

**Modern Digital Communication Techniques**  
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**Lecture - 37**  
**With Memory Modulation (Contd.)**

Welcome back to the discussion on Digital Modulation Techniques. And what we have been discussing is continuous phase modulation. So, we would digress a little bit instead of looking at constrained modulation forms where the phase is continuous to a little bit simpler form. So, as to get back to what could be a slightly different form of modulation and what is the essence because in the continuous phase form the expressions are not so straightforward whereby you could visualize what is happening exactly.

So, of course in the previous lecture, we have discussed about MSK where we had also seen a form of representation as if there are two carriers and you are modulating two carriers. However, the pulse shape is kind of half cycle of a sinusoid. And of course, they are shifted with respect to each other, so that is what we have seen. So, what we will do is we will look at a few basics where you need not necessarily have this half sinusoid and an equivalent representation, but we will rather straightaway start off with the BPSK and the QPSK modulation which is much easier to look at.

So, and just to remind you, we are mainly following the text on digital communications by a John Proakis, whereas this particular section that we are discussing here we would also be referring to the book on digital communications by Sklar both are excellent books. So, we would refer to each one as and when necessary. So, I would mention them at appropriate points.

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QPSK & Offset QPSK

$$d_k(t) = d_0, d_1, d_2, \dots \pm 1. \quad \left\{ \begin{array}{l} \frac{1}{T} \\ \text{duration } 2T \end{array} \right.$$

$$\begin{array}{l} \text{I} \\ d_I(t) = d_0, d_2, d_4, \dots \text{ even} \\ d_Q(t) = d_1, d_3, d_5, \dots \end{array} \quad \left\{ \begin{array}{l} \frac{1}{2} \text{ bit rate} \\ \text{duration } 2T \end{array} \right.$$

$$s(t) = d_I(t) \cos\left(2\pi f_c t + \frac{\pi}{4}\right) + d_Q(t) \sin\left(2\pi f_c t + \frac{\pi}{4}\right)$$

$$= \cos\left[2\pi f_c t + \theta(t)\right]$$

So, now moving forward, so we would like to look at the QPSK modulation and we will also look at the offset QPSK. So, the QPSK if we cover the Q, it is a PSK that is a phase shift keying. When we say Q, we mean it is quad quadrature that means there are four PSK. So, in case of PSK, we typically have let say the data sequence indicated by  $d_k$  being  $d_0, d_1, d_2$  and so on and so forth which we were using so far as  $I_n, I_1, I_2, I_3, I_4$ . So, this is similar, so similar to the earlier notation of  $I_n$ . And this could take values of plus or minus 1.

And the pulse stream that is the incoming stream it is divided into two parts the  $d$  of  $I$  and  $d$  of  $Q$ . So,  $d$  of  $I$   $T$  and  $d$  of  $Q$   $T$  such that the in phase consists of the even bits you can say the even symbols and this consists of the odd ones. So, we have already seen this in the previous lecture, but we are looking at it afresh because we are looking at QPSK. And these  $d_I$  and  $d_Q$  each have half bit rate that means, they have the duration which is  $2T$  and therefore, the rate is one upon  $2T$ , whereas here the rate is one upon  $T$ . So, however, since these two are together being sent on quadrature carriers they would come up with a rate of  $1$  by  $T$  and this one modulates a cosine this one modulates a sinusoid. So, we could write  $s$  of  $t$  is equal to  $d_I$  of  $t$  that means, this one times  $\cos 2\pi f_c t$  plus some phase factor, and in one of the representation, there could be an offset. Offset means you could be rotating it and you could have  $d_Q$  of  $t$  times  $\sin 2\pi f_c t$  plus  $\pi$  by  $4$

So, if d I takes a value of 1, d Q takes a value of 1, you have one particular combination; so 1 1 minus 1 minus 1 1 and minus 1 minus 1. So, these are four combinations of d I and d Q that you get and you could also write this in form of  $\cos 2\pi f_c t + \theta$ . So, where this  $\theta$  would be choosing one of the four possible phases that are required in a typical QPSK.

