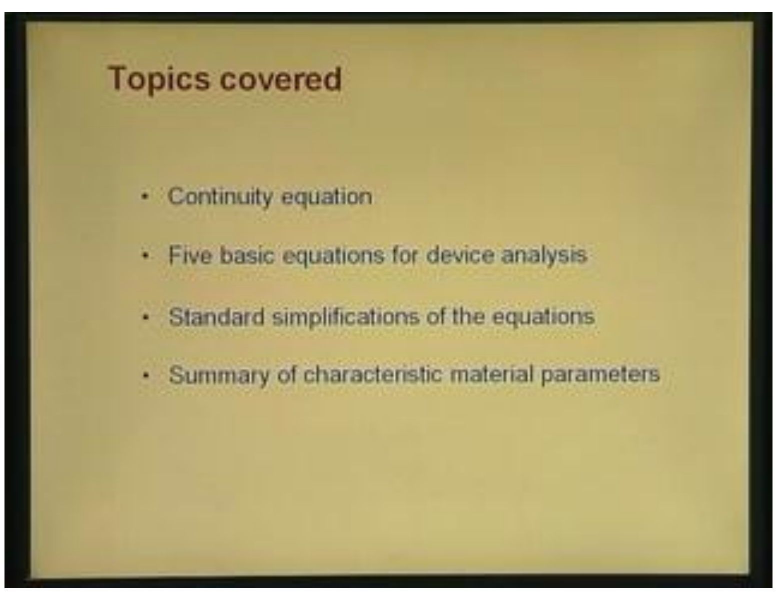

The topics we will be covering in this lecture are;
The continuity equation which describes the distribution of electrons and holes when there is excess carrier generation recombination and carrier movement, then we will see the five basic equations for Device Analysis. We will consider some of the simplifications which are used for deriving the characteristics, how the equations can be simplified. And finally we will summarize the characteristic material parameters which describe a semiconductor device. So let us begin with the continuity equation.

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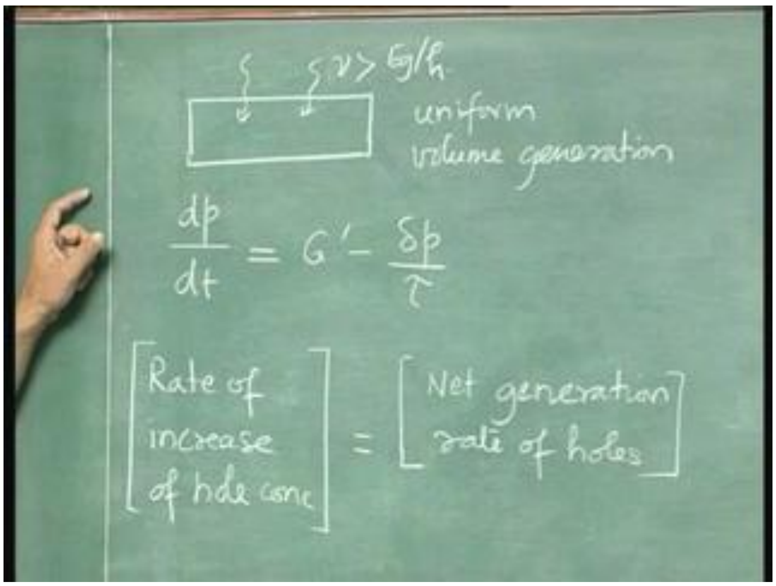

In the last lecture, we had shown that if there is uniform volume generation within a sample due to optical generation e.g. then the rate of increase of the carrier concentration can be described by this equation which says that rate of increase of concentration is equal to net generation rate of holes. Here we are showing the equation for holes and similar equation will be there for electrons. We had emphasized while giving this equation that since there is uniform volume generation there is no movement of carriers within the volume from one point to another point. However, in general this is not true as there will always be some movement of carriers. The situation will not be uniform throughout the device. So we need to have an equation which will describe the distribution of electrons and holes in a general situation. Now this is what we are doing in continuity equation. So the general situation is depicted here.

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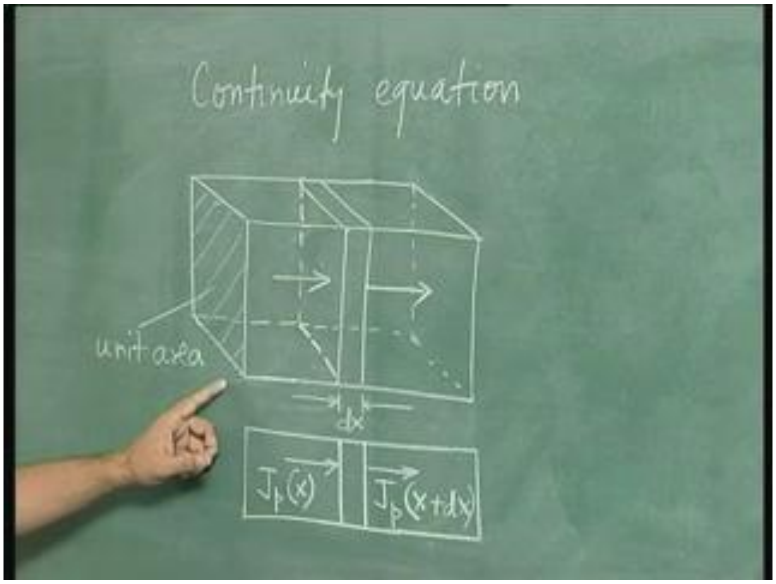

We are assuming a one dimensional case and we are considering holes. So you have holes moving from left to right in this particular semi-conductor block. We will assume that the area of the restriction of the block is unity. The current density J_p is different at this phase of a differential volume that we are considering and the other phase of the differential volume. So the two current densities are J_p(x) and J_p(x plus dx) which means the current entering the volume is not necessarily equal to the current that is leaving this differential volume which is the volume of our observation. Now, we need to write an equation describing the rate of increase of hole concentration within the volume.

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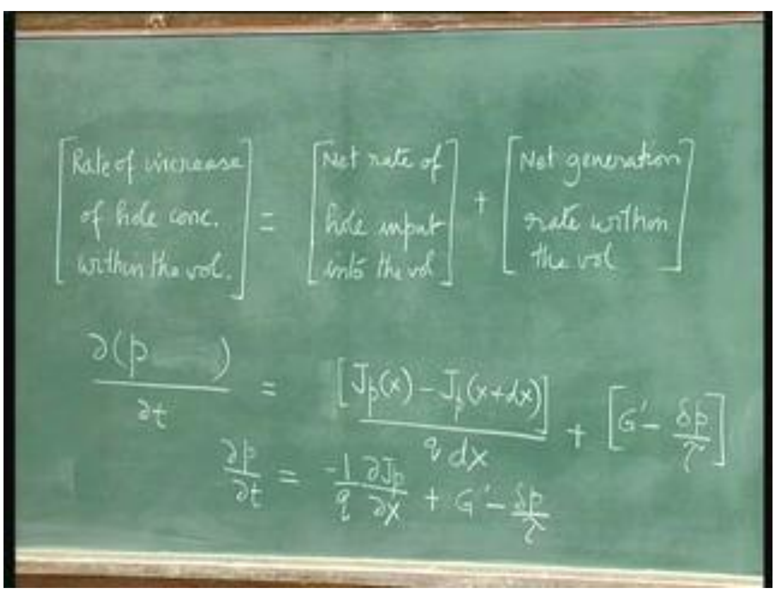

This can be written as follows; rate of increase of hole concentration within the volume can be written as the net rate of input of holes as described by the difference between these two currents $J_p(x)$ and J_p(x plus dx) plus any net generation within the volume. So rate of hole input into the volume is understood. This is the additional term that is coming in here as compared to the situation we described here where there was no movement of the carriers.

