VLSI Data Conversion Circuits
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Lecture - 58
Calibration and Randomization

This is VLSI data conversion circuits lecture fifty eight in the last class, we were trying to understand the effects of DAC nonlinearity in continuous time delta sigma modulators, and we saw that if there is mismatch in the DAC; then the levels are not what they are suppose to be and that causes arise in the in band noise floor as well as harmonic distortion.

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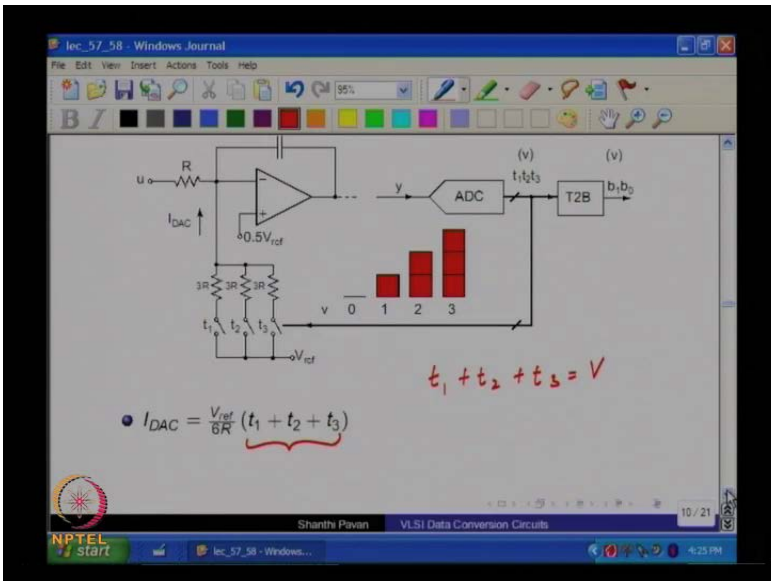

Now, we were discussing that the phenomenon may be understood in the following manner, we saw that in this particular example of four level DAC the sequence the output of the modulator is decomposed into a thermometer code represented by t1 t2 and t3, and by definition or by construction t1 plus t2 plus t3 equals V. And therefore, one can see the operation of the thermometer DAC in the following light, that the DAC is essentially decomposing the input signal V nonlinearly into three subsequences, t1 t2 and t3 converting each one of these subsequences into waveform and adding the waveforms R.

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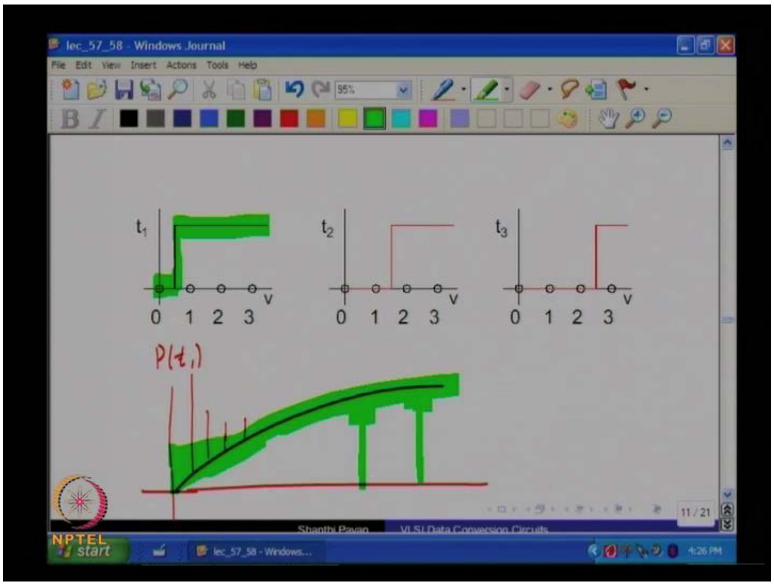

Now, since t1 t2 And t3 are related to V in a non-linear fashion perhaps by curve shown here. It stands to reason that the spectra of the signals t1 t2 and t3 will look like this apart from shape quantization noise in the input signal, they will consist of harmonics of the input signal plus the in band noise floor, which will be which can be expected to be considerably higher than what was there in V to begin with and the reason is that thanks to this stiff nonlinearity frequency components, you know out of the signal band will makes with each other and fall in band, now if t1 t2 and t3 over weighted with exactly the same factors. In other words since by definition V is t1 plus t2 plus t3 it follows that the power spectral density of V must be the same as the pass spectral density of t1 plus t2 plus t3.