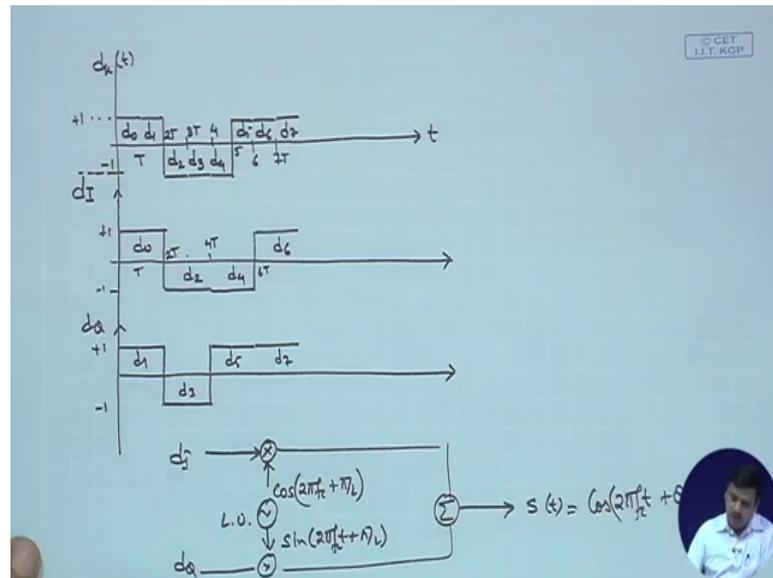
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$d_k(t) = d_0, d_1, d_2, \dots, \pm 1.$   
 $d_I(t) = d_0, d_2, d_4, \dots$  even  
 $d_Q(t) = d_1, d_3, d_5, \dots$  odd  
 $s(t) = d_I(t) \cos\left(2\pi f_c t + \frac{\pi}{4}\right) + d_Q(t) \sin\left(2\pi f_c t + \frac{\pi}{4}\right)$   
 $= \cos\left[2\pi f_c t + \theta(t)\right]$

Constellation Diagram: A 2D coordinate system with four points labeled 01, 00, 11, and 10.

So, what we could think of is this expression that we had drawn is or we can use this space itself and then you could say that you have used these locations as constellation points. And this is  $\pi/4$ , this is  $3\pi/4$  and this is  $-\pi/4$  and  $-3\pi/4$ . So, these are the four phases that you have of a cosine. So, you would select any of the four phases that you would require for transmission.

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And then we had already drawn the waveform earlier would still do it afresh because we are looking at QPSK now. So, this is some time  $t$ , and let us say we had original sequence as we are drawing it over here, where I would write this as  $d_0, d_1, d_2, d_3, d_4, d_5, d_6, d_7$ . And of course, this is  $T$ , this interval is  $2T$ , then there is  $3T, 4T$ , there is  $5, 6, 7T$  and so on and so forth. And here we would like to draw  $d_I$  and you could mark this as plus 1, you could mark this point as minus 1. So, now, here it was  $d_k$  of  $T$  from a previous page.

