Modern Digital Communication Techniques
Prof. Suvra Sekhar Das
G. S. Sanyal School of Telecommunication
Indian Institute of Technology, Kharagpur

Lecture – 19
Characterization of Signals and Systems (Contd.)

Welcome to the lectures on Modern Digital Communication Techniques. In the previous lecture we have discussed a few important relationships relating some functions, important functions and they are Fourier transforms. And we also looked at the some of the important properties of Fourier transform which we are going to use very shortly. Before we proceed just one tiny piece of reference that we need to just make it proper.

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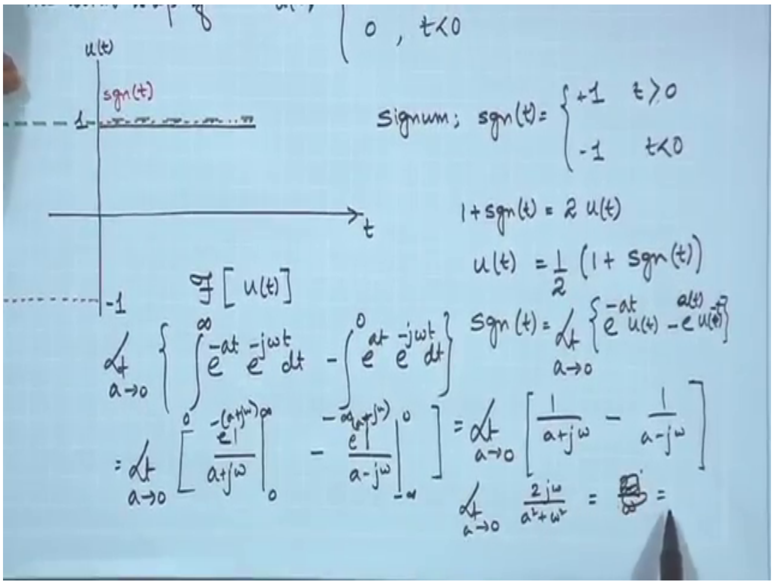

So, when we derived this particular relationship there should be a negative sign because as we take this up, here it is minus j omega minus j omega there should be a negative sign over here, there should be a negative sign over here.

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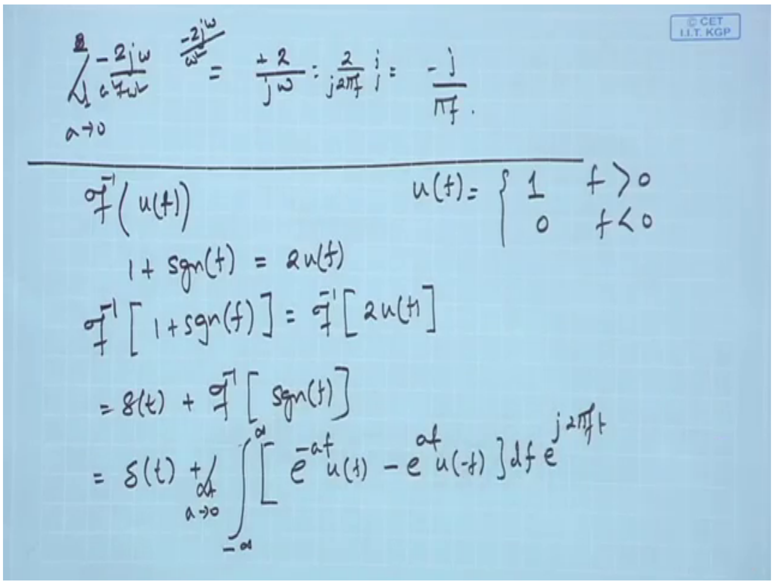

So, as limit a tends to 0, this is 0 and you have minus 2 j omega upon omega squared. So, if you would multiply by j on the numerator and denominator you going to get a j in the denominator and a minus 1, so that will be 2 and omega, and omega squared would reduce to omega in the denominator. And here again back to it multiplied a j on the numerator and denominator. So, that will result here because you have 2 by j 2 pi f. So, this 2 and these 2 cancels out. I again multiplied j, j in the denominator numerator when you get a minus sign here, so that minus sign and this j translates over here. So, that brings us that makes things sets things proper in our relationships.