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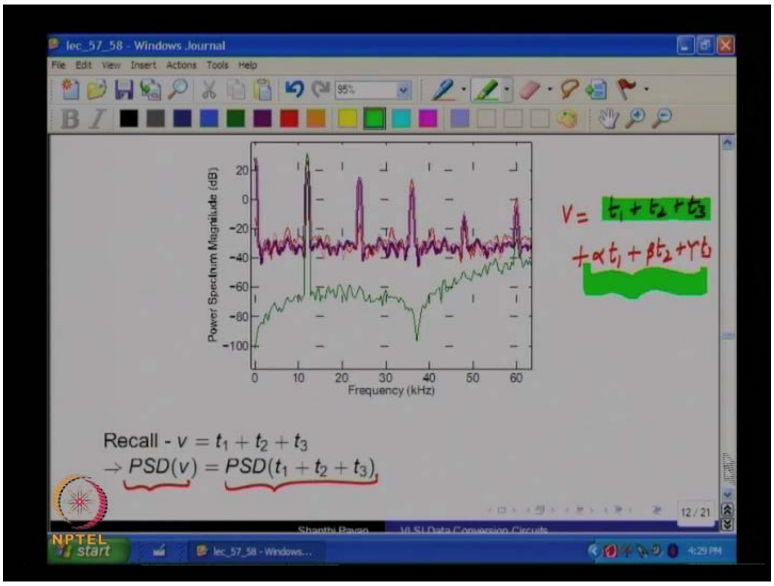

So, what is actually happening is that even though the individual signals sequences t1 t2 and t3 are horribly non-linear functions of V, when you combined them together with the same weights then all these harmonics are cancelling out and. So, is the in band noise floor right. So, that you are actually left with the true spectrum of V. So, now, it becomes quite easy to understand why when there is mismatch in the elements there is both an increase in the noise floor as well as harmonic distortion signals. Which were precisely suppose to cancel out each other’s harmonic components are slightly mismatch; because of the passive element mismatch, I mean you know a resistors are off or capacitors are off a transistor current sources are off.

So, when they are combine the factors with which you combine them for example, can be say what you are actually combining is 1 plus alpha into t1 plus 1 plus beta into t2 plus 1 plus gamma into t three. So, you can interpret this as t1 plus t2 plus t3 gives you your desired signal, but these culprits do not really cancel out. So, you are left with some remnants and. So, you can expect that they will be residual uncancelled distortion components and some residual uncancelled noise floor right. So, in the signal band when there is mismatch you can expect a noise floor as well as some uncancelled distortion does it makes sense all right.

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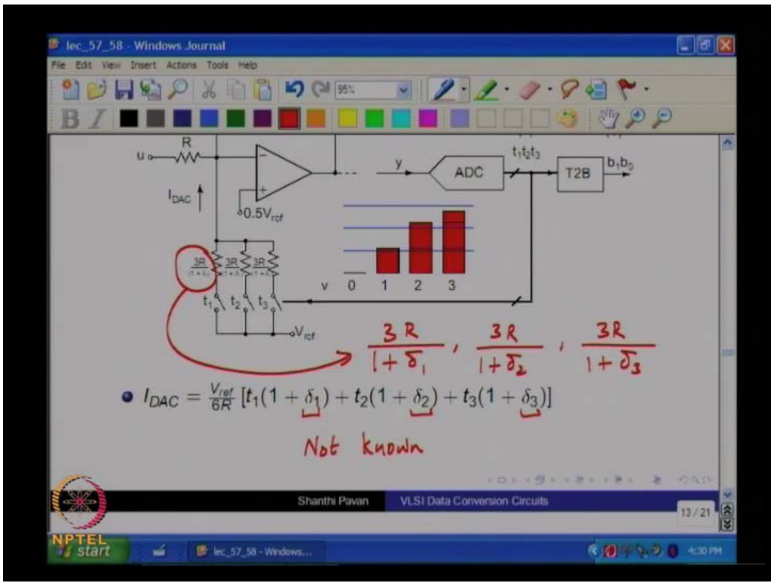

Now, So, to prove this point as I mentioned you earlier things is not very cleanly visible. If you now assume that each of the resistors are 3 R by 1 plus delta 1 3 0R by 1 plus delta 2 and 3 R by 1 plus delta 3, then the currents will be V ref by 6 R times t 1 times 1 plus delta 1 plus t2 plus times 1 plus delta 2 and t3 times 1 plus delta 3, and clearly delta 1 delta 2 and delta 3 are not known a priori not known a priority.

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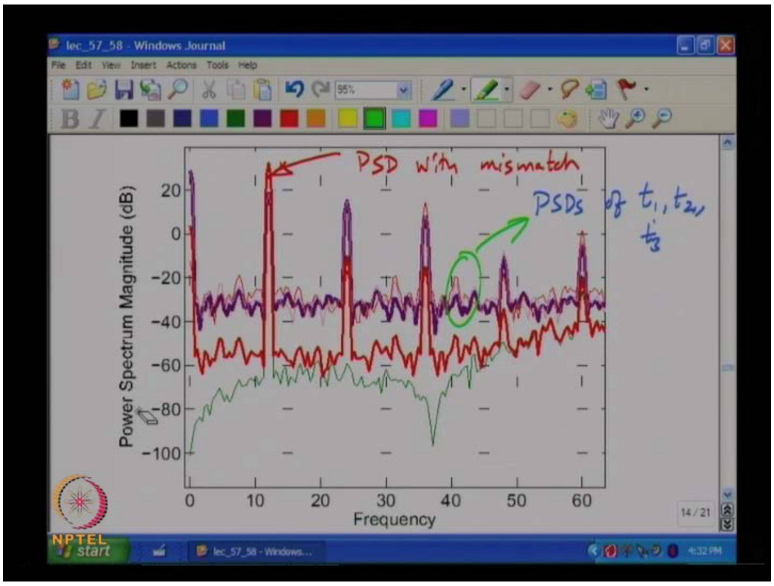

