Modern Digital Communication Techniques
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Lecture - 15
Analog to Digital Conversion

Welcome to the lectures on Modern Digital Communication Techniques. Till now we have discussed different source coding techniques where we have assumed that there is a discreet source.

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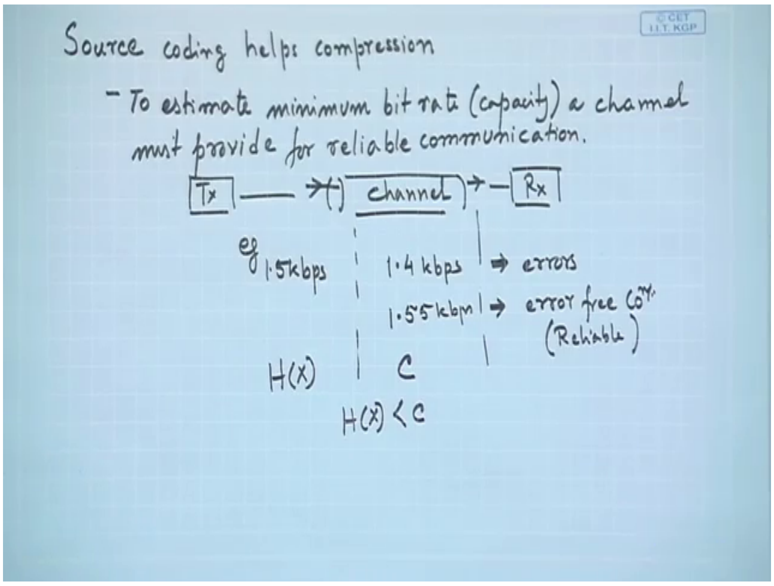

And we have seen various methods by which source can be compressed or the signal rate at which things are coming out from the source can be represented in the better way, and this we have termed as source coding and the reason why we should do source coding we have said before through an example that source coding it helps in compression. So, even today various techniques are been tried so that you get a better output from the source better in the sense lower bit rate.

Why this is important? This is important so that in order to estimate minimum bit rate or the capacity this term might be clear later on the channel must provide for reliable communication. So, when we write this what we actually mean is that we are estimating the rate at which bits can come out from a source. So, suppose I have we have estimated that on an average we require 1.5 bits per source symbolsm and the source generates let us say one thousand symbols per second so; that means, you need to have the rate at which bits come out form the source is 1.5 kilobits per second at this point to have a reliable communication between the source and the destination you would require the link between the transmitter that is the source and the receiver that is the destination. So, there lies the channel between them right. So, this should be able to carry that data rate through this channel.

Now, suppose example the transmitter produces let us say 1.5 kilobits per second whereas, the channel capacity is one point let us say 4 kilobits per second, this implies that there will be errors which will be this implies there will be errors which cannot be reduced; that means, you cannot make it 0. However, if the channel capacity is let us say 1.55 kilobits per second, this implies that one would be able to achieve theoretically at least error free communication by this I mean communication.

So, what it means is that suppose I have a channel for which I know the capacity now when I would like to send some source through this channel I would be tell up priory, whether I will be able to get error free or reliable communication through that link or not right. So, this is one of the important things that we can get when we study source coding. So, and typical number which we can use we can compare is the entropy of the source.

The entropy of the source we have calculated is given in bits per source symbol multiplied by the symbol rate gives us bits per second and we also will get channel capacity in bits per second. So, if we get H x and we compare c of the channel we should have H of x should be less than c of the channel right. If this is not true you going to get errors if this is true then you can establish reliable communication. Now with this we move further and what we now take at this point is with the background of heron law communications that how would one get discreet output from sources which are analogue in nature for example, a voice or the speech or music.

So, example I have this micro phone over here. So, when we are recording the voice signal then how do we get it converted to the discreet source or this analogue source should get converted to the digital discreet source so that we can use the techniques that we have studied before.