Now, we will use the relationships that we have derived in the previous lecture, to look at signals that we are going to encounter for proper representation of signals for digital communication system.

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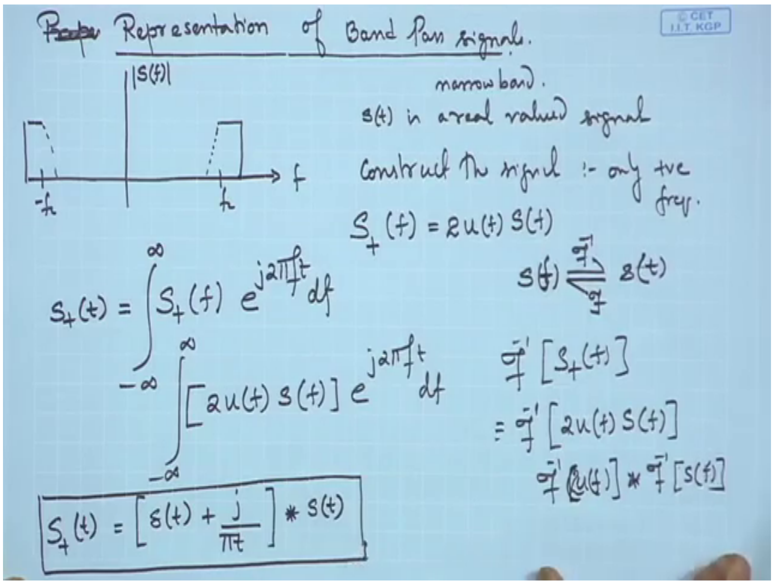

So, now we are interested in representation of band pass signals. So, we are slowly getting into communication systems. Once we are with this few expressions that we set up then we will simply move ahead with those particular things.

So, now, we considered let this be the f axis and let this be the mod of s of f. So, if we are looking at signal which is narrowband. So that means, it is centered around the carrier.

So, beyond the carrier there is no signal and if it is a sideband it was it would look like this. So, we would call this let us say f c as the center frequency and we going to have something like this here, as well on this side this is minus f c, So, s of t is a real valued signal with us and we have seen that for real valued it is even symmetric that we have seen in the previous lecture and that is the representation over here that we have. So, we have also said that if s is real valued then we can use only the positive set of frequencies to represent s of t because, the magnitude is even function and if you are going to look at the phase it is odd function.

So, you can reconstruct the signal only with the positive set of frequencies. So that means, the positive, so you know to reconstruct. So, if you have to construct the signal, you can use only the positive frequencies that is efficient. So, if that is so we could write that s plus of f indicating the signal with the positive set of frequencies as 2 u of f times s of f. U is the unit step s f is s f and 2 is a normalizing constant other justification could be since we are using half of the signal we would increase the signal strength. So, we are using 2 over here and of course, we have the relationship that s of f is related to s of t.

So, s of f is related to s of t via the Fourier relationship, this is the inverse Fourier this is the Fourier transform, this relationship. So, we need to look at what is s plus of t. S plus of t we obtained by taking the inverse Fourier of s of s plus of f. So, to obtain that we take integer integral minus infinity, infinity s plus of f e to the power of j 2 pi f t df, if we have s plus of f we have to replace this by the same term. So, going to have to u of f times s of f and e to the power of j 2 pi f t df, integral minus infinity to infinity.

So, we could also say that since we are doing the inverse Fourier of this. It is the inverse Fourier of this function, so that means, we are saying we are doing inverse Fourier of s plus of f. So, or we could write inverse Fourier of 2 u of f times s of f. If so, we have a convolution this is a constant. So, constant does not affect. So, we have inverse Fourier of u of f convolved with inverse Fourier, so you can keep the 2 of s of f right. And we have actually done this in the previous discussion when we are doing. So, we had established this relationship. That is what we are going to use over here. That is why we did that. So, we are going to use delta t plus j upon pi t. So, we had a 2 and there also we had a 2.