And therefore, when you combine these three signals together, you find that in the presence of mismatch the spectral density actually looks like this. So, this is red curve which is the PSD with mismatch and these are the PSD's of t1 t2 and t3 and this is what 1 would have got, if the weights of precisely equal to one all right. So, now, we have a hang of the problem right, and we trying to figure out how 1 can solve this. So, what is the within the codes what do you think is the most straight forward way of doing this ensure that there is one mismatch and of course one thing.

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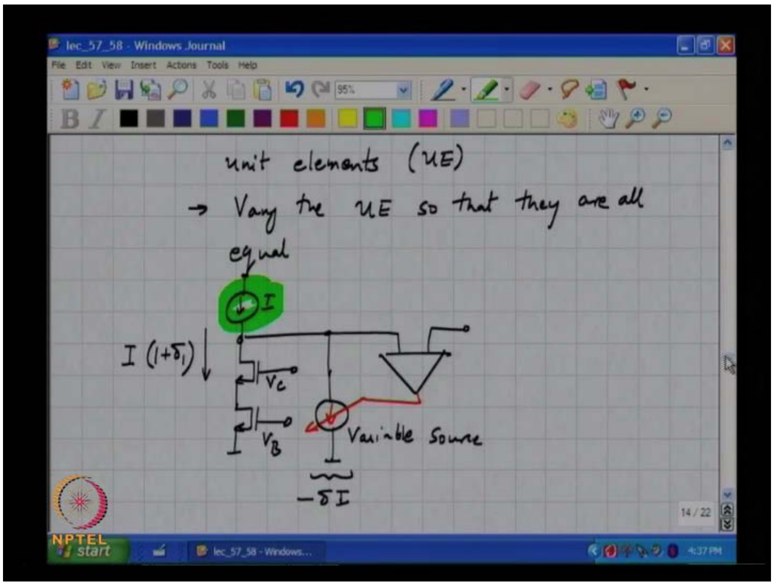

If I somehow figure out delta 1 delta 2 and delta 3 right and go and cancel them off then you know this will be quite. So, in other words this is what is called calibration of the DAC where you take somehow measure the mismatch between unit elements and somehow vary the unit elements. So, that they are all equal a common way of doing this is to say let say this is simplified equivalent of a current source. So, assume there is a biasing and all this stuff now clearly, because of device mismatch the actual current is not I 1, but it is I 1 times this not I, but I times 1 plus delta 1.

So, in principle what do you can do is you know that you wanted it to be equal to I correct. So, what you can do is you compare it let see you have a master current source, which is equal to I right you compare it with I and you add the it is actually I into 1 plus delta 1. So, you know that is off by a fraction of value of delta one. So, if you had an auxiliary current source if we have found an auxiliary current source right. So, this is V b and V c these are fixed add some variable current source I compare use, and op amp and compare this current source with a master current source. And tweak this variable current source such that the two currents are equal, and somehow I must have some way of storing that information is not it.

So, in other words this must provide the minus delta times I now this is a multiple DAC so; obviously, there will be many current sources. So, you compare every current source with the same master current cycle through all the you know n current sources, that you have store in a how much each current must be corrected. Once you finish three calibration cycle in principle, you have a DAC which is mismatch free right, I mean this is what you call a group force way of I mean I suppose not quite as group force as simply making the current sources bigger, but this seems like a very straight forward way of solving the problem. So, this is done sometimes where the DAC is calibrated of course, you have to worry about now when you calibrate the DAC, because when the DAC is being calibrated; obviously, you cannot use a modulator.

So, this must be done in the either you have a separate phase where the modulator is not being use, in that use that time to calibrate the modulator or you can do it at power up all right. So, when you power up the whole chip for some time you will not have the modulator in action because all these calibration loops are run the next technique.