Similarly, we can translate this particular statement into an equation as follows: The rate of increase of whole concentration within the volume can be written as p into d_x into 1 p is the concentration of holes, d_x is the width of this volume and 1 is the unit area of cross sections so it is the differential of this with respect to time. We are using a partial derivative since p can vary with x as well as t. So this is equal to net rate of hole input into the volume. So the input current density minus the output current density divided by q since the current consist of the charge also whereas we are the talking of only the concentration of the holes and not the charged concentration.

We shall multiply this by the area of cross section to get the current where area of cross section is unity. So this is the net rate of hole input into the volume. Please check that this term and this term have the same dimensions p by (cm cube into d_x (cm), 1 is cm square so this is simply concentration into volume which is just the number; so it is 1 by time that is the dimension of this. Here you can see current density is ampere per cm square, multiplied by 1 cm square so it is amperes by Coulomb so again this is per second. So dimensionally these two terms are identical plus the net generation within the volume can be written as G prime is the excess generation rate, this is the recombination rate and we will be assuming low injection level so this is the net generation rate multiplied by the volume which is (d_x into 1) so this is the net generation rate within the volume. Now we can simplify this equation by dividing throughout by d_x into 1.

If you do that what will happen is this term will become doe p by doe t. This term when you divide by $d_{x,}$ you find that, here I remove this d_x into 1 and this d_x comes here, this one unit area of cross section goes off, this d_x into 1 will vanish when you divide so this is the doe p by doe t term. This term can be written as (minus 1 by q) into (doe J_p by doe x) as d_x tends to 0. You are getting a minus sign because when you write the derivative, the convention is (J_{px} plus d_x) minus (J_{px} by d_x) that is the derivative. So here since these terms are reversed you get a minus sign plus this G prime minus delta p by tau. This is the so called continuity equation for the holes. It describes the distribution of holes as when there is movement of holes or there is a hole current you will have excess generation and as well as recombination.

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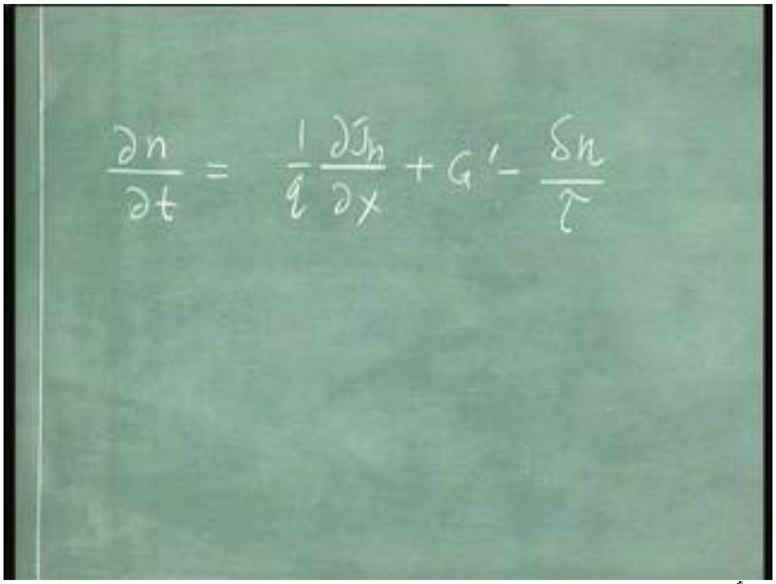

We can write a similar equation for electrons and that would be $\frac{\partial n}{\partial t} = \frac{\frac{1}{q}\partial Jn}{\partial x} + G' - \frac{\delta n}{\tau}$ tau where tau is the lifetime of minority carriers. Note that this minus sign has been removed as compared to this equation. Here you had a minus sign and that minus sign is gone there because if you have an electron current here if electrons are flowing into the volume then the current is from right to left so the current is in the opposite direction to the flow of electrons. That is why this negative sign is gone when you replace J_p by J_n or when you write the equation for electrons. Now we can combine this continuity equation with the other equations we derived in the earlier class; the transport equations. And then the basic equations would look something like this.

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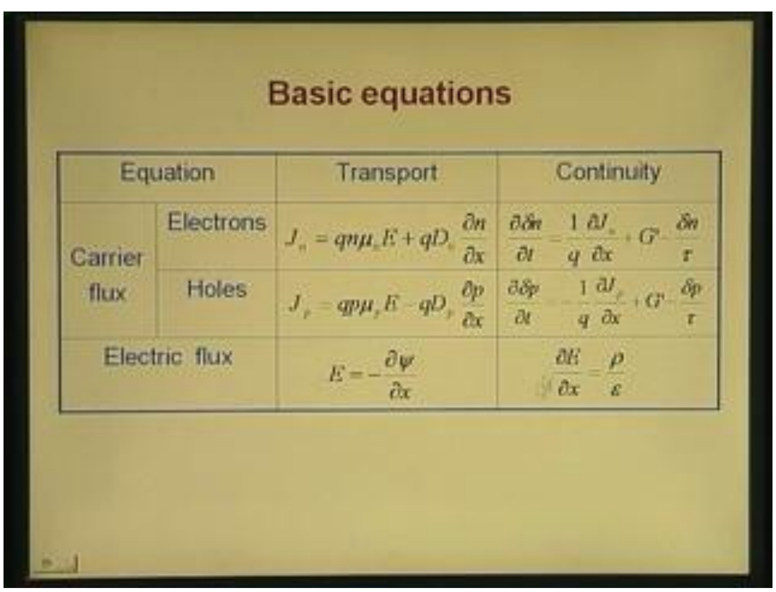

So you have the transport equation for electrons which combines the drift and diffusion. Then during the flow you have the continuity equation describing the distribution of electrons when there is generation, recombination and movement. Now note here that we have used $delta_n$ instead of n. This is to have only one type of variable in the equation. Since we have $delta_n$ here we have put $delta_n$ here instead of n so note that it makes no difference since $delta_n$ is nothing but n minus n_0 where n_0 is an equilibrium concentration. So when I differentiate this with respect to time the differential of n_0 with respect to time is 0 so these two are identical and we can replace this by this as it has been done here.

Similarly, you have the equation for holes, the transport equation, drift and diffusion and then you have a continuity equation we derived just now. This completes the picture for equations associated with carrier flux. These are the equations which show why the flux is caused and these are the equations which show that the particles are conserved during flow. The continuity equation is nothing but a conservation law. Now in addition we have the electric flux.

In fact the electric flux is causing the drift current here e so you have equations associated with this flux also. The transport like equation, this is not called transport equation in practice, people do not call this as transport equation but here we have listed it under transport equation because it is transport-like equation. This equation shows that the electric field is caused by gradient of potential just as this equation shows that the electric current or carrier flux is caused by the gradient of potential or electric field. Electric field itself is caused by gradient of potential.

Now, since electric flux can be regarded as a flow of electric lines you have a continuity equation associated with this called the Gauss's law, which shows the increase in the electric flux due to the space charge. So this is conservation of electric flux during flow just like the conservation equations for carrier flux. This is Gauss's law, we have come across earlier in Physics so we are not describing it in great detail here. We will provide a brief explanation of the Gauss's law.