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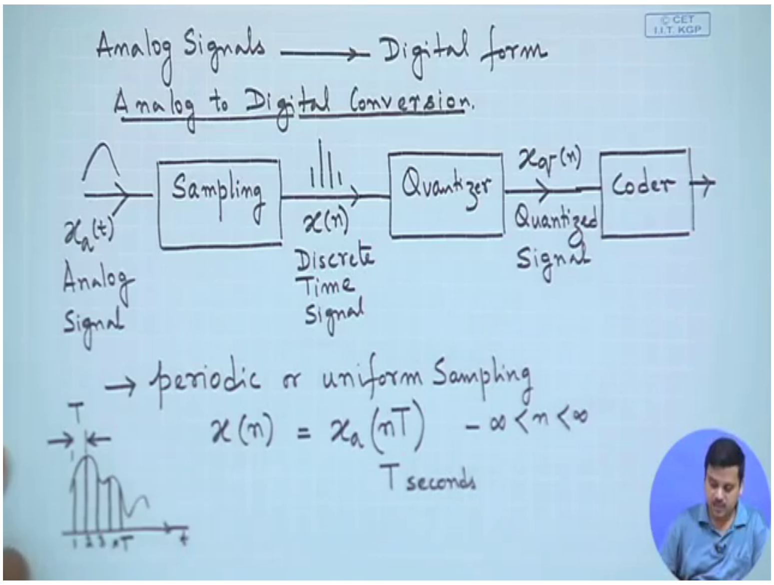

So, to do this we usually convert the analogue signals is first converted to digital form and definitely at the receiver you would like to convert this digital form back to analogue. So, this particular process is usually described by the terminology analogue to digital conversion that is what we are going to discuss in this particular lecture. So, this analogue to digital conversion we have seen a picture before. So, has three steps the first step is the sampling, and the input signal is x analogue of T which we can say is the analogue signal, then this sampled signal which we can represent by x n or the discreet time signal.

So, this is still continues in amplitude, but discreet in time. So, if we have some wave form like this and then we would have to have a Quantizer which will see in the later discussion. So, at the output of the quantizer we have x q of n which is representing q representing quantized. So, you have quantized signal and this quantized signal is then passed to the coder to the encoder, which converts this to the digital signal. So, we will discuss about this particular sampling which takes in analogue signal it samples the signal and it converts it into a discreet time signal.

This analogue to digital conversion or the sampling can be done in many ways. So, the one that we are going to consider is called periodic sampling or it is also called uniform sampling. So, this is what we will restrict ourselves and this can be established by the relationship that x of n that we have over is equal to x a of n of T where n lies in the range minus infinity to plus infinity, and x a of T has been sampled. So, x a of T has been sampled at every interval of T seconds. So, every T seconds we are taking samples of it so; that means, if there is an analogue signal, we are going to take samples at every fixed interval. So, this interval that we are talking about is T in our expression. So, x 1 is x analogue at time instant T x 2 is basically the analogue signal at time instant 2 t.

So, if this is 1 t, 2 t, 3 T and this is time. So, multiplied by T. So, this is the way we get the x n from x a of T.

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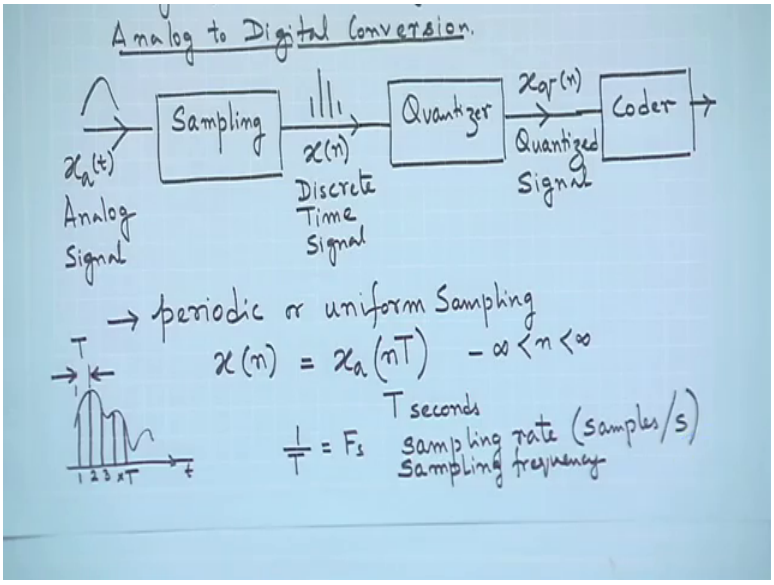