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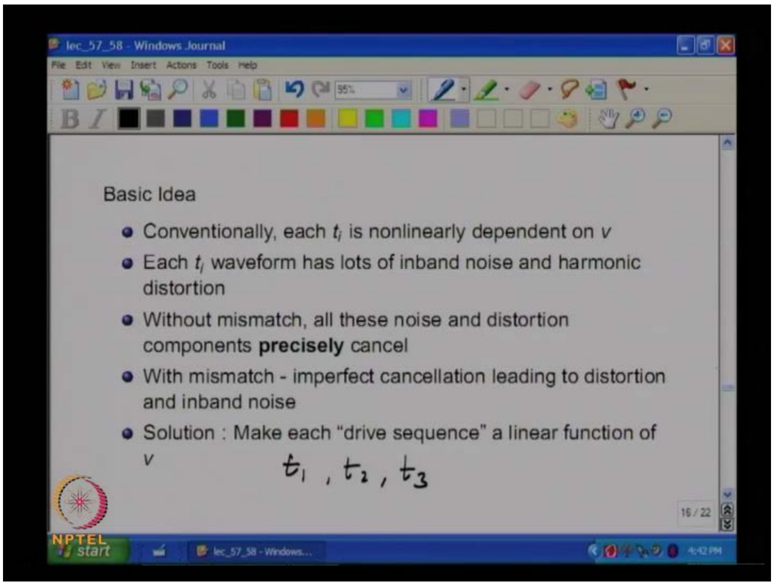

So, as you can see what we want to do right as for a change not match fixing, but mismatch fixing right you need to get rid of the problem, and as you will see it is indeed possible to do it without any knowledge of the of the deltas. It would seem at least on paper at first sight, that if I knew the mismatch values then perhaps they could be some way of fixing them correct, but as we will see quite surprisingly it is possible to get rid of this mismatch error to a large extent even when you do not know these mismatches a prior.

So, and the problem as we all seen is that with mismatch why is there an increase in the in band noise floor and distortion, because we are separating the I mean see we can t avoid the fact that we are going to add the outputs of several unit element DAC's correct. Now the problem is that if the weights are perfect then when you add the outputs of these unit elements DAC's, you know all the distortion components magically cancel out; however, the problem is that when these weights are not known or the sight errors in these weights the cancellation is not precise enough you understand. So, if you want to be able to add these things with slightly imprecise weights, and still not see harmonics or an increase in the noise floor or such a drastic in increasing in the noise floor what do you think you can do what is the problem.

The problem is that we were separating the input V into three sequences each of which control one of the unit elements in a non-linear fashion the moment, you pass the signal V through some kind of non-linear function the result is bound to have harmonics. And an increased in band noise floor and no surprise at when you combine them, you know with the wrong weights you will see residual distortion and noise. So, if I was able to separate them linearly right, in other words make t1 t2 and t3 linear functions of V, then one would be able to the individual sequences would have no harmonics. So, when you add them you cannot relaying on cancelation. You understand you are not relaying on cancelation. So, it if the weights are slightly off you are not you do not see any distortion you understand.

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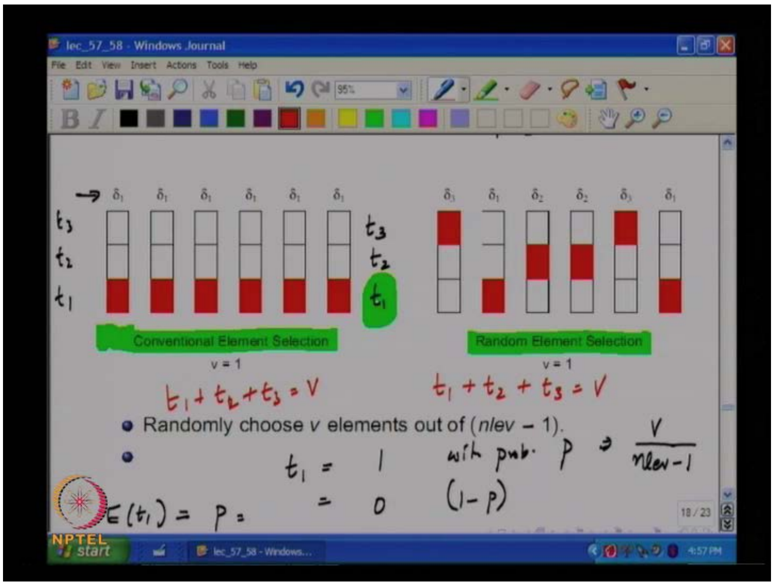

So, let us see what all we can do the first historic technique, that people used is what is called dynamic element randomization, again I will use a four level DAC example. So, will be three unit elements. So, each one of these rectangles corresponds to an element if an element is being used it is shaded in red. So, if you had a conventional thermometer DAC when V was 1 we choose one element and we choose the same element regardless of how many times V equal to 1 occur you understand. So, this would reflect the choices of unit elements and on top I have plotted the I have mark the error with respect to the I mean the ideal should have been 1 its actually 1 plus delta 1.

And the error will be delta 1 always, you on the other hand if V was 2 the error will be delta 1 plus delta two delta 1 plus delta 2 since delta 2 is not the same as delta 1 clearly. This is a I mean using a thermometer is a nonlinear process, because the input as doubled, but the error sequences not doubled, because delta 1 is not equal to delta 2 this is clear.