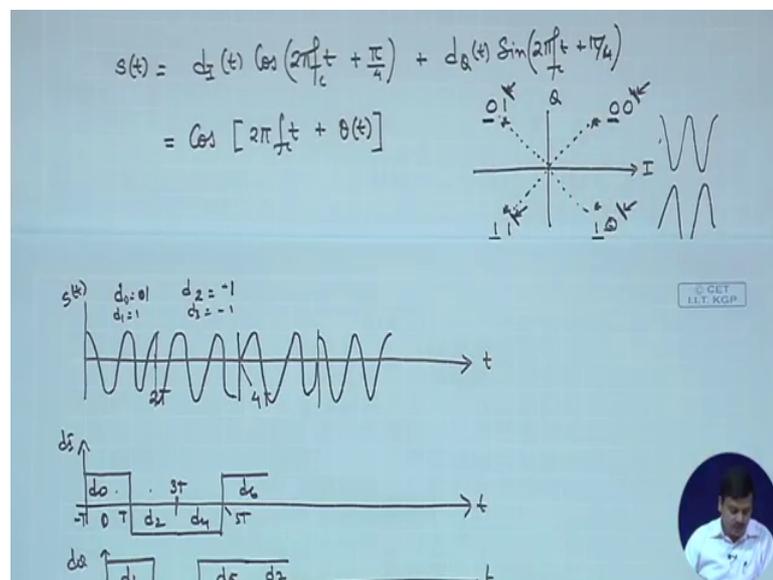
So, here what we are going to do is we are going to stretch  $d_0$  for two time intervals then we are going to have  $d_2$  for two time intervals,  $d_0$  was stretched over here,  $d_2$  is stretched over here. So, in which  $d_4$  would stretch  $d_4$  for two time interval then you reach to  $d_6$  stretch it for two time intervals. So, now, this is  $d_0, d_2, d_4, d_6$ . Similarly, when we go there we would have  $d_1$  and  $d_1$  would stretch from  $0$  to  $2T$ . So, this is  $T, 2T, 3T$ , this is  $4T$ , this is  $6T$ . And then we have  $d_1$ , then we have  $d_3$ , then we have  $d_5$ . So,  $d_5$  is here on top and then we have  $d_7$  that is how we have.

So, this we could write it as  $d_Q$ . And similarly you could write this as plus 1, minus 1, plus 1, minus 1. And you could say that you have as if there is a local oscillator which is feeding  $\cos 2\pi f_c t + \pi/4$  given the constellation that we have this is L.O. And it is feeding on the other side a  $\sin 2\pi f_c t + \pi/4$ . And then you are feeding on this

side d I there is a multiplier you are feeding d Q and then you are adding them up the similar diagram we had drawn and this is  $\cos 2\pi f_c t + \theta$ .

So, if we see this signals and this is selecting these four phases and you could mark these four phases with choice of bit sequences 00, 01, 11, 10 for instance let us say. So, when d 0 and d 1 are 00 you would choose this phase and you would keep on changing. So, when you are at a signal interval, which is right. So, we have the full screen covered here. So, at every  $2T$  interval your d 0 and d 1 are both able to choose distinct values. So, if they are both able to choose distinct values. So, from 00, this number could become 01, if d if d I sequence continues to this or it could become 10 or it could become 11.

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So, at every interval, the phase change that can occur could be very large that means, we could now draw a phase change diagram for QPSK. So, if this is our time axis  $T$ , and in this axis, we would like to draw  $s$  of  $t$ , so in the interval  $0$  to  $2T$ , you could get a wave shape which is let us say this right in the next interval. So, here you have  $d_0$  equals to  $0$ ; and  $d_1$  equals to  $1$ . In the next interval, you are going to get  $d_2$  sorry this is equal to  $1$  and the is equal to  $1$ ,  $d_2$  equals to minus  $1$ ,  $d_3$  equals to minus  $1$  and your wave form could take a shape which is looking like this. So, this point is  $4T$ .

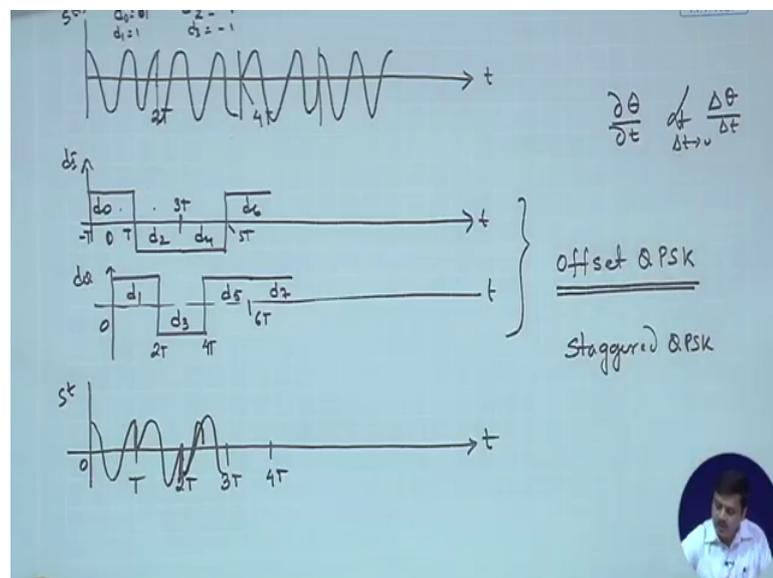
So, what you see over here, there is a huge change in phase that has occurred at this point and then you could get a wave form which looks like this and so on and so forth that means every  $2T$  interval you are going to get one of these wave forms. So, you are going