And we also have the relationship that 1 upon T is equal to Fs which is called the sampling rate or samples per second. So, through this we can get the analogue samples the analogue samples, but at discreet time. So, this is also called the sampling frequency just briefly let me tell you at this point that these particular details you can easily capture on a course on digital signal symbol per discreet time processing where one of the initial things that one learns is about the analogue to digital conversion.

So, here we will take a cursory look into the principles which governs this particular operation.

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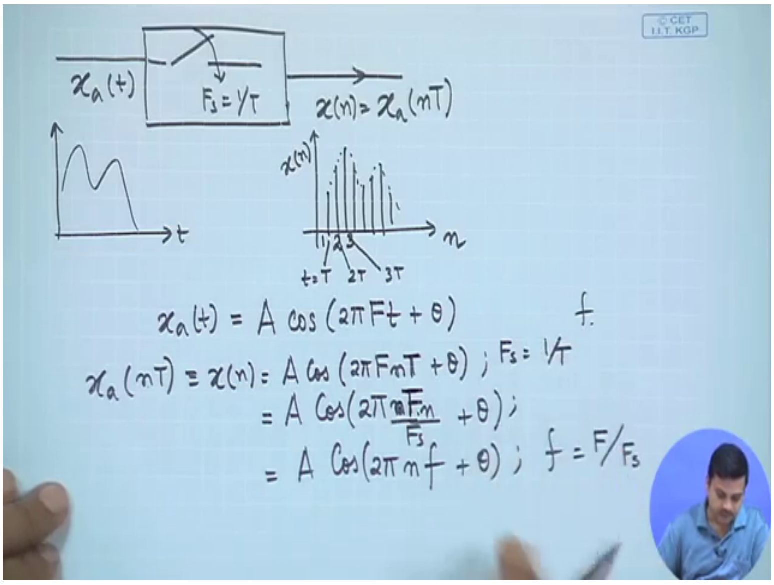

So, for this generally what we can think of is there is this analogue signal x a of t and then there this sampler which is usually represented by a circuit like this; that means, it closes there is r c circuit you can imagine and that can hold It and this switch closes at a rate Fs equals to 1 by T. So, this is typical representation of the sampler and what you get produced is x n which is nothing but x a taken at intervals of time which is integer multiplies of capital T, and the signals that you have here we have already drawn it let us say this is T this is x a of T and here what you get you will have n and you are going to get the same signal, but sampled version let us say with sample at these intervals. So, this is T, this 2 T this is 3 T and so on and so forth or this interval is 1 2 3 and so on. So, this happens at a time when this is equal to T 2 T and so on and so forth and this is usually denoted as x of n.

So, to understand this better let us take x a of t is equal to A cos 2 pi F t plus theta where F is the analogue frequency of the signal. So, this could be also related to the discreet frequency which we will see as f. So, we have to establish the relationship to the discreet frequency F through this kind of a relationship. So, what we see is that since Fs is equal to 1 upon T what we could write is that x a at n T is basically x of n which is A cos 2 pi F. So, at n T. So, T gets replaced by n T plus theta now since Fs is equal to 1 by T this you get as A cos 2 pi will replace f n T remains n T as sorry we will replace T I will replace T by F times n upon Fs or by Fs is the ratio that we have. So, what we usually write this as further a cos 2 pi and for convenience you would write it as n f plus theta where f is equal to F by F s.

Now, this is the important relationship that we have between x a of T and x n of T related through Fs and f and one upon n t.