Now dynamic element randomization is a strategy where you say if V equal to 1 that clearly how many ways of picking one element from three. The three ways of the three ways of picking one element. So, all that 1 says is that hey I am going to pick at random every sequence, I mean at every instant of time I am going to pick any 1 of these at random all right. So, in the first instance of time I pick the third one second instance I pick the first and. So, one and the error sequence now is different for instance. I mean it is clearly different for each instance and you can see there is delta 3 delta 1 delta 2 delta 2 delta 3 delta 1 and. So, on, but what I want it you to understand is can you comment on t1 t2 and t3 in the conventional case versus the randomize case, how does a sequence t1 look like here it is always one while t2 and t3 are zero always what about here.

Student: Operational matrix of 2 m 2 by.

So, this the t1 sequence here is 0 1 0 0 1 the t2 sequence is 0 0 1 1 0 0 the t3 sequence is 1 0 0 0 1 0 all right of course, the way of the drawn the diagram here it also appears as if in you know for this small sample the average of t1 is same as t2. What is the average of t1 2 by 6 which is one-third, the average of t2 2 by 6, the average of t3 is also 2 by 6. So, at least it seems as if t1 t2 and t3 are linearly dependent on V you understand, because the average of t1 is the same I mean it has to be if the sum of t1 t2 and t3 have be have to be equal to V. It must follow that the average of t1 plus the average of t2 plus the average of t3 must be equal to V. So, apart from a scaling factor the average of t1 t2 and t3 are proportional to V. I mean you can make this you can come to this for through another argument right for a fixed V right. So, if you look at only the rows here right the probability that a particular cell is red is what 1s one-third.

So, in other words t1 for example, is 1 with probability with probability P which in this case is basically V by n level minus 1 correct all right, and is 0 otherwise with probability 1 minus P correct, because at every instance of time you choosing this you picking whatever in this particular case one cell out of three cells all right. So, what is the average of what is the expected value of t1.

Student: V by n minus 1 V by n one.

Where is simply p which happens to be V by n level minus 1 similarly. So, E of t1 is p all right.

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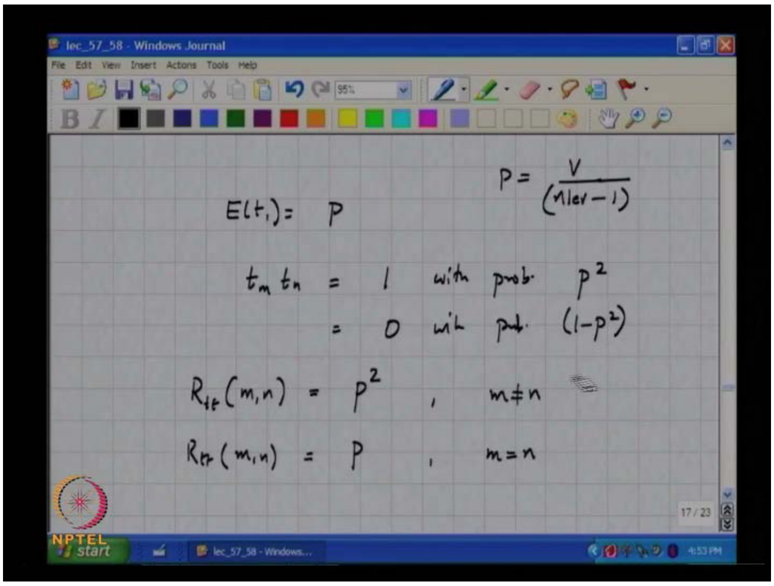

The autocorrelation and let us remind ourselves that P is nothing, but V divided by n level minus 1 what is the autocorrelation of this sequence t1 is the expected value of t. In the n-th instant and t in the n-th instant all right and this must be equal to please not that at the m-th and the n-th instants. The decision of which to choose is completely independent of what happened earlier correct, because the choice is purely random, now that means, that this E of the expected value of t m times t n, what is the probability or rather the probability density function of t m t n is what this will be 1. This can also take on only t m times t n what values can this take on one or zero. So, this is 1 with probability what.

When both of them are one and that happens with probability P square right. So, t m t n is 1 with probability P square is 0 with probability 1 minus P square correct. So, what is the autocorrelation function P square minus 1 instead of summation. So, R t t of m comma n is nothing but P square. Simply the expected value of t m t n which is P square this is for m not equal to n for m equal to n what is it.

It is simply P where m equal to n clearly this does not depend on m or n or you can see that the cross correlation does not depend on m, the autocorrelation does not depend on m or n. So, you can as well say that this if you look at the autocorrelation function we can now plot it simply as a difference between m and n right for 0.

