Digital Speech Processing
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Lecture - 13
Uniform Tube Modeling Of Speech Processing Part – V

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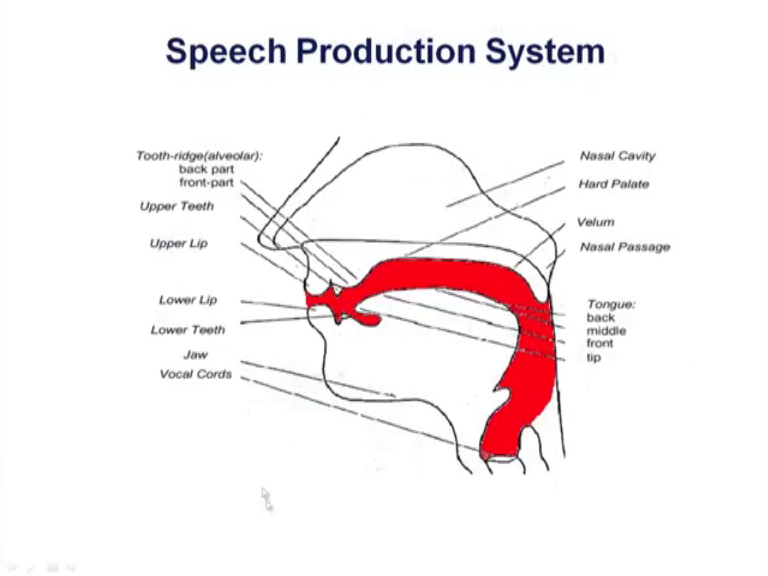

So, we have discussed about the single tube model. Now, if you see the speech production system of human being, so red color mark is that your vocal track. So, this is the vocal whole vocal track is like that. If you see the cross sectional area of the vocal track is not uniform throughout the tube here (Refer Time: 00:37) area very small. So, what about the constriction is made, the cross sectional area of the tube along you can say the x equal to 0 to x equal to a l the lips the tube cross-sectional area is not uniform.

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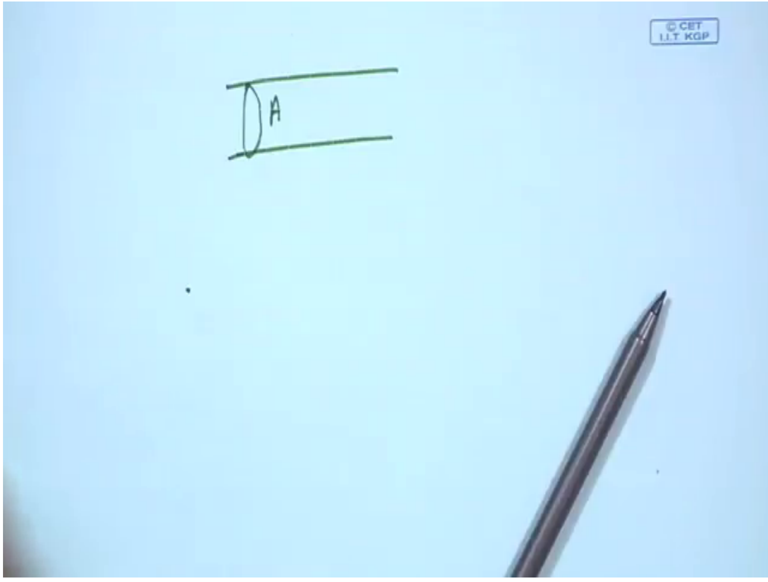

So, instead of a single uniform tube with a fixed cross sectional area which is A, we have done instead of that; now we want to make the model with a multiple tube of different cross sectional area connected to each other.

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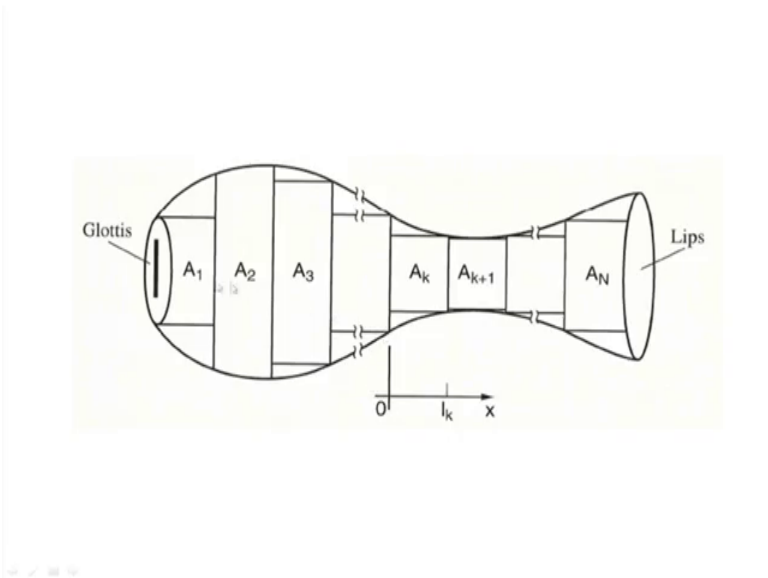

So, if I see let this kind of things that let this is the glottis and this is the lips, this is the structure of my vocal track. Now, I can say whole vocal track instead of modeling by a single uniform tube can be model as a multiple number of uniform tube connected with each other. So, let us the whole track is model by a N number of tube with a fixed length, length is fixed, lets the length is l. So, I can say lets l k length tube and n number of tube is connected with each other.

