Spread Spectrum Communications and Jamming
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Lecture – 09
Spread Sequences and Waveforms

Hello students, today we will start discussing about the spreading sequences and waveforms. We have understood in the last few modules that in spread spectrum communication systems, the heart of the system design is the spreading sequence design. So, this our typical module is not dedicated on the design of this, but we will take an example of the most used some of the spreading sequences. And we will look into the generation mechanism all those sequences, and we will try to also observe the typical properties of those sequences and the waveforms.

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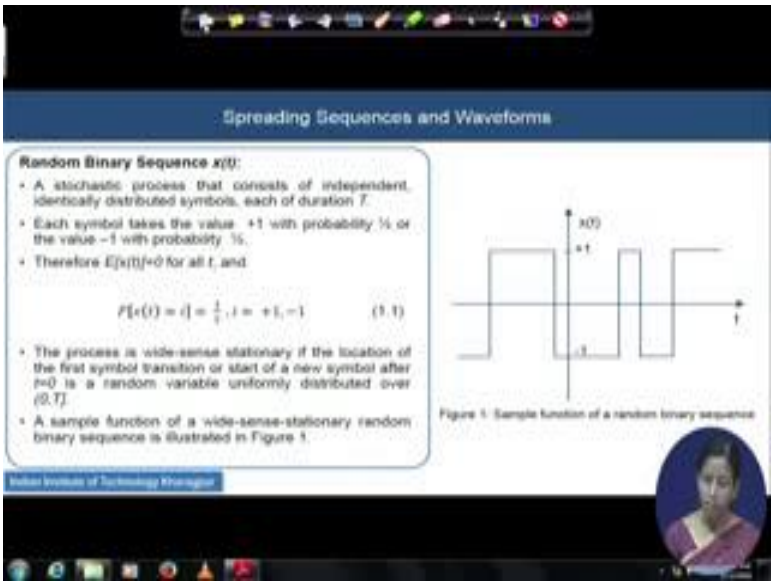

We will start with the random binary sequence. Random binary sequence is the most popular kind of the sequence from where almost all the spreading sequences, which are used these days in practice they are generated form. So, it is very essential to understand the property of the random binary sequence. The sequence here we are mentioning let it be x t, the random binary sequence this x t, it is basically a stochastic process where this process is consisting of several independent and identically distributed symbols. Each of this symbols we will see they are having a duration of capital T. So, as they are at the symbols the typical value of each symbol either can be plus 1 or it can be minus 1.

So, hence as it is a random process, so occurrence of this symbols will be random and hence their occurrence will be related with their probability. Probability that the symbol value will be plus 1 or the symbol value will be minus 1 having equal. As the probability is equal hence they can be equated is equal to half. For this equal probability, we can write down the expression for this probability as a x t to be equal to i, i is either value will be plus 1 or equal to minus 1, this probability value will be equal to half as explained.

Hence if I draw the sample function of this random binary sequence of x t here in figure 1 with respect to the time axis t, the values are the value will be plus 1 or minus 1 and that figure will look like this. The x t, now we understand that the duration of this typical each of each and every symbol of plus 1 as in the minus 1 is equal to capital T. As we are seeing that the probability of occurrence is also half, and the values are either plus 1 or minus 1, so average over the time we will get the expected value or the mean value of x t is equal to 0.

The process can be wide-sense stationary process. In communication system, mostly we are interested in this wide-sense stationary of any process and hence we would like to know what is the meaning of this wide-sense stationary process under what condition we can declare a stochastic process to be a wide-sense stationary. We write that wide-sense stationary process as WSS process. This wide-sense stationarity demands that if the arrival of the first symbol or any new symbol after t equal to 0, the arrival of any new symbol after t equal to 0, if it is obviously it is random, but that random value variable the occurrence of this new symbol which is random variable. If that occurrence is uniformly distributed over an interval open interval of 0 to capital T, then we can declare that the process is wide-sense stationary.

I repeat to declare that typical stationary stochastic process x t to be wide-sense stationary, you need to look that either the first symbol of that x t or any new symbol after t equal to 0, which is a random variable and the occurrence of this random variable is uniformly distributed over an open interval of 0 to the T. T is the duration of that symbol. If it holds good, then you can declare the process to be wide-sense stationary process.