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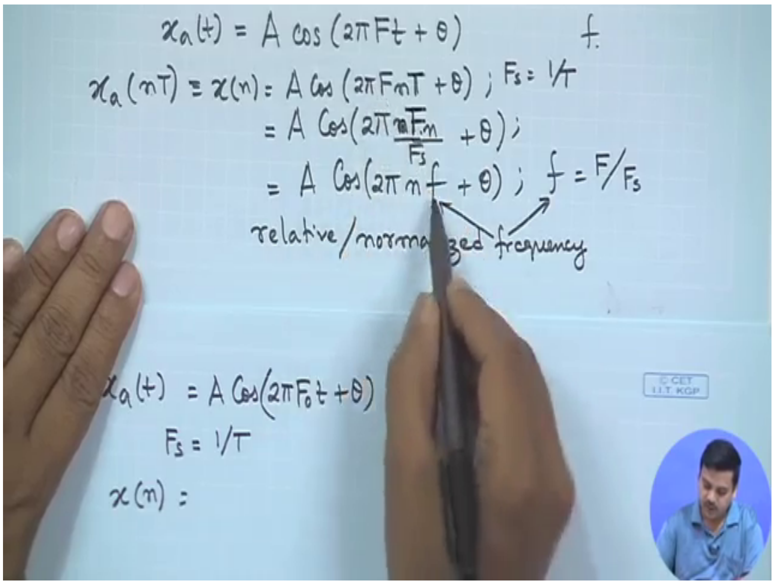

So, f that we have over here is this f that is used over here in this 2 expression is known as that relative or the normalized frequency; this is clear because you have capital F over small f. So, this is this f is relative to this particular sampling frequency now since these are ratio what you can guess from this is that for certain values of f which less than Fs we have this f lying between to be less than one and when f is more than Fs we get the value of f which is greater than one. So, because of this kind of relationship we have certain constraints on the values that f and Fs can take and that will be clear very soon.

So, suppose at this point we say that x a of T is equal to A cos 2 pi F 0 of T plus theta; that means, I have taken a particular value of frequency instead of the generic value that we have taken here. See if we take this as 2 pi f 0 T and we decide that Fs is equal to 1 by T we can easily write that x n using our previous relationship.

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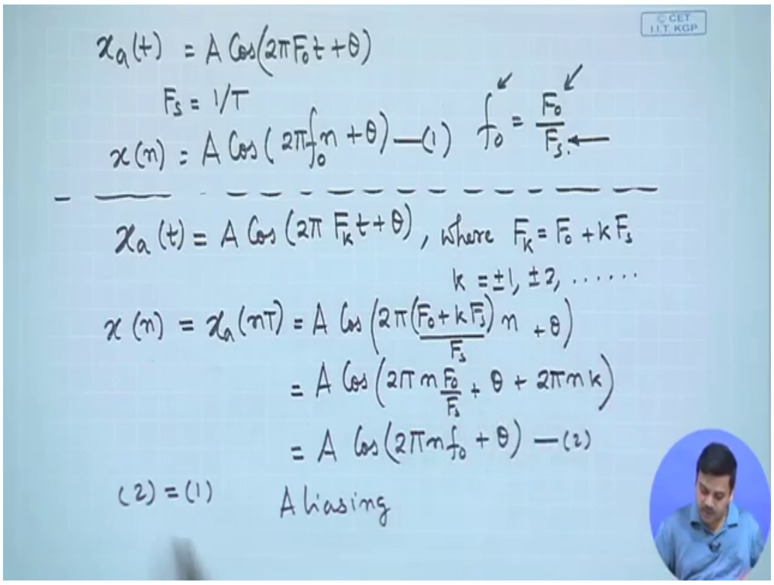

That means, the relationship that we have over here. So, since f is replaced by f 0 our small f would also get replaced by small f 0 A cos 2 pi small Fs 0 n plus theta. So, what we see is that there is lot of similarity in this relationships in this three expressions; however, T is getting replaced by the index n which is here on the left hand side and capital f 0 which is inherits is getting replaced by f 0 which is in the in the digital frequency domain and f 0 is equal to F 0 upon Fs.