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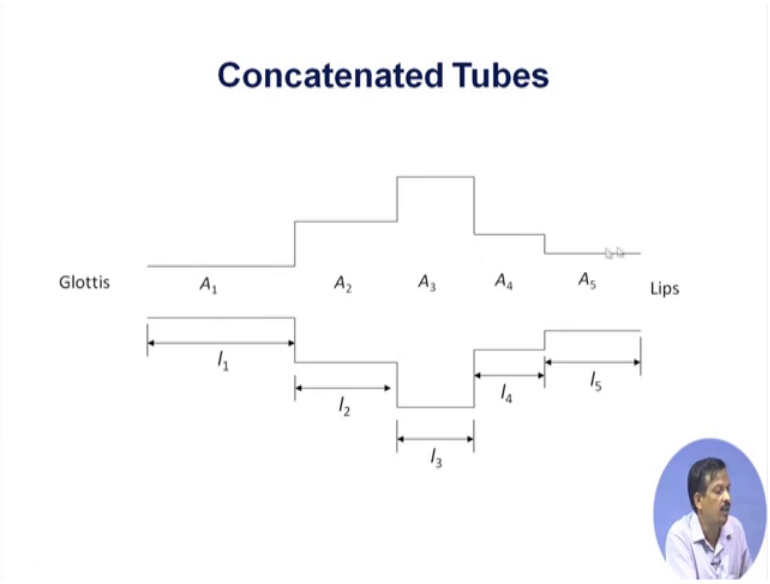

So, I can say the vocal track is model like this way. So, I can say l 1 or initially I can say that whole vocal track can be model different number of connected tube with different length also. So, if I say the we are do that then I can say this is the l 1, l 2, l 3, l 4, l 5 number of this is the length of the tube and five number of tube is connected to each other. Or I can consider a l 1 equal to l 2 equal to l 3 equal to l 4 equal to l 5, then the length of the tubes are same, but n number of small tube I divided and whose cross sectional area is A 1, A 2, A 3, A 4, A 5 all are not same, different cross sectional area.

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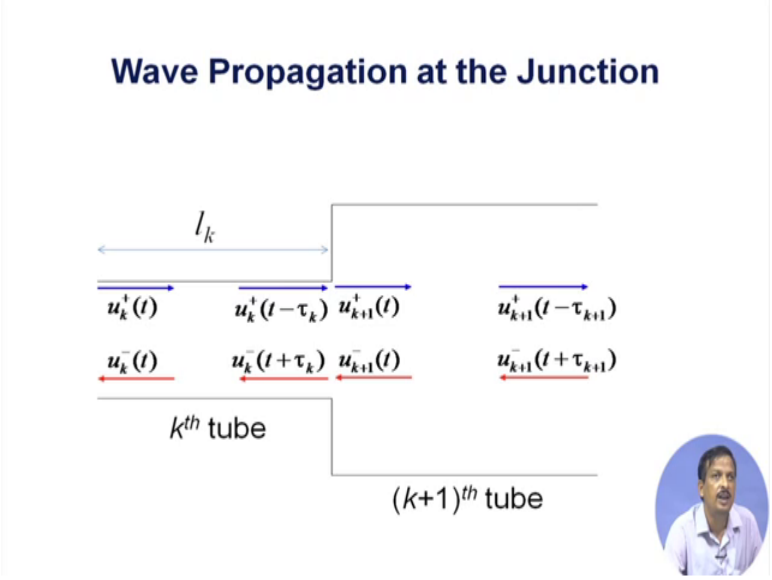

Now, if I consider that then what is the problem, if I consider that whole tube is a connected tube instead of a single tube. So, the problem is that I know what will happen when it sound will travel throughout the tube, if it is a single tube, I know input and output. It is in tube actually in a nothing but a delay, I can derive the equation for signal flow equation from the input and output, and can find out the transfer function of the tube. There will be effected junction also. So, the main purpose for multiple tube modeling is that I have to find out what happening at each and every junction of the tube.

So, the cross sectional lets this one is very small cross sectionally, this is wide cross sectional area. So, what will happen; volume velocity and pressure wave equation in this junction, what will happen all tube will be connected here this junction. So, let I want to know what will happen at kth junction. So, kth junction is nothing but a kth tube and kth plus 1 tube junction that junction I want to know what will happen.