Because 2 u of s is equal to 1 plus sine of f convolved with inverse Fourier of s f is s of t. So, we could write s plus of t by this relationship. So, moving forward if we see what is delta convolved with this function it is basically the same.

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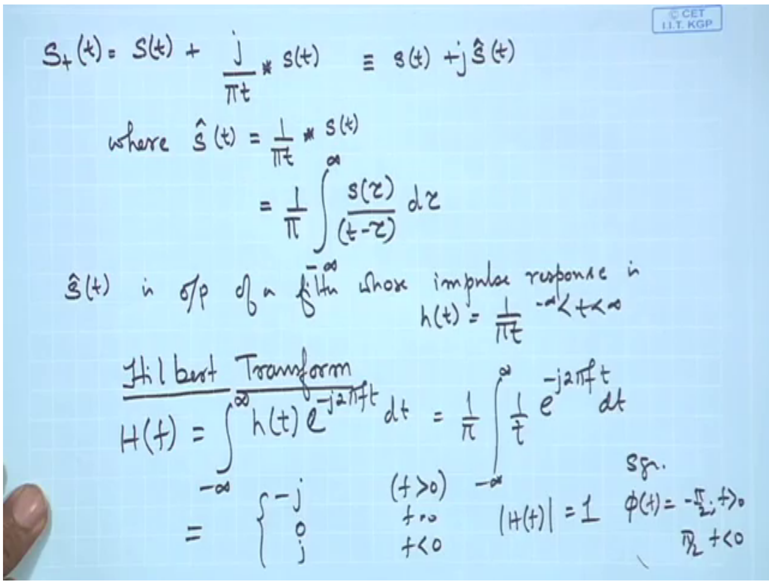

So, it is s of t plus j upon pi t convolved with s of t. So, we could say that let us considered this as represented by s of t plus, s cap of t where you can say s cap of t is 1 upon pi t convolved sorry, plus j is cap of t, convolved with s of t, that you could write as 1 upon pi because pi is away is a constant, minus infinity to infinity s of tau t minus tau d tau. So, what we can say from here is that s cap of t is the output of a filter whose impulse response is given by h of t which is equal to 1 upon pi t for t, ranging from minus infinity to infinity. Such a filter that we have is known as a hilbert transform. And you could recall from our earlier discussions as we just made this would help us, this is 1 upon f.

So, it would hint where it has something to do with u of f, it is something to do with the sine function because we were doing the inverse Fourier of we are doing the Fourier transform of sine function. And when we did that we arrived at an expression which had 1 upon f, So 1 upon pi f rather. So, similar thing is what we are going to encounter here. And here as well, what you can see is this delta function remains as it is and when we take this sine we have 1 upon pi, 1 upon pi t. So, this gives as an hint that when we are dealing with this 1 upon pi t there is something to do with the sine function that will be there in the Fourier domain or the Fourier transform relationship would be like that. So, we can state using our knowledge that h of f is equal to the Fourier transform of h t that is 1 upon pi minus infinity to infinity 1 by t e to the power of minus j 2 pi f t dt.

So, basically we are taking the Fourier transform of 1 by t. So, definitely this will be the sine function, and this you could say it is minus j as we had seen before. For f greater than 0 because there was a j involved if you would recall this again, we had this j involved right. And we started off with this function. So, if you see carefully this particular part we had j involved there. So, that will help us. So, equals to minus j for f greater than 0, it is 0 for f equals to 0 and it is j for f less than 0. And h of f mod value mod value is equal to 1. And phi of f is equal to minus pi by 2, for f greater than 0 and pi by 2 for f less than 0. So, this is the relationship that would help us. So, if we have this we can move forward and we can say that we are, we are interested in the last expression that we have here. We have s plus of t, so we would be interested in writing this particular expression.