to have a certain wave form over here, you are going to have another wave form. So, all these different wave forms they are going to differ from each other with a phase of  $\pi$  by 4,  $\pi$  by 2. So, there is a  $\pi$  by 2 phase shift relatively between them. So, you could get you get another wave form.

So, similarly there would be two other wave forms. So, for every two possible inputs you would choose any of the four wave forms that are available with you. So, what could happen at every two interval at every  $2T$  intervals? Your phase of the carrier could change from any point to any point. What is worse is when the phase changes by 180 degree when the phase changes by 180 degree there is a sudden 0 crossing right this causes this may cause your amplitude to go through 0. So, instead of being constant amplitude you could be going through a 0 crossing suddenly and there is a huge phase shift.

And remember we had discussed when we talked about angle modulation, a huge phase shift within a very short duration of time means you have a huge spectral side lobe.

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So, to avoid this, some minor changes could be done in the form that we would now instead of doing this kind of a waveform, we could say why not stagger this or why not shift this a little bit. And if you shift this a little bit the waveform that you would get would look like let say the I channel we draw over here first  $d_0$  then we have a  $d_2$ . Then we have a  $d_4$  and then we have a  $d_6, d_0, d_2, d_4, d_6$ . And the other signal that we have

is a  $d_1$ , yes, and a  $d_3$  or  $d_5$  and a  $d_7$  and this interval is  $T$ , this is  $-T$ , this is  $0$ , this is  $T$ , this is here, there is  $3T$  in between and here there is  $4T$  in between. So, here you have  $6T$ . So, this is your time axis.

So, by looking at both of them, you could figure out what is where. So, here is  $4T$  here there is  $2T$  here of course, we are able to get all the time lines. So, if you see this, what is happening is only one of these two channels this is the I channel and this is the Q channel is changing at interval of  $T$  not both of them change simultaneously. So, when they even changes, the odd continues to remain in the same phase. And here when of course, we have a similar situation here, what we find over here again, the odd one that is the Q channel changes whereas the even continues.

So, what this means is that in this diagram, you are changing either the I bit, so you could identify this as the I bit, and you could identify this as the Q bit. The second bit has the bit for the Q-axis and this for the I-axis. So, what you would see is if one of them changes so that means, if this remains constant only I changes, so I would change from  $0$  to  $1$ . So, there is no problem over here, you are not going through a  $180$  degree phase shift, you are just changing from there to there and; that means, you are going to shift by  $\pi$  by  $2$ .

If now we say that the I remains constant and Q changes, so you could go from  $0$  to this one. So, you are not going through  $0$  not both of them are going through  $0$ . And again if we say that the Q remains constant and I changes you would go there right. So, like that you are going through a change of  $\pi$  by  $2$  and not  $180$  degree phase shift that can happen that the maximum phase shift that can happen is  $\pi$  by  $2$  in this case. And we could draw the wave form in this situation  $s$  of  $t$  at every  $T$  interval we are interested to change. So, space is small, I would rescale let us rescale, so that is why we need to write things back again, this is  $0$ . As if there is one waveform, there is another waveform, then you are going to get from here one waveform and so on and so forth.