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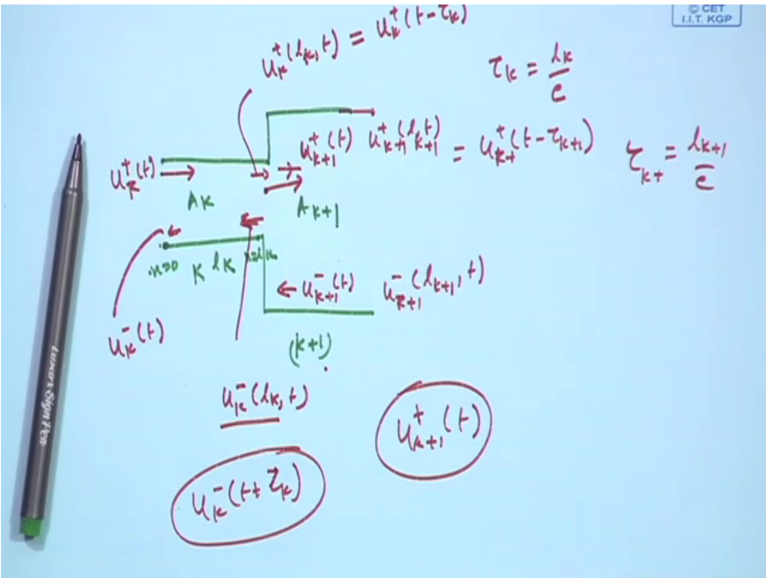

So, instead of single tube, let us consider I have a tube the different conserve different tube are connected with each other with different cross sectional area So, let us the k th junction I taken. So, this is the kth junction of the tube. So, this is the k plus 1 tube and this is the kth tube, this is the junction. So, here if it is l k then here l this if I say this is a beginning of the kth tube, so x equal to 0; and here it is x equal to l k. Here will be analog junction. So, let us solve for the single junction this will be replicated at every junction. So, cross sectional area this tube is A k, this tube is A k plus 1.

Here, so volume velocity in here which will travel along the tube forward wave this is nothing but a u t plus or you can say u k t. And here it is nothing but a u k plus l k, t; backward wave in here it is nothing but a u k minus l k, t; and here it is nothing but a u k minus t. Input of this tube here forward wave is nothing but a u k plus 1 t; backward wave in here it is nothing but a u plus u k plus 1 minus t. If it is a end of tube in here then it will be u k plus 1 plus l k plus 1 t and here will be u k plus 1 minus l k plus 1 length of the tube is l k plus 1, here is l k. Now, what will happen, if you see the slides, I have tau k.

So, here I have written as tau k, what do you mean by tau k if it is see this is nothing but a u k plus t minus tau k where tau k is nothing but a l k by c. Here it is k plus 1 t, here again it will be this will be nothing but a u k plus 1 t minus tau k plus 1, where tau k plus 1 is nothing but a tau k plus 1 is nothing but a l k plus 1 divided by C. Backward wave same, I can say the backward wave like this way.

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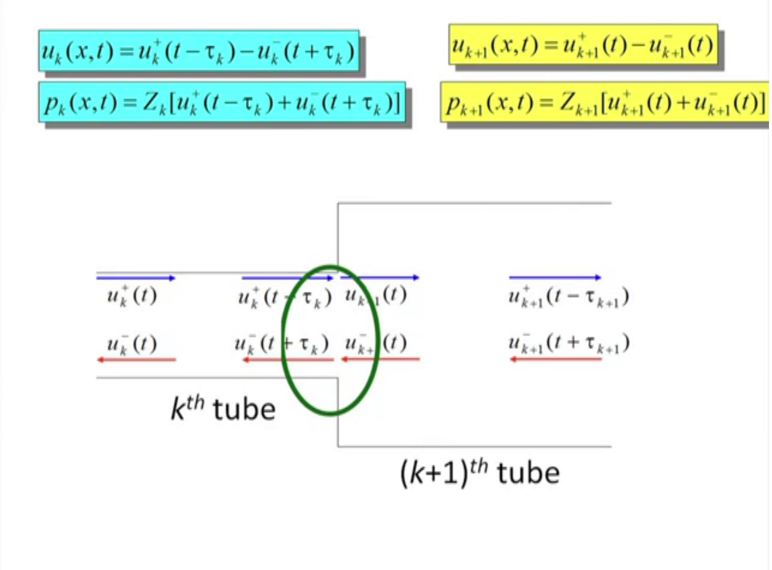

Now, I have to know what happening in the junction, this junction what is happening what will happen in this junction. So, this junction what will happen? Some; I have to find out sum if you see that the; if this is analogous to the water pipe also.