Here we have with us, this is this is the relationship that we were working on; this is the last term that we have trying to explore. And simply we have expanded here that this s cap of t is the Hilbert transform which is given by this relationship of a filter, where h of f is minus j for f greater than 0, j for f less than 0 and phase is given by minus pi by 2 and pi by 2 for different conditions of f, whether it is positive or negative values. It is it is a phase rotate you can say.

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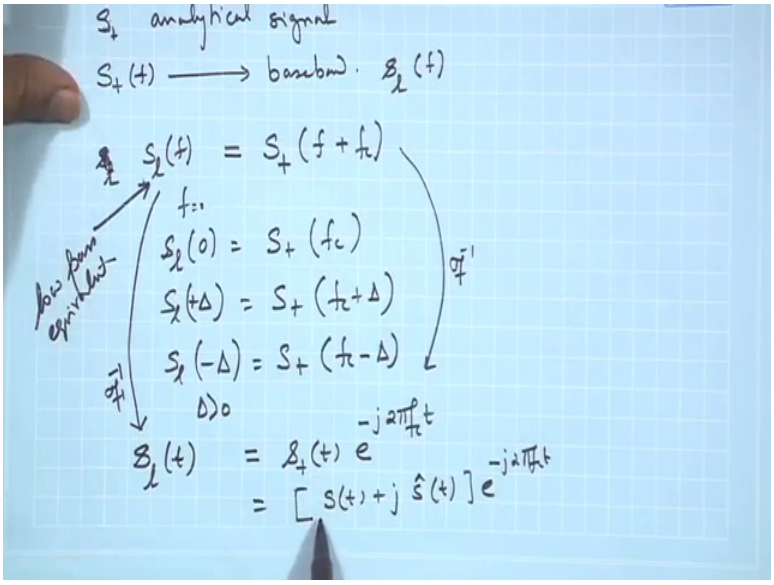

So, what we now have with us is this s plus of t. S plus of t is also called the analytical signal, is also called the analytical signal or the pre envelope.

So, we could say that s plus of f that we have seen could be translated to baseband, you would like to translate to baseband and you would represent it by s l of f that is the low pass equivalent of the positive set of frequencies of s f that is what we are talking about. So, we could write s l of f is equal to s plus of f plus f c this is nothing, but the frequency translation. So, if you would put f is equal to f c we going to get s l of 0 right. So, if f is equal to f c sorry, if f is equal to minus f c even we get s plus of 0 and so on. And so, forth if you put s f equals to 0. So, if I am going to put f equals to 0 I am going to get s l at 0 is equal to s plus at f c. And s l at let us say plus some delta is equal to s plus f c plus delta and so on.

So, basically if we go by our picture here we are talking about bringing this spectrum here right. We are talking about bringing the spectrum here, as given by this relationship. And if you put s l of sum minus delta where delta is a positive quantity you going to get s plus of f c minus delta. So, again if we look at this image then things will be clear as to what is meant by s l of f. It is basically bringing it down here and accordingly we have used this notation l to represent low pass equivalent right. That is that is an important relationship. So, yeah so, that is what is s plus of s l of f. So that means, we could write s, If we have this thing we could say that s l of t because finally, we are represented in the low pass, interested in low pass equivalent of the time domain signal.

This we obtain by getting the inverse Fourier of s l of f. So, that would mean that we are going to get s plus of t inverse Fourier of this, but there is a scaling, and that scaling would mean e to the power of minus j 2 pi f c t. So, this is also, why inverse Fourier. S plus would relate to this thing as well as this would lead to scaling, we have already seen such relationships here. Now we did not give it but of course, that is not a complicated one let me see if we have, we have not given it, but it is evident from our earlier expressions that we have looked at. So, this is the inverse Fourier relationship it is a frequency scaling operation as well as this is the corresponding time domain signal.