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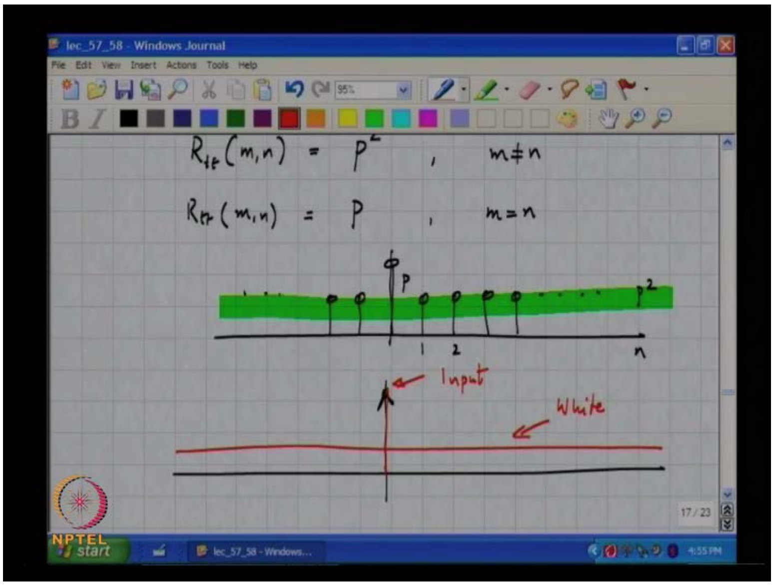

It will be P and for n not equal to 0, you will get P square does make sense. So, how will the PSD of this look like how is the autocorrelation related to the power spectral density, it is Fourier transform this is a d c sequence plus an impulse, dc sequence will give you an impulse response at the origin correct. And you will get a white noise does make sense the dc component corresponds to the input signal correct you understand; that is P sorry P square right, that corresponds to this square value of the input signal all right along with it there is a white noise does it makes sense.

So, this is the input and this is white component please not that we have still not about talked about mismatch at all this is simply the transfer function or sorry this is simply how t1 relates to V.

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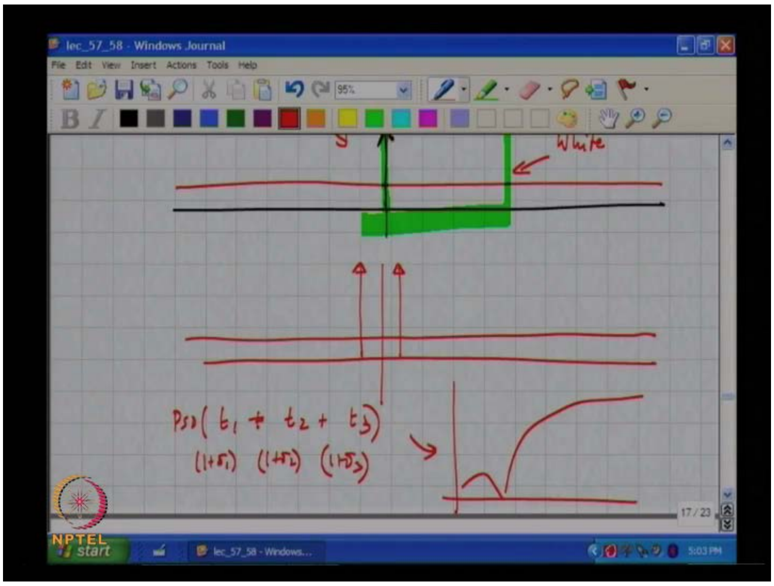

Now, please note to that whether you are doing the conventional type of elements selection or whether you do random elements selection always t1 plus t2 plus t3 equals V you understand. So, it must follow that if the weights are exactly 1 right whether you decompose the signal using the conventional thermometer decoder or whether you randomize the selection, if there was no mismatch the PSD of t1 plus t2 plus t3 will look exactly the same and will be equal to that of b is this clear. So, the only difference between randomization and the conventional way of choosing t1 t2 and t3 is that, now all the drive signals t1 t2 and t3 are linear functions of V right, I mean we have just seen. In fact, this is not rigorously it is not rigorously proven that they are linear functions of V all that we have seen as that the mean of P 1 is the same as.

As V I mean V by 2 within a fraction right and we have seen this only for d c all right, now we have getting into too much mathematical detail it now follows that if V is varying slowly, where the same argument you can expect the what can you expect for the mean of t1. If V was d c the mean of t1 t2 and t3 is the same as is you know whatever V by n n level minus 1 correct. So, intuitively all the three paths are contributing to the output right regardless of whether the signal V is small or large, you understand this is completely in contrast with the conventional thermometer way of decoding, where is the signal is small only t1 contributes right if the signal is I mean t3 gets into the act only when.

The signal is very large all right. So, so what I wanted to point out was that the spectral content of t1 t2 and t3 consist of the mean of V we saw this for d c and now its stands for reason that if V was varied slowly the mean of t1 would.

Student: means.

Would also track V right I mean if V was 1 the mean of t1 is one- third if V was 2 the mean of t1 will be two- third. So, if V change you know slowly over time then it follows that the average value of t1 will also slowly track the average value of V does make sense. So, since the individual drive waveforms have spectra which will therefore, look like let say they input V was sinusoid you will expect that. What will be the spectrum of the drive waveform of spectrum of P 1 for instance, guys if the input is d c this is the PSD of t1 t2 t3, now if the input was varying slowly; that means, as a sinusoid for instance what will be the spectrum of t1 t 2 or t3.