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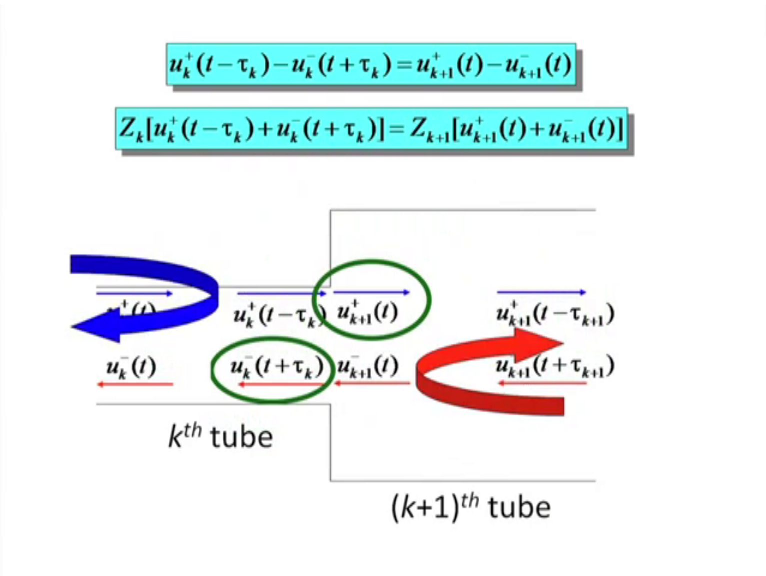

Suppose, there is a pipe water pipe different cross-sectional area water pipe have joined together. So, the water is flowing here some of the water if you this junction, so the volume velocity this point is the volume velocity which is injected in the second tube and the volume backward volume velocity in the kth tube is nothing but a volume velocity which is injected due to the backward wave. So, I have to find out the expression of the forward wave of the kth plus 1 tube; and backward wave at the kth tube, if I able to find out the this 2 expression, then I know what will happen in the junction, I can draw the equivalent signal flow diagram at the junction.

So, my whole purpose is to find out how much forward wave is injected by the kth tube to the kth plus 1 tube, and how much backward wave is injected by kth plus 1 tube to the kth tube that I have to find out. So, I want to know the expression of u k minus t minus t plus tau k this is t plus tau k this one. And I want to find out u k plus 1 t 1 u k plus, these 2 expressions, I want to find out.

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$$u_k^+(t-\tau_k)-u_k^-(t+\tau_k)=u_{k+1}^+(t)-u_{k+1}^-(t)$$

$$Z_k[u_k^+(t-\tau_k)+u_k^-(t+\tau_k)]=Z_{k+1}[u_{k+1}^+(t)+u_{k+1}^-(t)]$$

$$u_k^+(t-\tau_k)-u_k^-(t+\tau_k)=u_{k+1}^+(t)-u_{k+1}^-(t)$$

$$\frac{Z_k}{Z_{k+1}}[u_k^+(t-\tau_k)+u_k^-(t+\tau_k)]=u_{k+1}^+(t)+u_{k+1}^-(t)$$

$$u_k^-(t+\tau_k)+u_{k+1}^+(t)=u_k^+(t-\tau_k)+u_{k+1}^-(t)$$

$$-\frac{Z_k}{Z_{k+1}}u_k^-(t+\tau_k)+u_{k+1}^+(t)=\frac{Z_k}{Z_{k+1}}u_k^+(t-\tau_k)-u_{k+1}^-(t)$$

Now, what is the problem how do I find out that thing. So, how do I find out?

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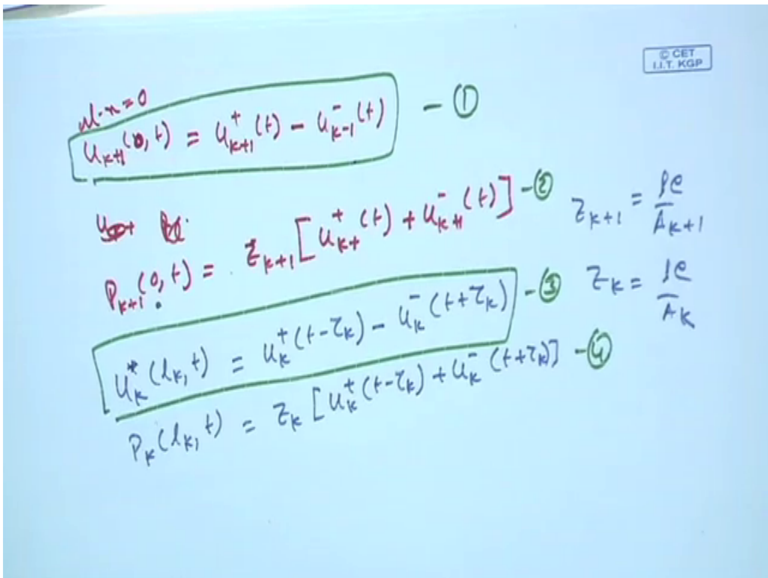

So, let us start that expression. How to find out these 2 equations, how to find out? So, I can say that u k plus 1 x t any point u k plus 1 x t is nothing but a u k plus 1 plus t minus u k plus 1 minus t what is x t at x equal to 0, so this will be 0 t. U k plus 1 or I can say p k plus 1 0 t is nothing but z k plus 1 u k plus 1 plus t plus u k plus 1 minus t at x equal to 0 at the k plus 1 tube. At x equal to l, at the k th tube, u k plus u k at l k, t end of the k th tube u k l, k t is nothing but a u k plus t minus tau k, why tau k is l k by c minus u k minus t minus t plus tau k, ok or not. And p k l k, t is nothing but z k into u k plus t minus tau k plus u k minus t plus tau k.