So, once we have this, now we can get back to our expression of s plus of t. So, s plus of t is this relationship right, that we have over here. So, we could simply say this is equal to s of t, let us let us see this. I do not know if it is visible, it is probably not visible. S f of t plus j s cap of t e to the power of minus j 2 pi f c t. Simply we have replaced s plus of t this expression into this expression over here and we have got this expression.

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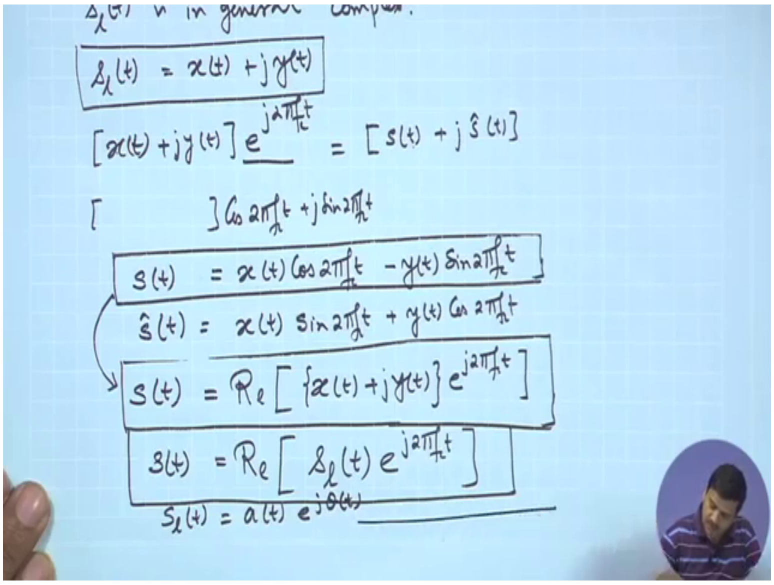

So, what we can see here is that s l of t is in general complex. It is a complex signal in general and then you could write s l of t as x of t plus j y of t. If this is so, then you could equate the terms that we would have x of t plus j y of t if I would take this exponential on this side, e to the power of j 2 pi ft is equal to s of t plus j is cap of t right.

And then you can expand this you are going to get this as cos 2 pi f c t plus j sine 2 pi f c t and you have this term. You want to get x cos 2 pi f c t minus y sine 2 pi ct plus j and j gives the minus sign. So, we will equate this real part to the real part. So, you can get s of t is equal to x of t cos 2 pi f c t minus y of t sine 2 pi f c t. And if we look at the imaginary term y cos 2 pi f c t with the j and x sine 2 pi f c t with a j and we can equate with this term. So, we are going to get s cap of t is equal to x of t sine 2 pi f c t plus y x with sine and y with cos y of t cos 2 pi f c t.

So, what you are seeing is that with the hilbert transform, is the ninety degree phase shifted version of s t that is what we are getting over here. Plus what we see is that s of t which we said is the real valued signal which goes out is of this form, where s l of t is of this form. So that means, we can also write that s of t is equal to real part of from this we can draw, we can write real part of x of t plus j y of t times e to the power of j 2 pi f c t. So, the real part of this is the co sinusoidal, what I would have got is x cos 2 pi f c t which you have said in analog communications. .

And here we have a signal form where we have a signal which is the baseband signal without carrier up converted, with a carrier and the baseband part on the imaginary axis up converted with this. So, this is another important representation of the pass band signal s t in terms of it is low pass equivalent and you could also write this as real part of s l of t e to the power of j 2 pi f c t, which is a very, very desired form. So, these are some of the desired forms which relates the pass band signal to it is low pass equivalent. So that means, if I have a low pass equivalent signal then I could relate to the pass band through this and we have arrived at a formal method of representation of the baseband signal of the pass band signal from it is equivalent chorus equivalent, low pass representation from this part pass band starting by, starting at a point where we said.