Now, at this it is interesting to note is that if f 0 is. So, basically we can find f 0 given small f 0 if we know this particular sampling frequency; however, there is little issue with this particular description is that suppose instead of taking this particular signal we take another signal or generic family of signals which we can write x a of T is equal to cos 2 pi f k of T plus theta where we define F k to be equal to F 0 plus k times Fs. Now suppose I take this particular signal instead of taking F 0, so, where f k is F 0 plus and integer multiple of Fs. So, of course, I should mention that k is equal to plus minus 1 plus minus 2 and so on and so forth So, what would happen as a consequence of this? As a consequence of this you are going to get x n is equal to x a of n T which is equal to a cos 2 pi f k I am replacing by this expression.

F k I am replacing by this expression and T of course, is replaced by n upon Fs because T is equal to n times capital T and capital T is equal to one upon Fs plus theta. So, these results in a cos 2 pi n f 0 upon Fs plus theta plus let us look at this 2 pi n we have 2 pi we have n and k because Fs and Fs cancels out. So, this is what remains which is equal to A cos 2 pi n f 0 plus theta. Now if you look at this expression and if you look at this expression if I mark this as let us say one and if mark this as 2, 2 is equal to one because this is a 2 pi n k n is an integer and k is an integer. So, we have 2 multiple of 2 pi.

So, since we have multiple of 2 pi cos of theta plus 2 pi cos is basically theta. So, you have this expression that means, you cannot distinguish between this sequence that we generate at this point so; that means, if we recall this particular picture we have x a of T here and we have x n over here. So, all we have established is if I have fixed a sampling frequency let us say Fs then there could be multitude of different analogue signals which could become the same discreet signal.

So, this kind of a situation is usually known as aliasing because x n is an alias of many different sinusoids one of them could be F 0 and others could be F 0 plus integer times this sampling frequency. So, your unable to distinguish between one signal and another signal which could be of different centre frequencies. In the reason for this is quite clear from the expression which is here which basically establishes that f which is the digital frequency is basically ratio of F times F s.

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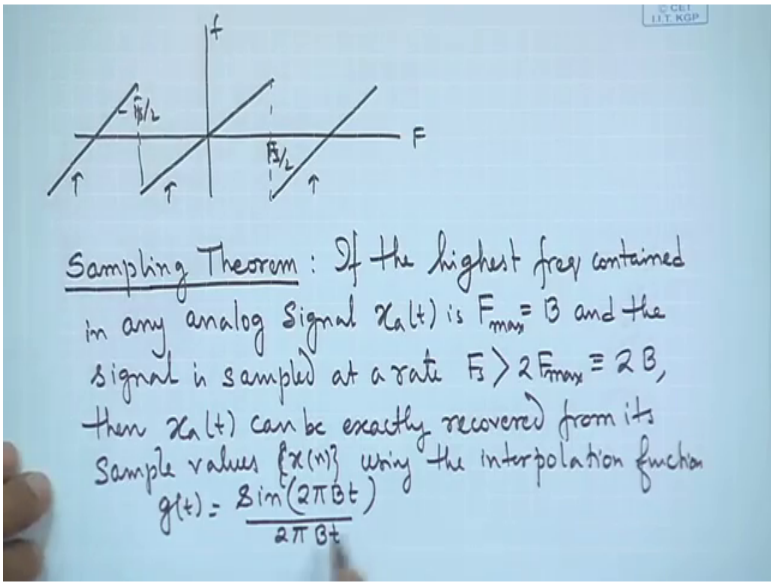

And since we are aware that if you try to map the analogue frequency to the digital frequency what you are going to get is this is a linear relationship from capital F to small f for Fs by 2 to minus Fs by t 2, but there after it repeats there after it raps around. So, because of this we are unable to distinguish between the frequency here and the frequency here and the frequency here.

So, at this point what we get is the important notion of the sampling theorem, this are very important theorem which comes from the abode description what it says is that if the highest frequency contained in any analogue signal x a of t is f max which is equal to b and signal is sampled at a rate Fs which is greater than twice f max which is equal to 2 B which can be defined as 2 B then x a of t can be exactly recovered from its sample values that is x n using the interpolation function g of t is equal to sin of 2 pi B T upon 2 pi B T so; that means, that if we have x a of t.

So, we can use this particular sheet again which tells that what we have see over here is x n is identical for different values of capital f k.