Where z k plus 1 is nothing but a rho c by A k plus 1 and z k is nothing but a rho c by A k cross sectional area of the tube is A k, ok or not. So, I am not animation I am not showing the animation. This is the 1, 2, 3, 4 equation I will get, I know what I have to find out. I know, I have to find out u k plus t plus tau k and u k plus t, this I have to find out, this is the four equation I get. Now, if I say this one is equal to this one ok or not, the volume velocity, which will input to the second tube is nothing but a volume velocity, which is exited from the first tube. So, the volume velocity which is exited from the kth tube will be the volume velocity which is the input of the k plus 1 tube.

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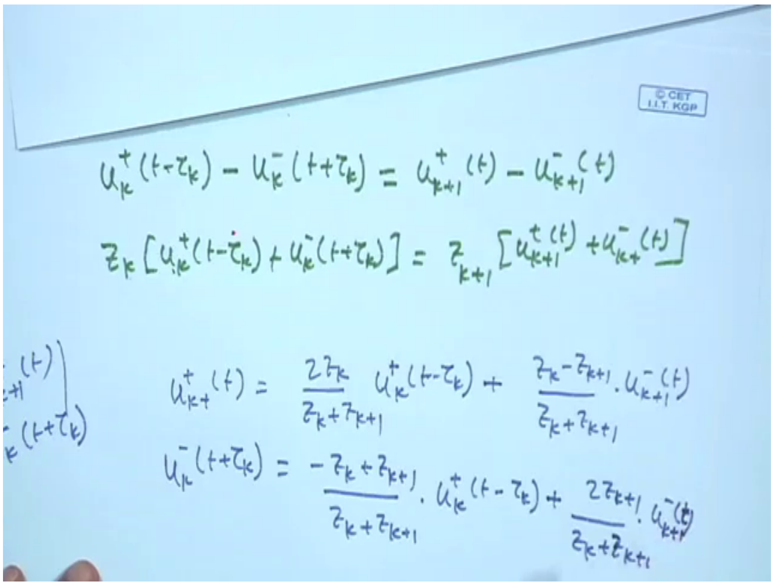

So, I can say u k kth tube volume output volume velocity is nothing but a u k plus t minus tau k minus u k minus t plus tau k which is equal to the volume velocity input of the k plus 1 t. So, it is nothing but a u k plus 1 plus t minus u k plus 1 minus, is it ok or not, ok. Second one is that output pressure of the kth tube will be equivalent to the input pressure of the k plus 1 tube. So, what is output pressure of the kth tube, z k u k plus 1 t minus tau k plus u k minus t plus tau k which will be equivalent to or equal to z k plus 1 u k plus 1 plus plus u k plus 1 minus t. This is the 2 equations, I will get from the condition.

So, I said that if is the my first tube and it if it is the k th tube and if it is the k plus 1 tube, red one is the k plus 1 tube. So, the volume velocity which is inject at the output of the kth tube will be the equal volume velocity at the input of the red tube. Similarly, the output pressure of the black tube is equivalent to the input pressure of the red tube that is why make. If it is that, then can I solve this 2 equation for this 2 I have to find out u k plus t and u k plus 1 t and u k minus t plus tau k. The amount of f expression for the reflected backward wave at the kth tube expression for the forward wave k plus 1 t, these 2 things I have to deduce from this 2 equation.

Now, if I take this one then I can say that using these 2 equation I can find out the expression. I am not deriving you can derived it and find out the expression. So, the expression of u k plus 1 plus t will be 2 z k divided by z k plus z k plus 1 u k plus t minus tau k plus z k minus z k plus 1 divided by z k plus z k plus 1 in k plus 1 minus t.

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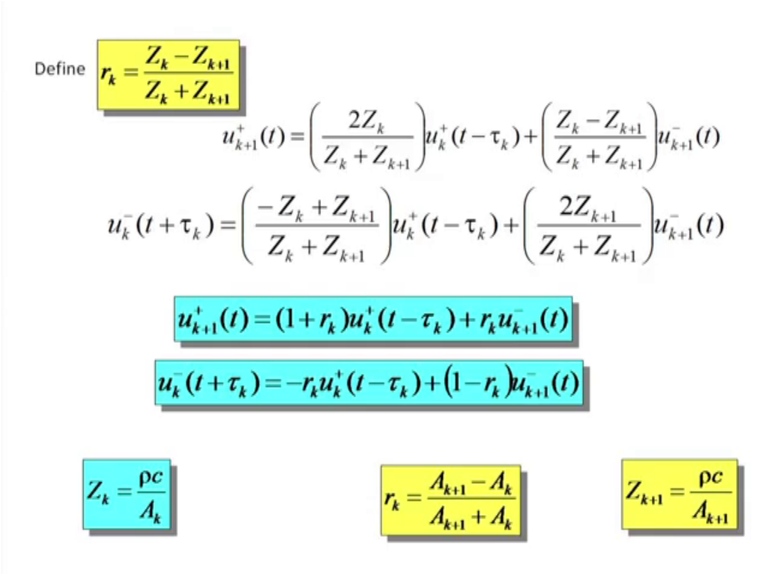

And u k minus t plus tau k will be minus z k plus z k plus 1 divided by z k plus z k plus 1 into u k plus t minus tau k plus 2 z k plus 1 divided by z k plus z k plus 1 into u k plus 1 minus t. This is the 2 expressions; I will get from these 2 equations.