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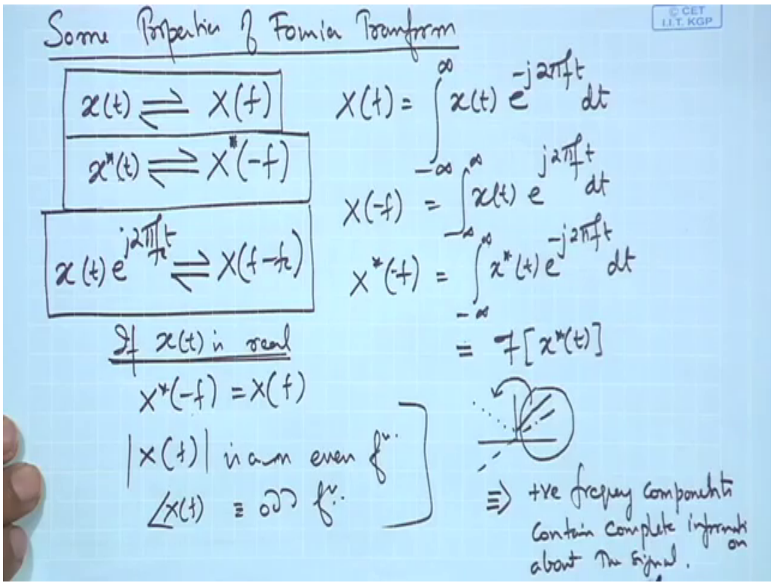

Let s t be a real valued signal and we said earlier based on our food relationships which we had established here, since we said that for real valued signal only positive set of frequencies contain the complete information, you said that let us construct let us take only the positive set of frequencies and represented by s plus of f where these are the Fourier relationships. And we then went to the time domain version, which we call the analytical signal and we represented s plus of f as s f times u of f times 2; 2 because we are taking only half the signal power. So, we need to take twice that to normalize it and. When we derived it we had this we wanted to take the inverse Fourier of s plus of f that is what we did over here, but we broke it down into factors, this factor we had derived earlier this we simply write it as s of t as in here right.

So, we found that it is s t plus j times some convolution with s of t, which we write as the filter one as the hilbert transform with the impulse response 1 upon t which we described. And then we said that s plus of t could be represented as s t plus j, s cap of t and we have slowly realized what is s cap of t through a few relationships as given in this expression. One could also represent s l of t as a of t, e to the power of j theta t.

So, if you do this in that case one could represent in this.

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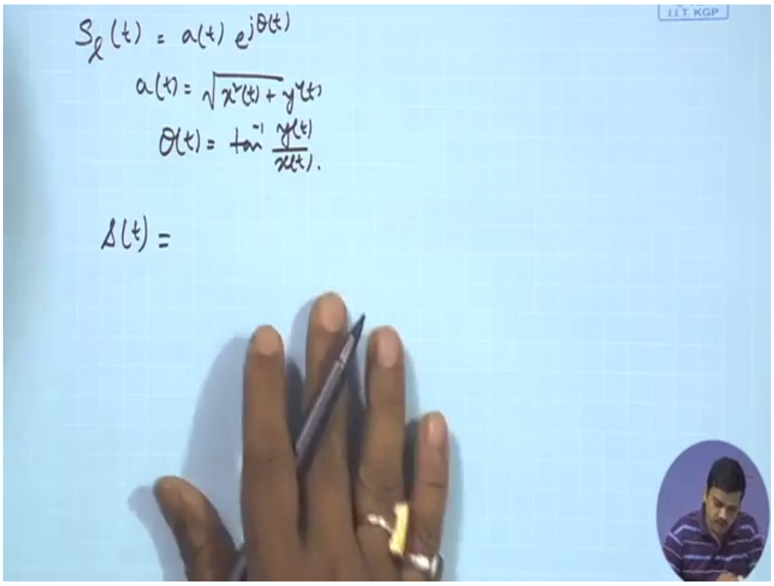

In this case one could represent, one could say at well known result is equal to square root of x square t plus y square t, plus theta t is equal to tan inverse of y t upon x t. So, one could write s t.