Student: During white on performance.

You can expect it to be the sinusoid right plus hat was plus white noise all right this is a 2 side spectrum I have drawn, but what we have seen in the diagrams all along is something here correct all right, now if t1 t2 and t3 were combined precisely with weights of 1 what will the resulting spectrum look like. It will be look like if you still did t1 plus t2 plus t3 you should get something like this correct; however, if you combine them with slightly different weights what will you get or what do you expect to get, when the in band noise will be there I mean. Earlier what had we see harmonics, you will see harmonics noise floor and increase why would you see harmonics,

Student: Because they hardly.

Because they individual spectra of t1 t2 and t3 had harmonics, so when there not added together in precisely the right way there will be some residual harmonics left now what do you see in the individual in the spectra of individual drive waveforms there are no harmonics. So, even if you add t1 t2 and t3 with slightly erroneous weights 1 thing you can be sure about is that there will be no harmonics is this convincing what about the noise floor now.

Student: It is noise floor.

In band noise floor.

Student: White noise.

So, if there added with precisely the right weights the noise source is also cancel of because by construction t1 plus t2 plus t3 always equals V correct; however, if these are not combined in exactly the right way 1 can conclude that instead of seeing the spectrum of V you will also see in addition some uncancelled residual noise which happens to be white right. Because the individual drive waveforms have spectral characteristics which are the input sinusoid plus a white component which means that. If you know there is mismatch and there would not cancel perfectly some of that residual white component will show up in the output all right, and clearly as the mismatch becomes smaller and smaller the this residual white noise will become smaller and smaller.

So, the total noise in band is a sum of the quantization noise of the modulator plus this uncancelled residual white component. If the mismatch is very small you might be dominated by quantization noise and you have nothing to worry about right, but as a mismatch becomes larger and larger the white 1 will dominate, and you will be you will be worried about it all right. So, the key point behind randomization as you can see is that it converts the sinusoidal tones into white noise.

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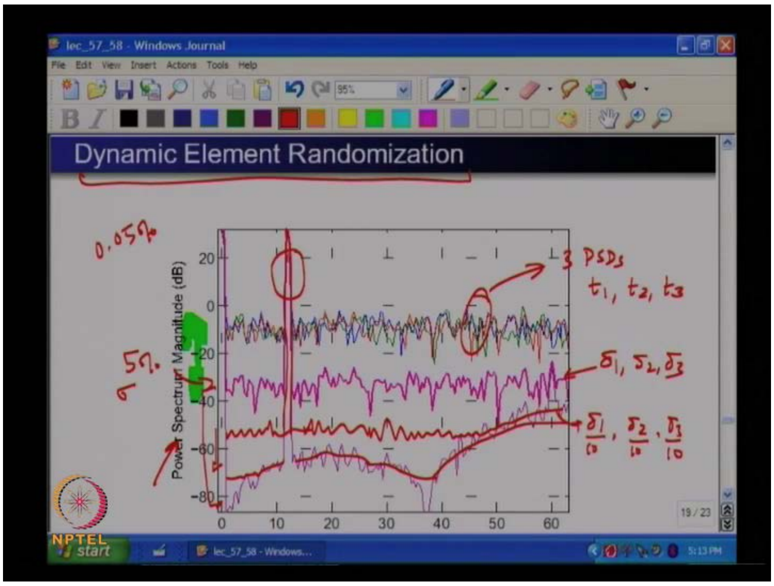

So, this is a simulation showing the spectra of there are three PSD’s here, and they are that corresponding to I am let me show you t1 t2 and t3 all right, 1 thing you can you should observe is that even though it is not very clear here it turns out that what is that component.

Student: Which is the signal component?

That is the signal component in t 1 t 2 and t3 and that happens to be the same for all the signals and that make, sense because t I mean each 1 of the drive signals is linearly dependent on the input in the same way correct. So, the signal components better be the same. In fact, we can see that the PSD's of all the three drive wave forms is a same all right, which I think I should draw your attention to the earlier waveforms. So, I think you are not able to see it, but for instance look at this harmonics here these are the PSD's of t1 t2 and t3 and clearly you can see that.

Student: They are not.

Each of these harmonics the peaks are different and that make sense because the three nonlinearities are different and the signal components. In fact, you may not be able to see that here, but the signal components here are also different all right. So, this is the case where a regular thermometer DAC was used the in contrast to that notice that when you do dynamic element randomization, you have the same spectra for all the three drive waveforms. And when you combine them again this is with the same five percent mismatch, that I had been using earlier you can see that the residual noise floor is down, but clearly you are still way over the quantization noise. Because the mismatch is apparently too high you understand, now let us say I reduce the deviation. So, let us see this corresponded to some delta 1 delta 2 and delta 3. So, if I magically made delta 1 delta 2 and delta 3 lower by a factor of 10 what do you think will happen the noise floor will go down by how much.