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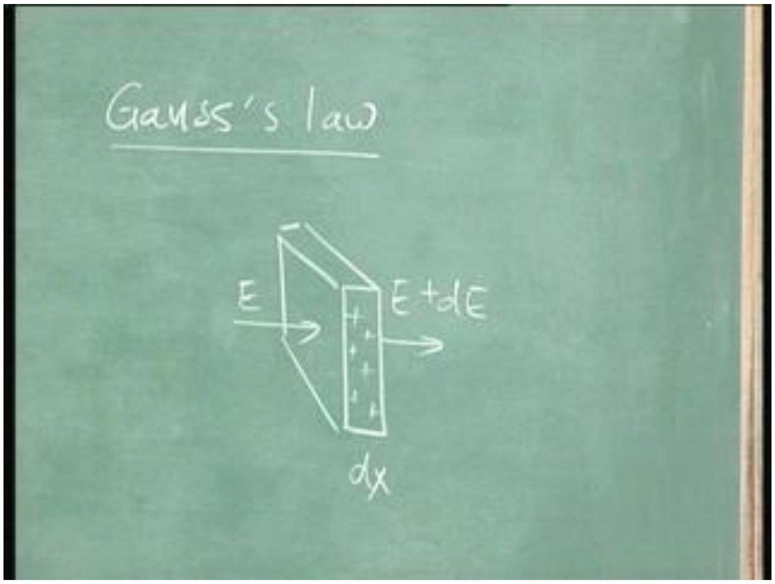

Here what you find is the one dimensional situation where electric field is going into this volume and coming out. Inside the volume you have positive space charge. The electric field that is coming out has more lines then the number of lines entering this particular volume because every positive charge is a source of a field line so that is why the electric field increases because of the volume space charge. We can write the increase as follows: The increase de is due to the space charge rho in the volume. Now the total charge in the volume would be rho into d_x into 1, we assume the unit area of cross-section. Now the total flux into this area can be written as E into d into the area because the flux is defined in terms of displacement vector d which is E_e so differential of this displacement vector is E de and then we can equate these two. Now when we rearrange we cancel this one and bring this d_x here and this E here and you get the Gauss's law. So that is the Gauss's law equation as shown here.

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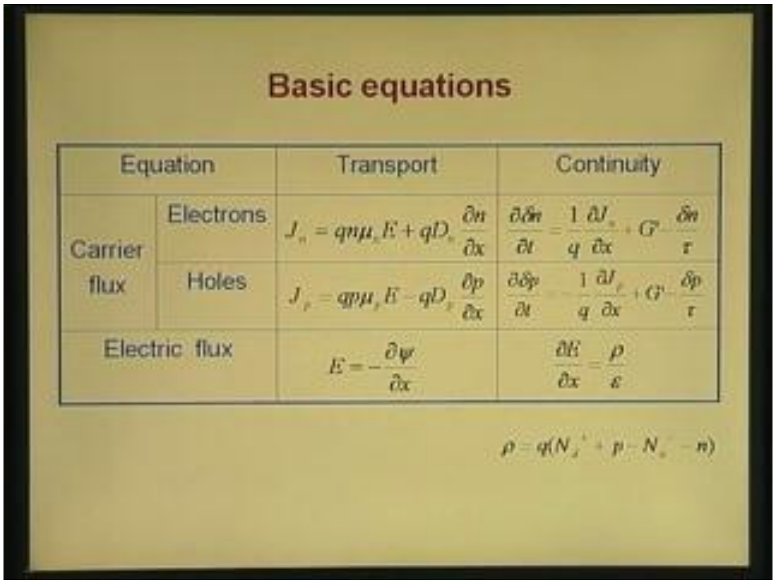

Now recall that we have already said that the space charge within the semiconductor rho is given by [q(nd) plus p minus (minus na) minus n] where nd plus is the ionized donor concentration and na minus is the ionized acceptor concentration. So sum of the difference between the positive charges and negative charges is the space charge. Now these equations have a close correspondence between the equations for carrier flux and that of the electric flux if you rewrite the equations as follows. The first step we can write this e in terms of the gradient of potential to see one to one correspondence between these two equations and this equation.

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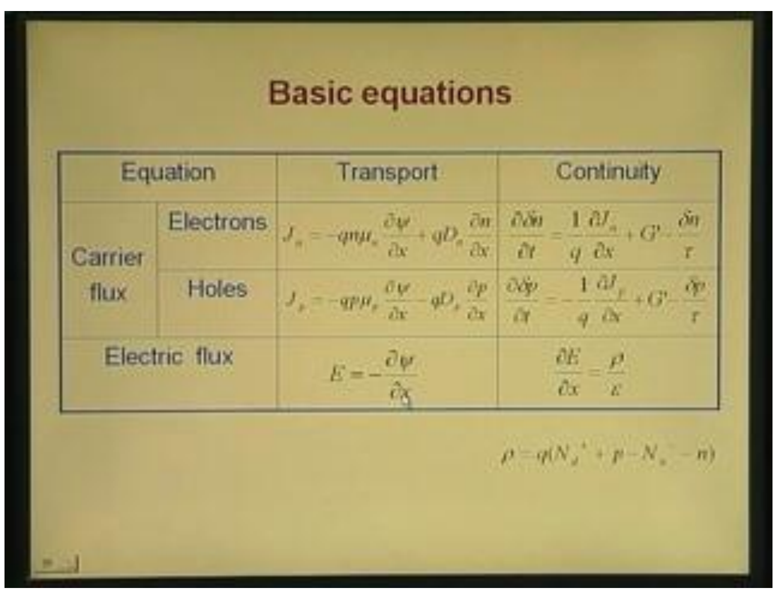

You can see here e is replaced by minus doe psi by doe x where psi is the potential. So here the current is because of concentration gradient and potential gradient while electric flux is because of potential gradient. We can achieve even better correspondence among this set of equations for carrier flux and the electric flux by rewriting the continuity equations by putting this particular term on the left hand side and the delta_n by doe t term on the right hand side. So when we do that, this is how the continuity equations look like.

And now we can see J_n is the flux and this transport equation shows the cause of the flux, continuity equation shows the gradient of this particular flux which is the cause of this gradient. So here you have the cause of the flux and here you have the cause of the gradient of the flux. So using the continuity equation you can find why there is a gradient in the flux. So now you can see the one to one correspondence e is the flux and doe e by doe x is the gradient of the flux for the electric fields.

Similarly, these two are the fluxes for carriers and these two are the gradients of the fluxes. So in this form the close correspondence between the six equations is seen very easily and therefore it is very easy to remember these equations. Now however what you find is all these six equations are not independent. In fact there are only five variables in these equations which are coupled that is the concentration n of electrons, concentration p of holes, J_n the flux of electrons and J_p the flux of holes and the electric field which again depends on the concentration of holes and electrons. So n, p, J_n, J_p and e are the five variables which are independent and psi can easily be obtained once e is obtained. And that being the case only the five equations have coupled so e is equal to minus doe psi by doe x has been removed and basically you only have these five equations to be solved simultaneously. Once these equations are solved you can always get the potential.

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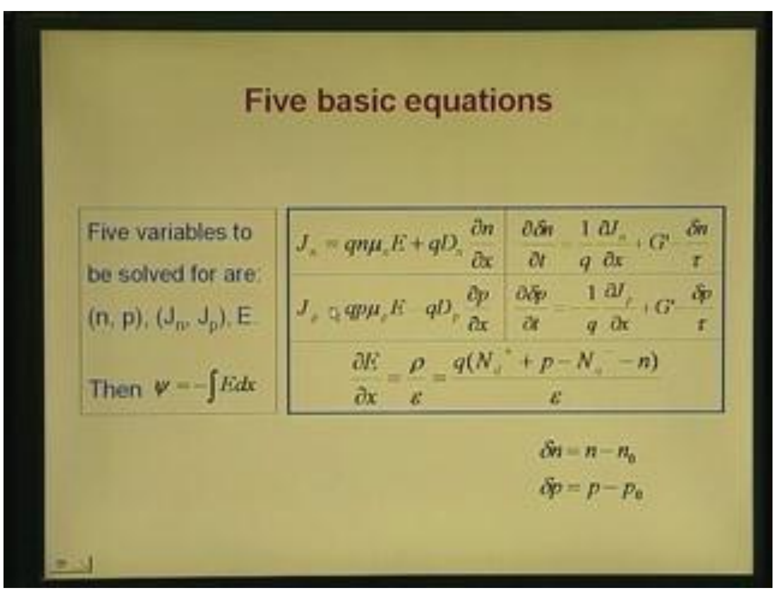

So this is what is shown here in a summary. You have the five basic equations j n transport equation, J_p transport equation, J_n continuity equation rewritten in the form in which it was derived and J_p continuity equation and then the continuity equation for the electric flux or Gauss's law. And so you have five variables to be solved for are that is n, p, J_n, J_p and e. Once you have solved these variables simultaneously using these equations then you can get ψ from electric field using this particular formula. Now these equations look fairly complex so we need to discuss how they can be simplified in different physical situations. Before we take that up it would be useful for us to see how these transport equations on the slide i.e. equations for J_n and J_p can be represented in a compact form using the Quasi Fermi-levels.