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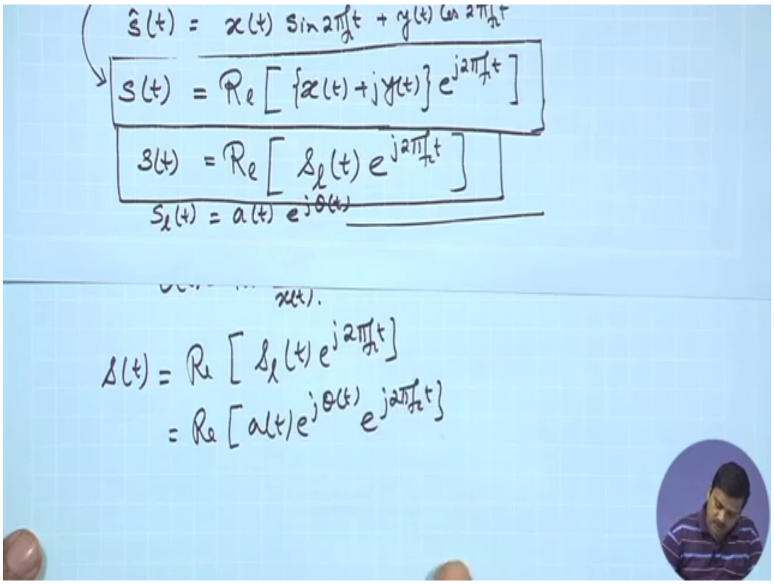

Going by this earlier relationships as real part of s l of t, e to the power of j 2 pi of c t. Which would now write it as at e to the power of j theta t e to the power of j 2 pi f c t, which you could write it as a of t times cos 2 pi f c t plus theta t.

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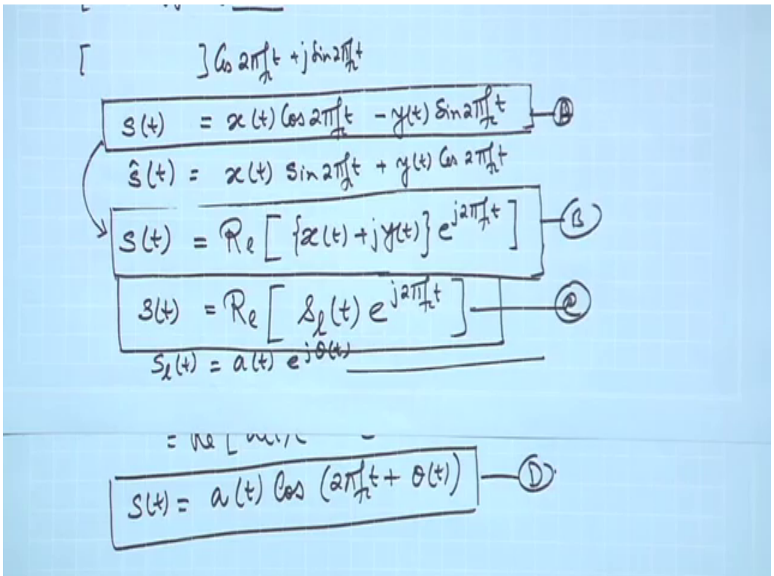

So, these few expressions as we have over here, we can mark this as a, b, c and d are some of the important relationships that we require in representing the low pass equivalent form for a band pass signal.

And with these representations, we can now go ahead and study communication systems. Because whenever will be representing signal will be doing in terms of low pass equivalent form. And the reasons will be even more clear in a few lectures when we will be able to establish, that you could study all the pass band effects in it is equivalent low fa low pass form without the carrier. So, only if you have to translate it to the carrier, you have to do a frequency translation with f c and it will go to the corresponding frequency representation.

So, in the next lecture we are going to look at the low pass equivalent representation of a system and then we will continue to discuss the, the response of band pass system, when excited by a band pass signal with it is equivalent low pass representation form.

Thank you.