Student: 20.

20. In the I mean.

Student: 5 and.

5 10 20, the waveforms the difference in the error waveform has gone down by a factor of 10. So, error power is gone down by a factor of 100, but you are plotting power

spectral density. So, you should go down by

Student: 40.

Power by a factor of hundred 20 d B. 20 d B right. So, if this was like this and right if delta magically became down by a factor of this thing you will see that you should go down by 20 d B all right. So, this corresponds to 5 percent sigma mismatch. So, clearly 5 percent sigma mismatch is not good enough correct because the noise due to mismatch is way higher than.

Student: Noise for quantization.

The quantization noise floor right it is about 40 d B let me say about 30 d B was correct, and a sigma of point 5 percent is also not quite as is not quite good enough because you will see that you are still about.

Student: 10.

10 to 15 d B above the quantization noise floor. So, if I reduce this by a further factor of mismatch by another factor of 10 you would see something like this then you will be dominated by quantization noise here. And then again you will see this noise getting filled up and you would see this all right. So, in other words in order that you are not limited by noise due to mismatch right in this particular scheme where this is called dynamic element randomization you need a row matching of point 0 5 percent that is what this is telling all right. So, please note that even though we have only four level modulator in other words the quantizer is only 2 bits, but to achieve good SNR.

The linearity of this DAC steps must be very, very high you normally think that 2 bit quantizer is a you know within codes a are very cool affair right we just need four steps, and which means that in principle to get you know half an l s b of d n l, you I mean the elements can vary whichever way you want and you still be able to get four bit linearity right, but this is saying that four bit linearity even though I am sorry to get two bit linearity even though the quantizer is a only two bits the linearity of this quantizer must be very, very good, I mean this is like at the at the twelve bit level point 1 percent is ten bits right this is of the order eleven bits actually you understand ok.

So, you should go home with the feeling that hey I know a sigma delta is a way of taking you know absolutely lousy quantizer and getting fantastic resolution that is true; however, when you have multi bit operation you must bear in mind that the feedback DAC even though there are few elements the matching between those elements is actually quite critical, if you were only I mean if you did just the normal thing of the just depending on matching.

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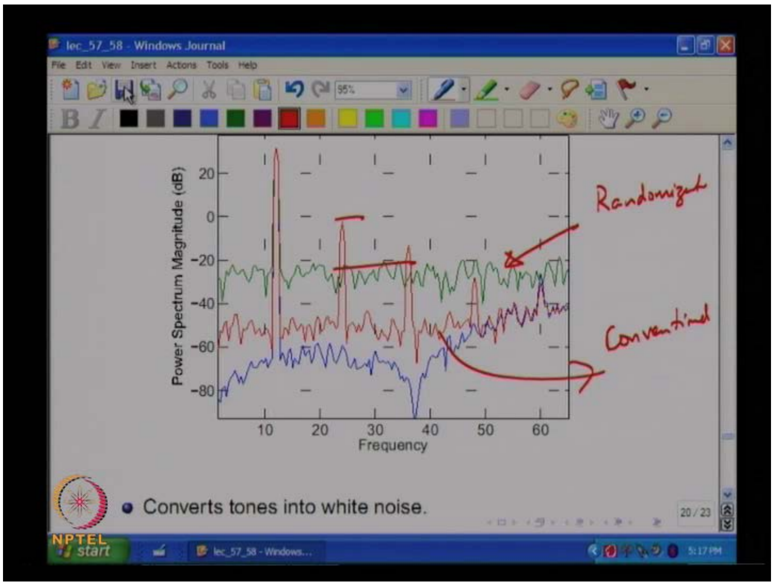

In the next class we will see that in the way before I let you off today I will may conclude this slide this is a with randomization, and this is with a normal thermometer decoded DAC and what you can see is that this randomization has significantly as eliminated tones at the expense of an increased noise floor right. So, now, the question is it possible to decrease noise floor also.

So, if you want to reduce the noise floor I mean, basically what you are doing is you are combining three signals right t1 t2 and t3 all right, if the PSD's of t1 t2 and t3 are I mean they have the signal component of course, but if the. So, called you know the other component the non signal component in the case of randomization was flat, if you want it to be if you do not want if you want much lower noise floor what must you what can you say about the spectra of the individual waveforms at you are adding, instead of noise.

Student: noise.

If you if you combine them and you expect to see very little noise in band it must follow that the individual PSD's before you add them correct. The drive waveforms of the individual or rather the PSD's of the individual drive waveforms must have very low noise floor within the signal band at least, if they have low noise floor elsewhere you know, it is not our concern definitely in the signal band they must have very low noise floor. So, that when you add them up even with mismatch the I mean the fact that they have inherently low noise floor means that mismatch will no longer be a problem. So, we will see this techniques in the next class. So, we will stop here for today.