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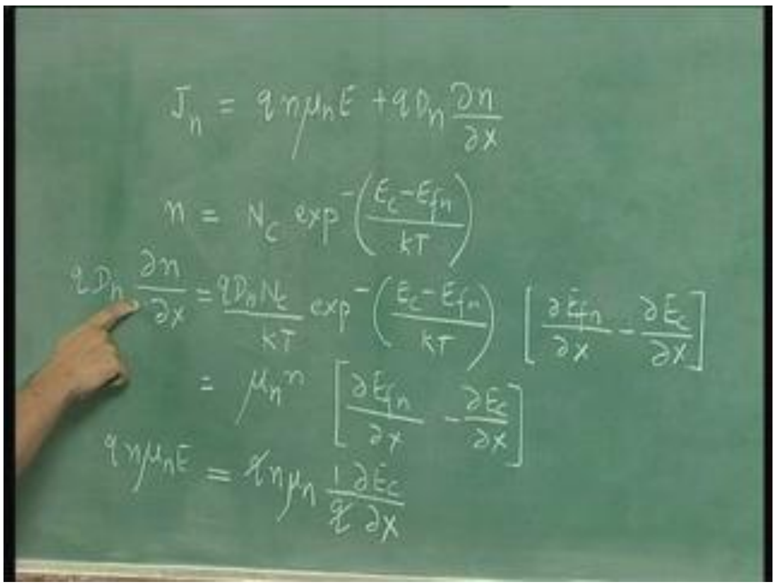

For example, let us take the equation for the electrons J_n is equal to qnmu$_n$E plus qDn ϱn by ϱx. How do I represent the right hand side using Quasi Fermi-level in a compact form. We start with the equation for n. Using the Quasi Fermi-level concept, we can write N is equal to Nc exp minus$\left(\frac{Ec-Efn}{KT}\right)$ this is the Quasi Fermi-level for electrons. Now let us perform this operation doe n by doe x and see what we get. So $\frac{\varrho n}{\varrho x}=\frac{Nc}{KT}\exp-\left(\frac{Ec-Efn}{KT}\right)$ multiplied by the differential of this with respect to x and that is $\left[\frac{\varrho Efn}{\varrho x}-\frac{\varrho Ec}{\varrho x}\right]$ so 1 by KT has been taken out already in this differential. Nc into exponential E_c - E_{fn} by KT is nothing but this n itself so this turns out to be nothing but$\frac{n}{KT}\left[\frac{\varrho Efn}{\varrho x}-\frac{\varrho Ec}{\varrho x}\right]$.

Now, if you multiply this doe n by doe x by q into dn to get this particular term we will multiply this by q into dn so here you will multiply by q into dn and here we will multiply by q into dn. Now what you find is this q can be shifted below and you can write this as KT by q this is nothing but the thermal voltage vt and dn by vt is nothing but mu$_n$

by the Einstein relation. So one can replace this term dn by kt by q by mu$_n$ and you get this as the expression for the diffusion current qdn doe n by doe x.

Now one can add this term to the drift term qnmu$_n$e where e should be written in terms of the conduction band energy. So we can write qnmu$_n$ into e the drift current can be written as $qn\mu n \frac{1}{q}\frac{\partial ec}{\partial x}$ we have shown this earlier that E_c by q is the potential energy corresponding to the electrons in the conduction band and gradient of this potential is the electric field. The negative sign is not there because ec is the electronic potential energy. This q gets cancelled and we find that this drift current is nmu$_n$ into doe ec by doe x. Now when you add up this and this to get this you find that nmu$_n$ doe ec by doe x will get cancelled here because this has the negative sign here and what will be left is only mu$_n$n doe E_{fn} by doe x.

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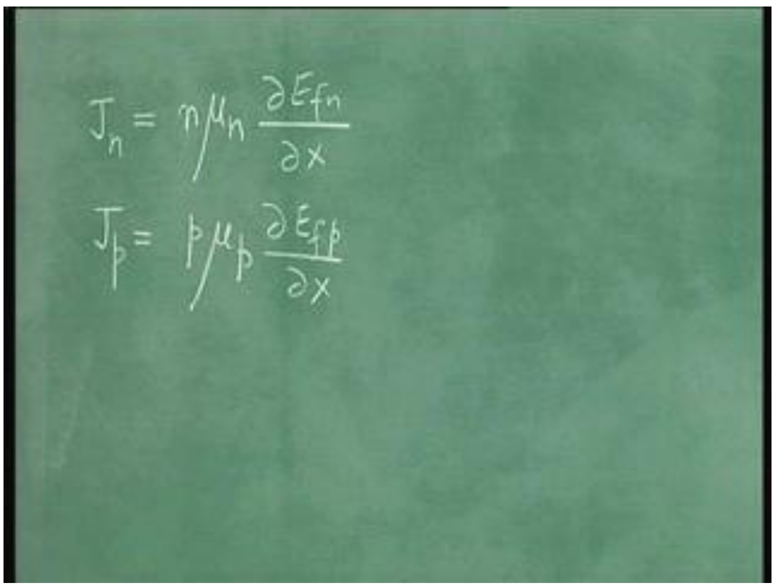

So let us write that here; you get the equation J_n is equal to nmu$_n$ $\frac{\partial Efn}{\partial x}$ that is the compact expression for the electron current in terms of the Quasi Fermi-level. Now you can understand why the Quasi Fermi-level is a powerful idea. So, the gradient of the Quasi Fermi-level contains the information about the drift as well as diffusion and that is what it shows. One can similarly show that J_p is equal to pmu$_p$ doe E_{fp} by doe x i.e. the hole current depends on the gradient of Quasi Fermi-level for holes. Now these are the compact forms of the transport equations in terms of Quasi Fermi-level. This will be useful for us as we will see subsequently. Now we will take up various physical situations in which we can show how these equations can be simplified.

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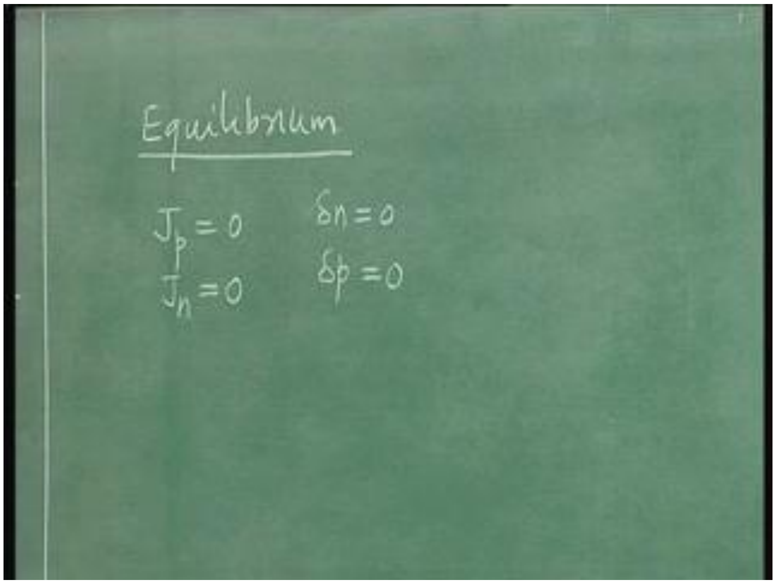

Let us take the condition of equilibrium. Under equilibrium J_p is equal to 0 and J_n is equal to 0 because according to the equilibrium state for every process there is an inverse process going on at the same rate. There cannot be any currents either for holes or for electrons. Further there cannot be excess carriers so delta_n is equal to o and delta_p is equal to 0.