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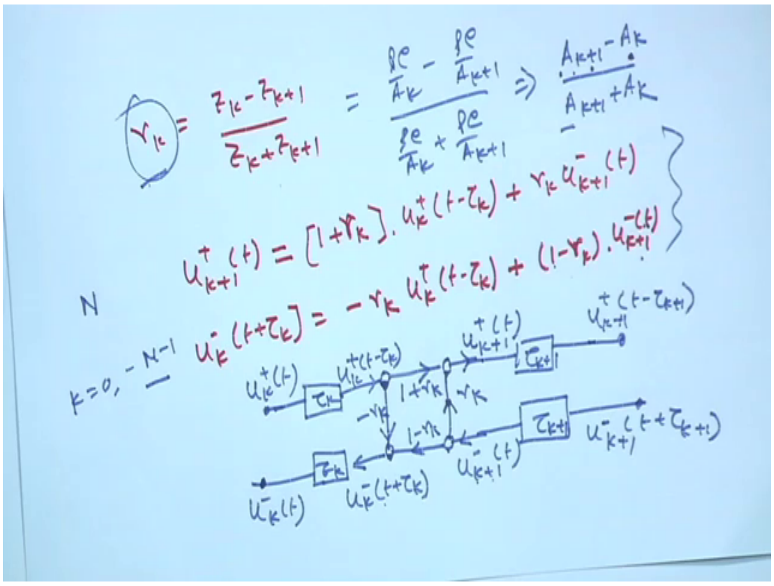

Now, just try to simplify these 2 equations. So, what I can write that, if I write r k is nothing but z k minus z k plus 1 divided by z k plus z k plus 1 z k minus z k plus 1 divided by z k plus z k minus is nothing but a r k. If this is r k hen I can say this expression u k plus 1 u k plus 1 plus t forward wave at the second tube will be or k plus 1 tube will be nothing but 1 plus r k plus this is small r. So, this is 1 plus r k into u k plus t minus tau k plus r k u k minus k plus 1 minus t. And u k minus t plus tau k will be equal to minus r k this one minus r k into u k plus t minus tau k plus 1 minus r k 1 minus r k into u k plus 1 minus t.

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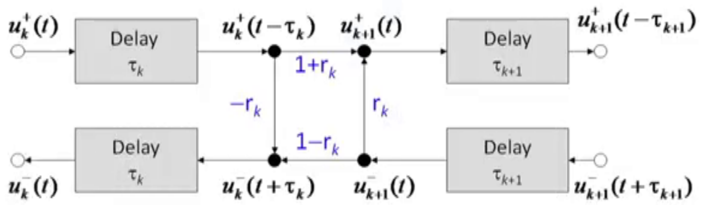
$$u_{k+1}^{+}(t) = (1+r_k)u_k^{+}(t-\tau_k) + r_k u_{k+1}^{-}(t)$$

$$u_k^{-}(t+\tau_k) = -r_k u_k^{+}(t-\tau_k) + (1-r_k)u_{k+1}^{-}(t)$$



Now, try to draw the diagram at the junction point. So, I want to draw the diagram at the junction point from these 2 equations. Let us try in here also here only. So, that it will be little bit of equation of there. So, now what I said I want to know let this is the junction, this is a four point at the junction. This is nothing but a u k plus 1 plus t and this is going this direction, and backward wave is going this direction at the junction. So, it is nothing but a u k minus t plus tau k. So, signal flow I can say this direction, this is the forward direction and this is the backward direction. So, this is nothing but a u k plus t minus tau k and this is nothing but a u k plus 1 minus t.

So, I can say u k plus or u k plus 1 t this is nothing but a 1 plus r k into, so this signal multiply by 1 plus r k. And it has to be go a signal from here which has to be added there which is nothing but a multiply by r k ok or not. Then I want to know this signal this signal is nothing but here which is there u k plus t minus tau k has to be multiply by minus r k; and this signal u k plus 1 t minus has to be multiplied by 1 minus r k, I get the junction diagram.