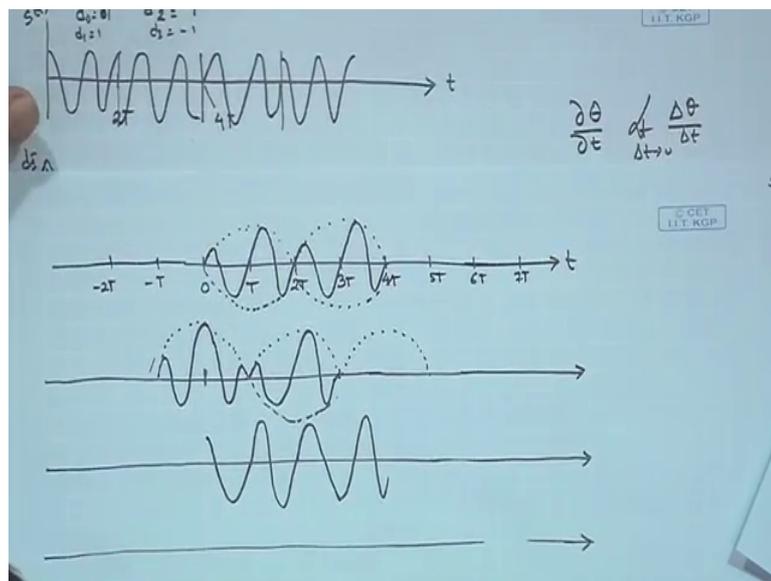
So, all that you are seeing is a phase shift which is here a phase shift which is here an exact picture will be given to you. So, all that we need to note over here is you do not have a huge phase shift. So, because your frequency change is proportional to  $d\theta/dt$ , the amount of change that is happening in the frequency or rather limit  $\Delta t$  tends to zero  $\Delta\theta$  by  $\Delta t$ . So, if this reduces the rate of change of  $\theta$  that means your

frequency side lobes would be different, and that would reduce the amount of spectral growth that may be available. So, this kind of shift is known as offset QPSK.

The advantage of offset QPSK is you are not allowing simultaneously I and Q bits to change thereby avoiding a zero crossing of the signal or maximum 180 degree phase shift of the signal is avoided. And since there is less phase shift per symbol duration therefore, the frequency growth or the side lobes that would come up would be less compared to that of a QPSK.

So, if we would compare this with what we had before with the MSK what we see is that there is a lot of similarity with what we are doing and that MSK or this is also known as staggered QPSK, because your staggering I and Q. The difference between this and the earlier one was if we would look at the expression you would be remembering that there is not this particular one of course, let us look at the different picture you have over here.

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So, if we look at this here also it is staggered and this staggering is expressed by this function. So, the difference between these two is here you have a rectangular pulse shape. So, here it is a rectangular pulse shape, whereas in the earlier case there is a half sinusoid pulse shape. So, the difference here is you still have a pi by 2 phases jump, but here that is completely avoided. So, instead of going for offset QPSK with rectangular pulse you could go for offset QPSK with the half sinusoid pulse which is also known as MSK, but offset QPSK with the rectangular pulse looks appears simpler and it is

straightforward to understand probably decode compared to a half sinusoid representation or the MSK representation.

So, with this we conclude our discussions on modulation techniques. And to summarize now we have covered memory less modulations in the initial parts, and we have also looked at with memory modulations in the later parts. And when we did with memory modulations, we did see that we needed memory to maintain phase continuity, there we studied CFSK continuous phase FSK which we could write it as a special form of continuous phase modulation.

Then we moved on to another special situation for binary FSK with  $h$  equals to 2 and we looked at the expression. And we then moved on to CQPSK and what is the problem with QPSK that there is a huge phase shift that happens at signalling intervals which could be reduced by offsetting the I and Q channels. And then we could compare MSK and offset QPSK, where we found that in one case it could be represented as a rectangular pulse, and the other situation it could be represented as a half sinusoid pulse.

The second case would give you a phase continuity, the previous case would not give you a phase continuity, but it would give you a maximum change between two symbols as  $\pi/2$  which is much better than a pure QPSK which would require a phase change of 180 degrees or  $\pi$ .

Thanks a lot.