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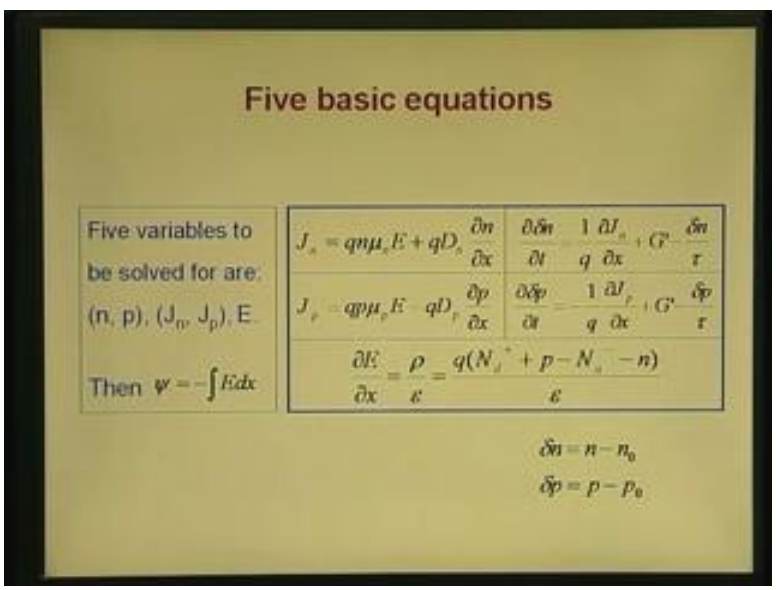

Now as a result what happens is that the continuity equations that you see here in the slide becomes trivial because delta_n is equal to 0 so this term goes off and this term also goes off, there is an excess generation so this term goes off so all the terms become 0 so this is trivial. But the transport equations may be non trivial because all that we are saying is J_n is equal to 0 i.e. left hand side is 0 in these two cases which means that there may be right hand side terms but they are canceling each other. So individually these two terms are 0, these two terms here are also 0 or these two terms and these two terms are canceling each other. So drift and diffusion are in balance if there is an electric field and concentration gradient.

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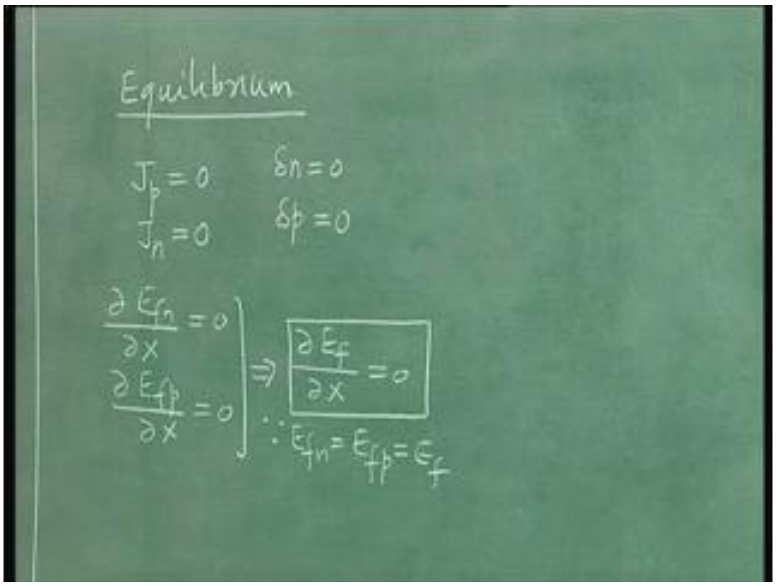

If you try to interpret this particular situation J_p is equal to 0 and J_n is equal to 0 in terms of the Quasi Fermi-levels then we get an important information that is the gradient of the Quasi Fermi-levels is 0. So doe E_{fn} by doe x is equal to 0, doe E_{fp} by doe x is equal to 0. This means E_{fn} and E_{fp} are constant with x. Now under equilibrium we know E_{fn} is equal to E_{fp} is equal to E_f so these imply that the gradient of the Fermi-level is 0 since E_{fn} is equal to E_{fp} is in equilibrium. As we will see this is an important starting point for drawing energy band diagrams under equilibrium for non uniformly doped semiconductors that E_f is constant with x. Incidentally here E_{fn} is equal to E_{fp} is equal to E_f. Now let us consider another situation and that is if the drift and diffusion currents which are exactly imbalance under equilibrium are individually very large but they are slightly in imbalance.

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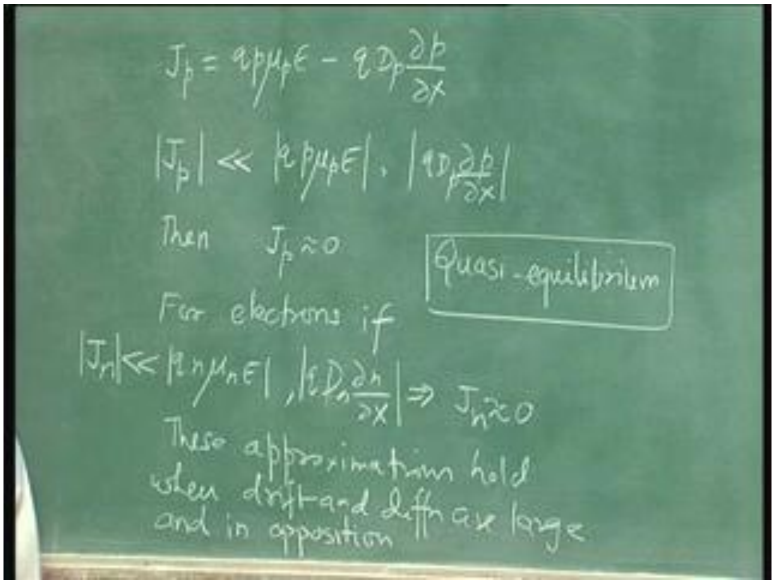

This means for example if I write for holes it is like this (J_p is equal to $qpmu_pE$ minus qD_p doe p by doe x). So imagine a situation where these two quantities are large but their difference i.e. J_p is very small. So if this is 1000 units and this is 995 units then 1000 minus 995 is equal to 5 so the left hand side is 5 units where 5 is very less than either 1000 or 995. Hence we are talking of the situation where J_p (the magnitude is less than $qpmu_p$ into e, the magnitude of this and the magnitude of q. I am concerned with the magnitudes because the signs have to be taken care of. So we are only interested in the magnitude. If this is satisfied then we can write J_p is equal to 0. So whether we say this is 1000 and this is 1000 in which case this is exactly 0 or in the other case this 1000 is 995 and in this also the difference is almost 0 compared to the individual magnitudes.

In our example it is 5 is equal to 1000 minus 995 that is what we are getting and we are saying that this 5 can be neglected and made is equal to 0. So we are writing this as this, the difference is almost 0. Please note that unless these two quantities are very large we cannot neglect the net result coming on the left hand side. So in some situations these kinds of approximations may enable us to find out these two quantities. So this kind of situation is called Quasi Equilibrium. You can have a similar situation for electrons.

For electrons again if $qnmu_n$e and qdn doe n by doe x and if the magnitudes of these two are much greater than J_n magnitude then we can write J_n is equal to 0. So these are small disturbances from this equilibrium J_p is equal to o and J_n is equal to 0. This is valid when drift and diffusion are large compared to the net current. So, these approximations hold when drift and diffusion are large and are in opposition and this situation is called Quasi Equilibrium because they represent a small disturbance from equilibrium. So this approximate equal to sign is interpreted as Quasi Equilibrium. So this is another kind of situation which can come about in devices. An example of the situation where the drift and diffusion are large and in opposition is near the PN junction. You have a PN junction and near the junction there is a region in which you find charges and electric fields giving rise to large drift currents but also there are gradients of concentrations giving rise to diffusion current. So under equilibrium in PN junction or even for small voltages applied to the PN junction you will find this kind of a situation.

We will discuss the PN junction in detail. I am only giving you an example of a situation where this approximation can be applied. Now let us interpret this J_p is equal to 0, J_n is equal to 0 in terms of the Quasi Fermi-levels. So if you interpret in terms of these you will get instead of this an equal to sign and approximately equal to sign. So under this kind of Quasi Equilibrium condition, you will have doe fn by doe x is equal to 0 and doe fp by doe x is equal to 0 which means for the Quasi Fermi-levels there will not be any gradients or the gradient will be very small. So E_{fn} and E_{fp} will be constant with x.