So, if you see that, this is the junction diagram, I have draw in here. So, once I know these 2 equations, I know the junction diagram 1 plus r k, 1 minus r k. Then what is r k, r k is called reflection coefficient. How it is come what is z k rho c by A k rho c by A k minus rho c by A k plus 1 cross sectional area of the A k kth tube cross sectional area of the k plus 1 tube minus rho c by A k plus rho c by A k plus 1. If I evaluate it, it is nothing but A k plus 1 minus A k divided by A k plus 1 plus A k, ok or not. So, this is called thats r k is called reflection coefficient, r k is called reflection coefficient. It is nothing but a cross sectional area of k plus 1 tube minus cross-sectional area of the kth tube divided by the cross-sectional area of k plus 1 tube plus cross-sectional area of the kth tube.

So, though the diagram. So, kth tube length is l k, so delay is tau k. So, I can say whole diagram. So, kth tube length is l k, so delay will be only tau k, here also will be delay tau k and here is the I get the input of the kth tube which is nothing but u k plus t and u minus k t. Here is the delay of the plus 1 tube tau k plus 1 and here will be delay of the k plus 1 tube tau k plus 1, and I can get that u k plus 1 plus t minus tau k plus 1 and here I get u k plus 1 minus t plus tau k plus 1. So, this is the complete junction signal flow diagram.

Think about similar kind of junction is repeated instead of k I can say put 1 k where is from 1 to lets n. So, if I if there is a n number of tube, then I can say k varies from 0 to n minus 1 tube where is from 0 to n minus 1 tube. Now, I just consider the value of what is should be the value of r k. The r k is A k plus 1 minus A k divided by A k plus 1 plus A k. Now, what is the maximum value of r k, what is the maximum value will be r k, what is the minimum value of r k.

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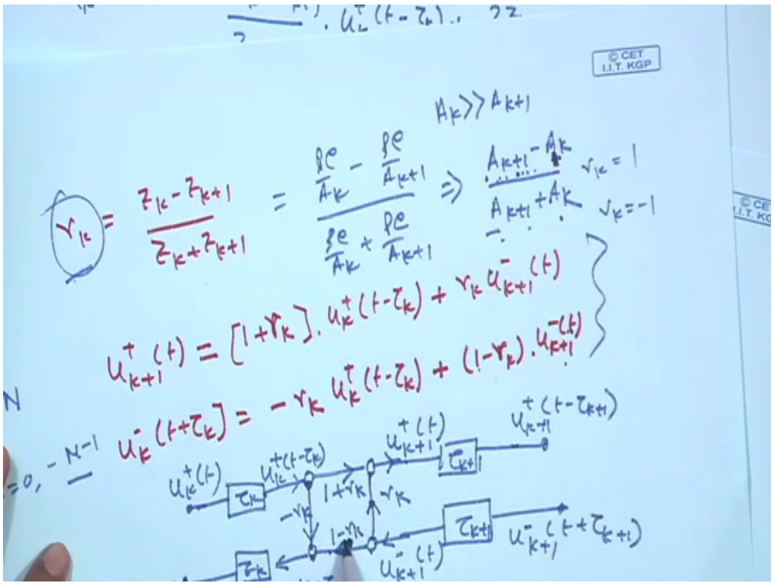

If A k is very small compared to A k plus 1, if I neglect A k then r k will be 1. If A k is very very big compared to the A k plus 1 or if A k is much much greater than A k plus 1 then I can say r k will be minus 1. So, if the cross-sectional area of the second tube is larger much larger than the first tube at the junction r k reflection coefficient will be plus 1. If A k is less A k is much much greater than A k plus 1 then r k is minus 1; if A k plus 1 is much much greater than A k, then r k is plus 1.

So, this is 1 minus r k. If r k is plus 1, then if you see this signal reflected the backward wave is multiply by this backward wave effect of backward wave is multiply by 0. If it is r k is minus 1 then 1 minus r k is positive. So, minus 1 2. So, this is the maximum value of r k will be either plus 1 and minus 1. So, you can prove it that r k value make maximum value will be plus 1 or minus 1.

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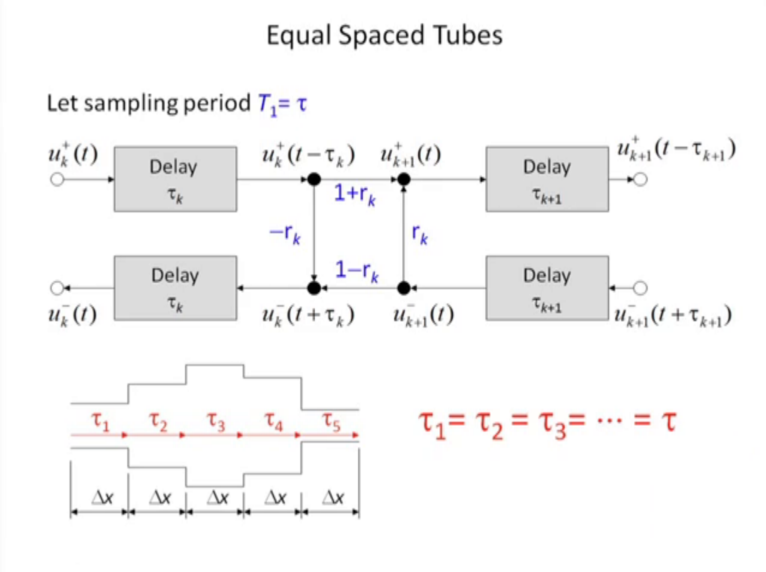

Now suppose I have a kth n number of tube with equal length. If I consider all tubes are equal length then what should be the diagram. So, I said this delay tau k tau k is equal to tau k plus 1, if all tube length are; so what is tau k l k by c. So, if all tube length are same then I can say the all tau k is equal to tau k plus 1 all are del x del x del x del x del x. So, delay circuit will be same. So, it is nothing but a delay, delay will be same, same delay will be happen. Now, of it is that then I can say lets tau is equal to sampling period t 1 z to the power minus 1 is one sample delay.