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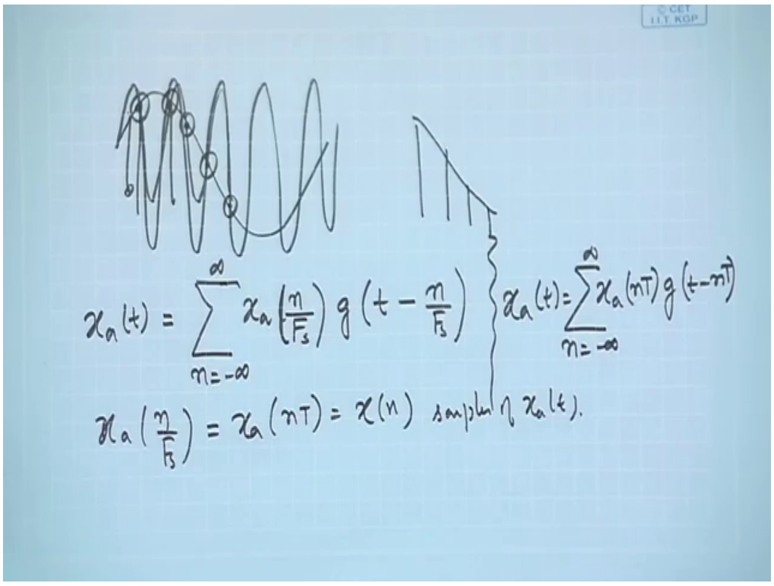

So that means, if we have signal which goes like this and suppose we have another signal sorry I should like full swing like this which is an integer multiple of this original signal in that case the samples. That It produces one will not be able to reconstruct the original signal one will not be able to reconstruct the original signal because all of them mapped to the same discreet sequence if you look at it. So, they all mapped to the same discreet sequence.

However what it says is that if you know the maximum frequency content as F max so; that means, if we go back to the signal if I know the maximum value of F k which is present in this signal then we should choose our sampling frequency Fs which is 2 times the maximum sampling frequency and if which is more than 2 times and if you are selecting your sampling frequency this more than 2 times the maximum frequency then you can reconstruct x a of t, x a t. That means, the analogue signal exactly without alias and that you can reconstruct using g T which is expressed by sin of 2 pi B B which is equal to f max upon 2 pi b t in in this case x a of T that is the analogue signal that you have can be expressed as n is equal to minus infinity to plus infinity, x a of n upon Fs because Fs is Fs one upon Fs is t.

So, basically x a of n T times g of t minus n upon Fs again you can instead of this you can look at it as x a of t is equal to sum of n equals to minus infinity to plus infinity x a of n T g of Ttminus n t. So, of course, we have said that x a of n by Fs you should write as x a of n T which is equal to x a of n which are samples of x a of t.

So, in this way what we have established is that if you are sampling a signal to convert it from analogue form to a digital form, then one should choose a sampling frequency which is at least twice or more than 2 times the sampling frequency 2 times the maximum frequency. So, if you are doing it then you would avoid aliasing what we did not in this particular discussion is that we did not prove that more than 2 times is sufficient to reconstruct a signal exactly without aliasing we will not take it up as part of this because that is usually handled in part of discreet time signal processing or digital signal processing communications.

In the next lecture we are going to discuss about how to take these discreet time signals; that means, we have signals at this point now we have the signal at this particular point. So, we have been able to do this. So, what we have said is suppose I know the frequency content; that means, for this if I know the frequency content or I know the F max then I will choose Fs as greater than twice f max then I can generate this signal and this signals will not be aliased and this thing comes from the sampling theorem so that we can reconstruct it back to its original form which is required at the receive at the receiver side you will require this; that means, at this end you are going to require this.

So, basically what we have ended up is why we need at least this kind of relationship otherwise we are going to end up in aliasing and now since we have got this discreet time signals which occur at discreet intervals, but take continues values in amplitude we should now discuss about how to quantize this or what is the meaning of quantizer so that we get a sequence which is a binary sequence which can finally go into the encoder. And some of the encoders we have actually discussed in our earlier lecture.

Thank you.