However, please note since this is not an equilibrium state $E_{fp} \neq E_{fn}$. So in terms of the Quasi Fermi-level this Quasi Equilibrium condition can be interpreted using this. This is also a very useful piece of information where drift and diffusion currents are very large and they are in opposition then if you were to draw the Quasi Fermi-levels in such a region they will be constant with x although they will not be identical so you will have to draw two parallel lines. We will see this when we analyze the PN junction. Now let us discuss another example of a situation where simplification is possible of the equations i.e. we can simplify the equations that is the steady state.

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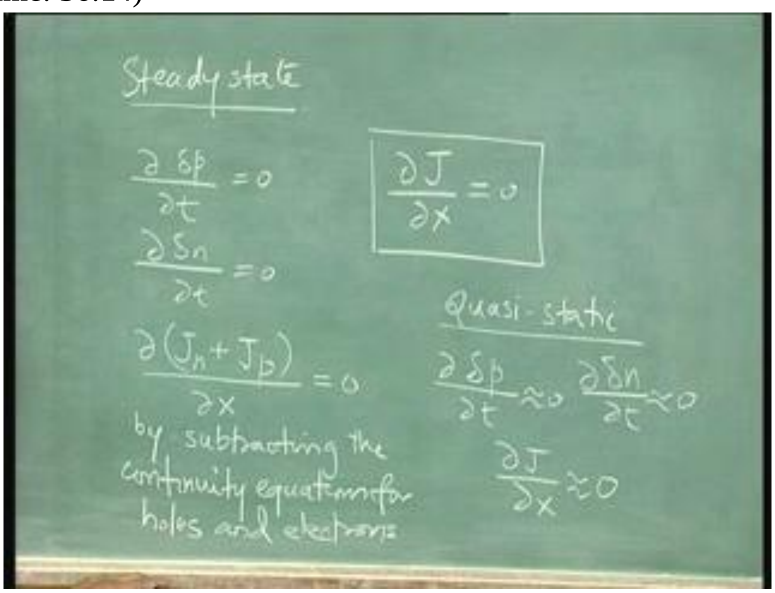

In steady state basically doe delta_p by doe t is equal to 0 and doe delta_n by doe t is equal to 0. This is the definition of steady state that is all processes are constant with time so there cannot be any quantity that is changing with time so derivative of quantities with respect to time will be 0. This means in the continuity equation shown in this slide the left hand side terms are 0.

Now what about the right hand side terms? They can exist but they will all add up to 0 and that is the meaning of the steady state. And important result for the steady state is obtained by taking the difference of these two equations. So because left hand sides are 0 when you take the difference what will happen is, since G prime for electrons is same as the G prime for holes because excess electrons and holes are generated in pairs, and similarly excess electron concentration is equal to excess hole concentration.

When you take the difference of these two equations the left hand side is 0 and in the right hand side these two terms will cancel leaving behind a gradient of the sum of J_n and J_p so you get doe J_n plus J_p by doe x is equal to 0. This is obtained by adding continuity equations or by subtracting the continuity equations for holes and electrons. So this is a very important result which means that if you write J_n plus J_p as the current flowing through the device the total current because sum of the electron and the hole currents we get the important result doe j by doe x is equal to 0.

So, in a one dimensional situation the current will be constant with x even though individually, the electron and hole currents may vary with x but their sum is constant with x. This is a very important relation in steady state which will help us in analyzing the devices. Now, like there is a small disturbance of the equilibrium state the Quasi Equilibrium you can have a small disturbance of this steady state that is called the Quasi steady state or quasi static state.

Quasi static or Quasi steady state would be described by equations namely this is approximately equal to 0 and these two are also approximately equal to 0 so now the meaning of these approximately equal to 0 should again be understood because in the continuity equation you find that these terms are obtained by summing up several terms.

Therefore when we are saying that this term is approximately equal to 0 it means that right hand side terms can be grouped together into one large positive term and another large negative term. The magnitudes of the positive and negative terms are very large compared to the difference between them which is giving rise to the doe delta_p by doe t and doe delta_n by doe t terms. This is what we had explained even in the case of Quasi Equilibrium i.e. the individual terms are much larger than the difference.

Now many transient situations can be analyzed using this Quasi static approximation. This enables you to use the picture obtained under steady state conditions even for analyzing the transient conditions. Next we will discuss another physical situation that we encounter in analysis of devices. In some regions of the device the diffusion current for a particular type of carrier i.e. either for electron or for the hole, dominates over the drift current. So this is called the diffusion approximation.

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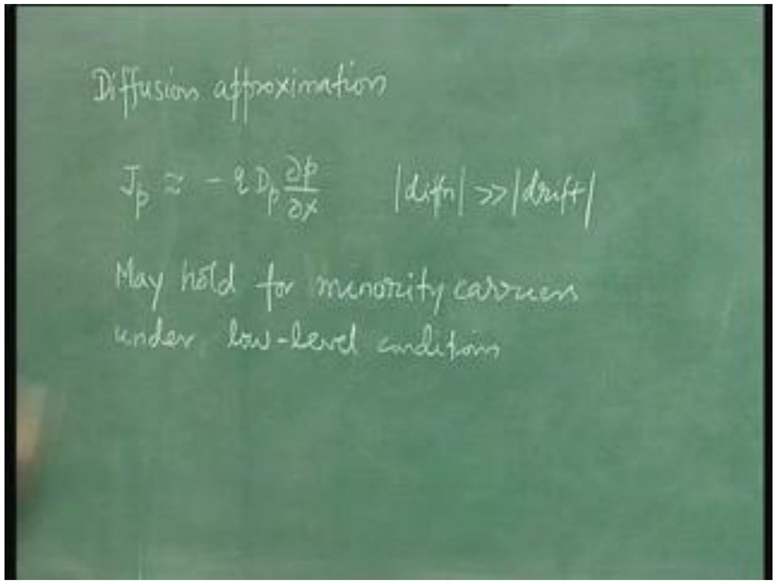

The diffusion current dominating over drift: Let us consider the example of holes. If diffusion current dominates over drift we can neglect the drift and we can write J_p is equal to minus qD_p doe p by duo x. So we will write here that diffusion is much greater than drift. We are taking the magnitudes so that the sign confusion is not there. Now an example of a situation where this kind of a thing is valid is when low injection level prevails in a region of the device we find that the minority carriers can be described by this particular approximation.

For example, this kind of approximation will hold for holes in an n-type semiconductor. This is because the drift current depends on the concentration of carriers and the electric field whereas the diffusion current depends only on the gradient of the concentration of carriers. So, if the concentration of carriers is small then the drift current tends to be small. So you can have a situation where the gradient of the concentration of carriers is large but the concentration itself is small.

This kind of approximation that drifts are very small compared to diffusion may not hold for majority carriers because the concentration of majority carriers is very high so the drift current for these carriers are high. So this may hold for holes in n-type semi-conductor under low level conditions. We can generalize and we can replace holes by minority carriers and we can say it may hold for minority carriers under low level conditions. This is a specific example of an n-type semi-conductor.

Now let us see what are the consequences of this?
If the transport equation can be simplified for holes in this form then what does it imply on the distribution of the holes or excess holes as the function of distance and time. Now let us look at the continuity equation for holes given here on the slide. Now we can simplify the term doe J_p by doe x term using the diffusion approximation and we will get doe delta_p by doe t which can be written as doe J_p by doe x.