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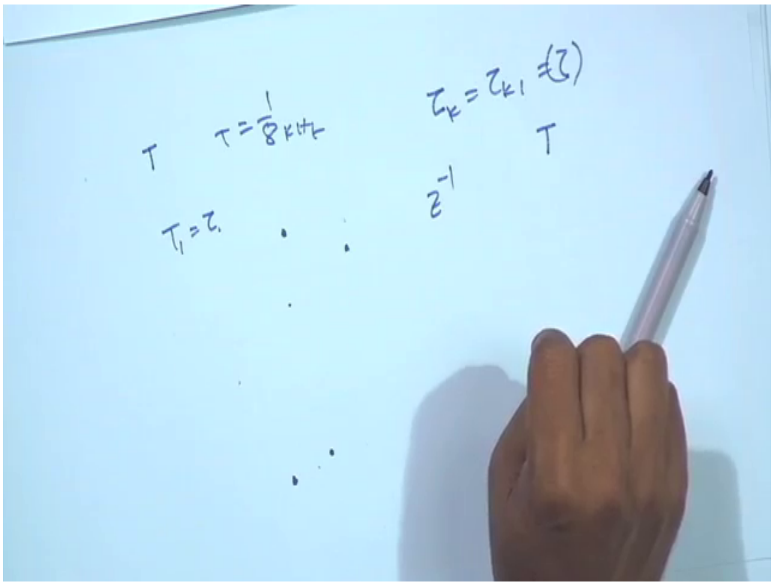

What is the delay? If it is 8 kilo hertz then what sampling period 1 by 8 kilo hertz is the sampling period? So, sampling period delay will be happen. Let the tau t one sampling period is equal to tau. So, this is one sample delay one sample delay in here. So, I can say this is replaced by z to the power minus 1.

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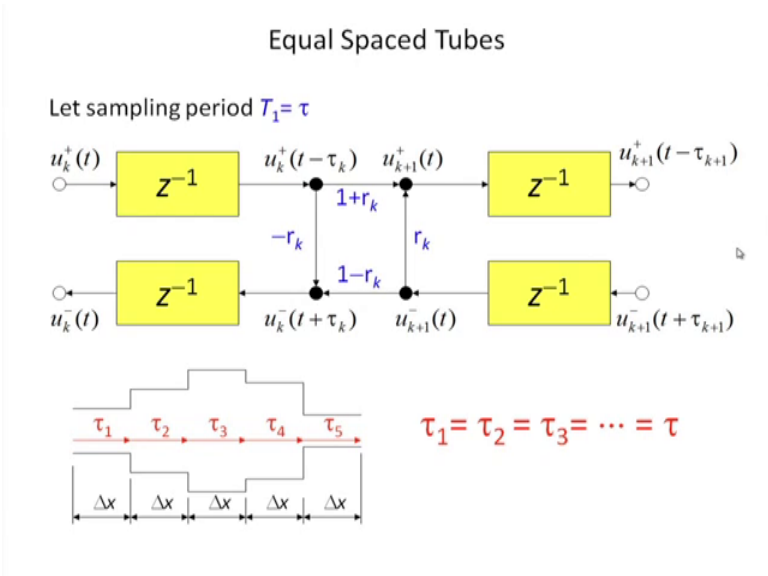

So, if it is tau k is the delay, so tau k delay is replaced by z to the power minus 1 and if tau is the tau k is equal to tau k plus 1 is equal to tau and this tau is equal nothing but a sampling period is equal to sampling period. So, in that case I can say the signal is one sample delay that is z to the power minus 1. Now, considering the boundary condition, I can derive the complete transfer function of this tube model in z domain, then I can implement it.

So, next class what I will to do I want to derive the total vocal cord transfer function for a n such tube model n such tube model. So, instead of you can say the single tube, I consider multiple tube of different cross-sectional area are conjuncted with each other, let there is a n number of tube then I derive the signal flow diagram for the single junction the same junction will be repeated for n minus. So, if there is a n tube n minus 1 junction will be there. So, n minus 1 junction will be there in the tube, this will be repeated. Then I can say that delay can be simply implemented by signal sampling delay. So, why if I replace this tau k delay by z to the power minus 1 sample delay then I have to find out the whole z domain equation of this vocal track transfer function. So, next class I will try to do that.

Thank you.