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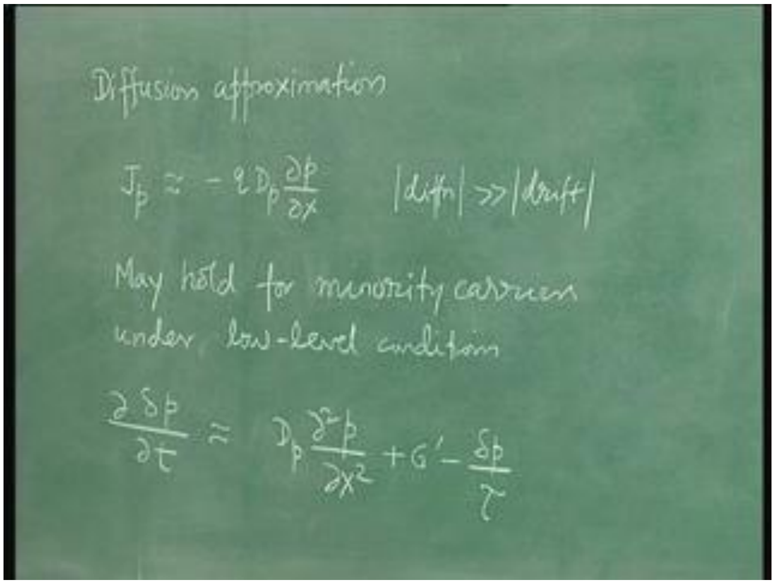

We have to differentiate this so D_p doe square p by doe x square and the q is cancelled and you have a negative sign here. Actually this negative sign will also get cancelled because there is a negative sign in the expression for J_p and there is a term negative sign here in the equation for the continuity against J_p so the negative sign gets cancelled and this sign will not be there so it is plus g minus (delta$_P$) by tau. Now further let us assume steady state condition in which case this term will drop out.

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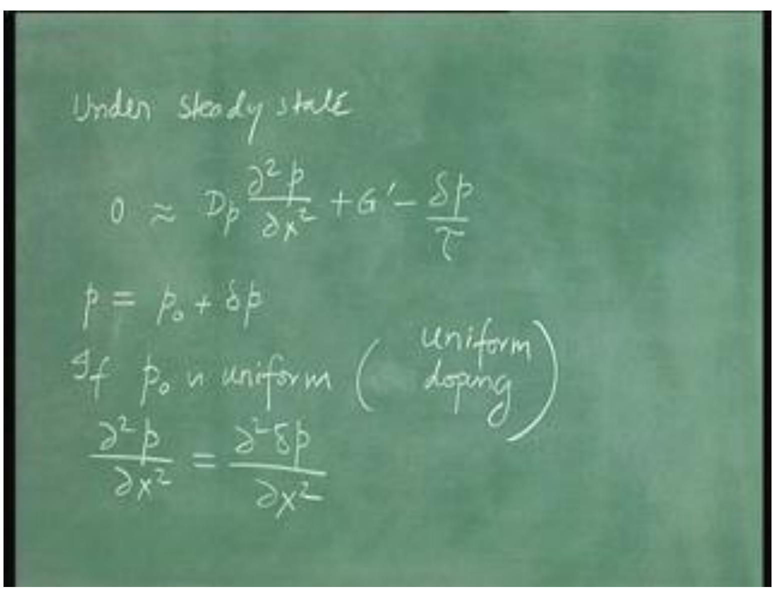

Under steady state this equation reduces to 0 is equal to (D_p doe square p by doe x square) plus (g minus) minus (delta$_p$ by tau), now this p is p_0 plus delta$_p$. If you further assume that the semiconductor we are talking about is uniform and as a result if p_0 is uniform then it is due to uniform doping. So that is the same as saying doping is uniform. If that happens then p_0 is uniform means p_0 is constant with x and when you differentiate this term with respect to x the differential of p_0 will go to 0. And then you will have doe square p by doe x square is the same as doe square delta_p by doe x square.

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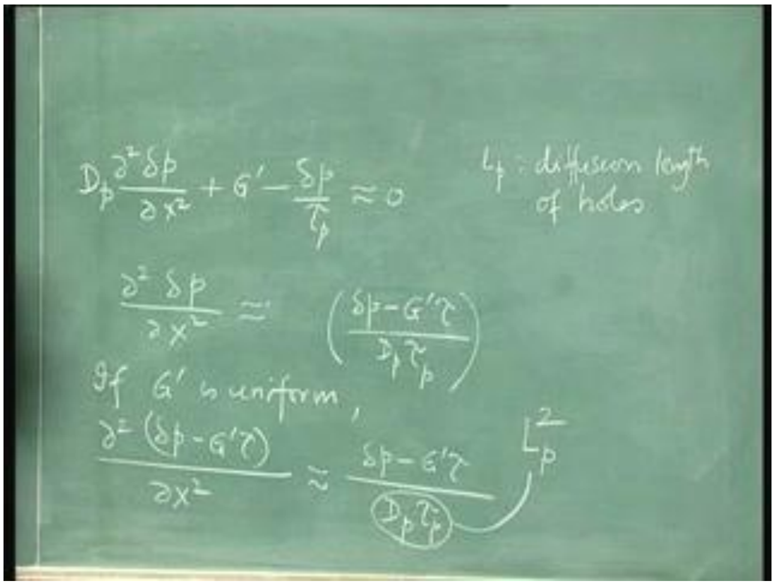

Therefore, this equation can be rewritten as D_p doe square delta_p by doe x square plus (G prime) minus delta_P by tau is equal to 0. Now we can simplify this particular equation and write it. So what we do is we divide by dp throughout and these two terms we club so we will get the equation of the form doe square delta_p by doe x square is equal to (delta_p minus G prime tau by D_p tau). Now we note that if G prime is uniform it does not change with x then we can replace this delta_p but delta_p minus (G prime tau) so everything will come in terms of one variable because the differential of G prime with respect to x would be 0. So if G prime is uniform then the above can be written as doe square delta_p minus (G prime tau) by doe x square is equal to delta_P minus (G prime T) by D_p tau.

Incidentally this tau is the lifetime of minority carriers and since we said that the diffusion approximation holds generally for minority carriers we can always identify this tau as the tau_p. So we can write this tau as tau_p the lifetime of holes because holes are going to be minority carriers because we are writing this diffusion approximation for holes we can replace this tau by tau_p.

From here by comparing the two sides it is very clear that this term has dimensions of distance square D_p is cm square by sec and tau_p is second and when multiplied you get cm square. This has dimensions of length and this length is denoted as L_p and square of that. This particular length can be physically interpreted as the length over which a hole travels by diffusion before it recombines. That is why this is called the diffusion length L_p is called the diffusion length of holes so this is the minority carrier diffusion length.

How do you show that L_p is the length over which a hole travels before it recombines when diffusion and recombination both are present? It can be done in the same way as we have done for lifetime. How did we show that if there is an exponential decay of a particular particle concentration then the time constant of this exponential decay is the average time for which a particle in this population that is decaying stays. In a similar way one can show that a solution of this differential equation is exponential in nature and in that exponent you will have the L_p coming in. So let us show how the solution looks like for this differential equation.

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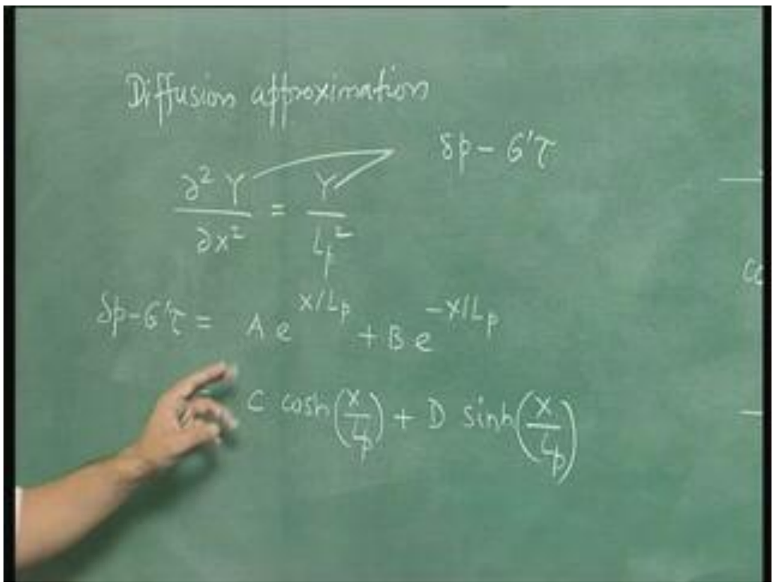

Now, for a differential equation of the form doe square y by doe x square is equal to y by L_p square the solution is given by y is equal to A e to the power x by L_p plus be to the power minus x by L_p. Or you can also write it in the form c cos (x by L_p) plus d sin (x by L_p) so you can use any one of these forms that is either in terms of exponential or in terms of hyperbolic sines and cosines. So, one can easily see by a substitution here that this is the solution of this equation. When you double differentiate the exponential you get an exponential itself and that is what the key is here.

What do you find here is when you differentiate the y function two times the form of the result should be the same as y itself and that property you hold for an exponential function or for a hyperbolic function which is nothing but sum of exponentials. Now these two ys are nothing but $delta_p$ minus (G prime tau). Therefore this y is $delta_P$ minus (G prime tau) which is the form of the solution. Now how do you decide whether you must choose the exponential form or the hyperbolic form? Let us look at the graph corresponding to these two exponentials and these two hyperbolic functions.

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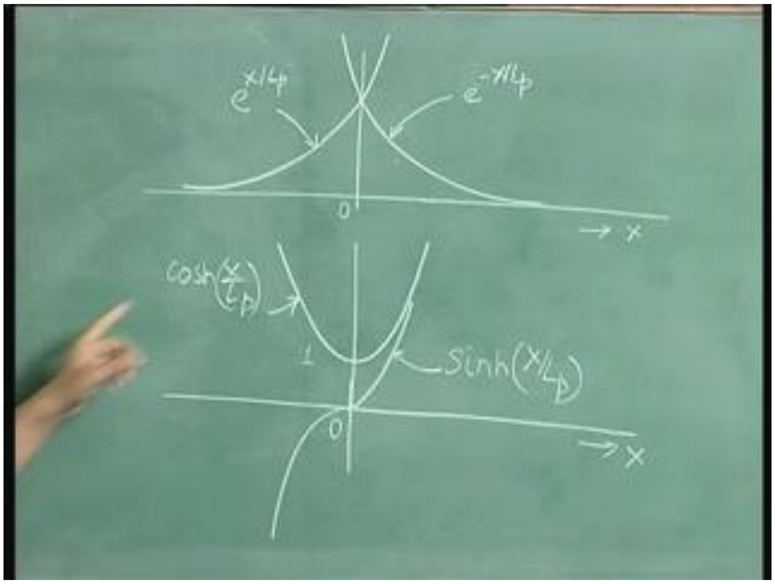

These graphs are shown here. So exponential minus x by L_p goes to 0 and extending to infinity, exponential x by L_p goes to 0 and extending to minus infinity. On the other hand the cosine hyperbolic function the value is 1 at x is equal to 0 and then it rises for x positive as well as x negative and this value is 1 here. The value of the sine hyperbolic function on the other hand is 0 at x is equal to 0. It rises to positive values for x positive and goes negative for x negative. So these are the features of this functions which will enable us to decide in a given situation which form of the function should be used.

For e.g. supposing you have an infinitely long semiconductor and it goes from x is equal to 0 in this direction to infinity. Now in this semiconductor the Excess Carrier concentration has to be 0 in at least one of the ends because if it is non zero over the entire block then this would mean that infinite number of Excess Carriers because length of the block is infinitely long and if the Excess Carrier concentration is non zero all over then there are infinite number of Excess Carriers and this is not possible. So generally for very long or infinitely long semiconductor blocks this particular function has to be 0 in at least one of the ends and therefore it is the exponential form of the function that must be used to describe Excess Carrier concentration in infinitively long blocks.

On the other hand, if the length of the block is finite, so if this end is x is equal to w where w is not infinite then it is one of these two hyperbolic forms which will be useful. So, if in this block you find the boundary condition is that the Excess Carrier concentration is 0 at one of the ends then evidently it is a sine hyperbolic function that is appropriate. On the other hand, if at both these boundaries the Excess Carrier concentration is not 0 then the cosine hyperbolic function is more appropriate. So in this fashion depending on the situation one must choose the appropriate forms of the solution. Although in principle you can use any of these forms to describe any situations. If you want a simple form of the solution i.e. you want one of these terms should drop out you do not want both the terms then you must follow the guidelines that we have just discussed.

So, for infinitely long blocks, exponential form of this solution it will be either this function or this term. On the other hand for finite blocks you use the hyperbolic form of this equation and it will contain either this term or this term depending on your boundary conditions. In the next class we will discuss further on the implications of this particular form of the solution for the diffusion approximation and we will consider the other approximations of the equations that are possible.

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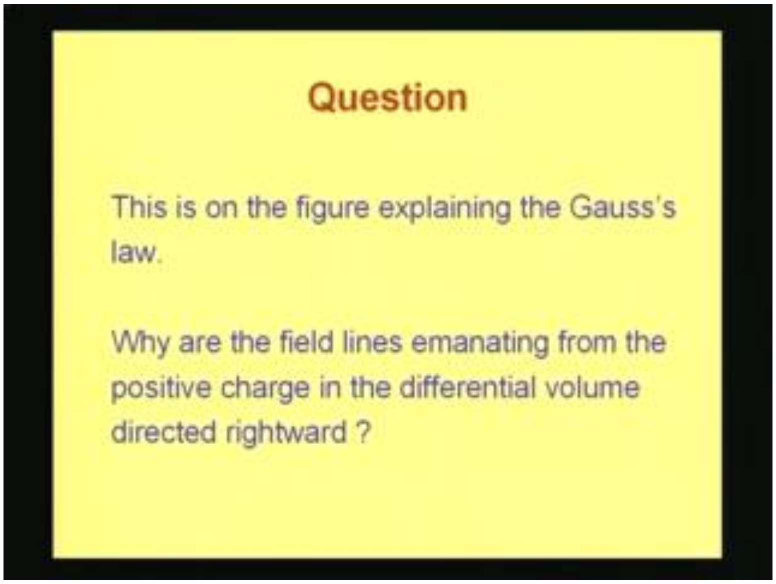

Student: Sir this is on the figure explaining the Gauss's law. Why are the field lines emanating from the positive charge in the differential volume directed rightward? Your question is about this figure.

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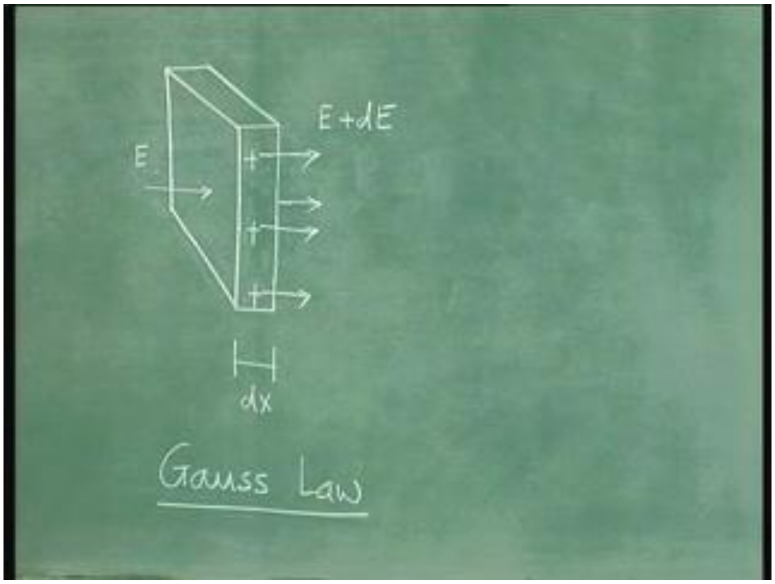

Why are these lines coming out of the positive charges within the volume in this direction? Why cannot some of these lines be in this direction? The reason is very simple. We are assuming that all the field lines are terminating on negative charge which is present on the right hand side here, and that is the reason.