Why we are talking about this wide-sense stationary process is we will it will be relevant to once we will declare, we will go through about the auto correlation property of the spreading sequence. Why we are interested in the auto correlation and cross correlation property of a sequence is we will learn the importance of these two property of the spreading sequence design, once we will discuss about the usage of these two properties in view of the communication system design.

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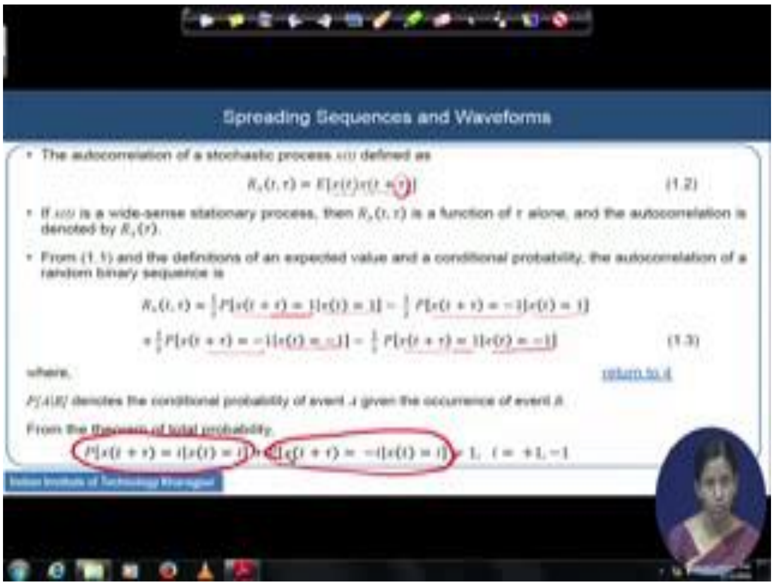

In the next slide, let us first enter into the auto correlation property. Remember, whenever we are spreading some signal in the transmitter with the main with the help of a spreading sequence, and in the receiver, we are trying to extract the transmitted information by mean of this spreading. We usually try to choose the auto correlation property of a sequence to be extremely good, which means the maximum power of this transmitted signal should be confined with the mean log of the auto correlation. And the side peaks or the side lobes will be typically extremely low compared to the mean peak value.

This is the typical demand for the auto correlation property of any spreading sequence, then those kind of the sequences are most preferable in kind of our system design. Only, due to the fact that - they give actually the very good demodulation as we dispreading aspect at the end of the dispreading you will get good difference and between the dispread signal with respect to the jamming one. And your signal energy would not be spread much on the side lobes.

Similarly, the cross correlation properties are very important in the system design, because a cross correlation property ensures if the cross correlation side lobes if the cross correlation property is perfectly zero that means, that means those two codes are perfectly orthogonal, and those codes can be nicely used for the users - different users, multiuser scenario. And in a single user scenario also the cross correlation property is ensure that you would not get the much interference from the similar kind of the code sets. Which are may be utilized for enhancing the data rate of the same user. So, wherever you are, either in a single user trans mission scenario or in a multi user transmission scenario, the auto correlation as well as the cross correlation property of the codes are really very, very important issues where we should look into.

So, it is really hard in practice to get or to design a code, where the auto correlation property and the cross correlation property as simultaneously good. I mean to say the auto correlation property is very high perfectly nice with a very low power in the side lobs. And simultaneously the cross correlation property are really very 0, really very low with close to the 0 values. So, getting such a code is really hard in practice we will see later on either you will get a code, we will having a very good auto correlation property and very less cross correlation or the vice versa.

Depending upon the user's scenario, we keep on changing the codes also that wherever the good auto correlation is required, we choose a several set of the codes who are having good auto correlation property. And for a scenario of a multi users case or a high delay spread wireless channel, where we need to restrict against the inter symbol interference or inter carrier interferences, we will prefer to use a set of the codes who are having very good cross correlation properties.

Let us start understanding about the auto correlation of a stochastic process. Defined process x t the auto correlation property are x t tau will be given by the expected value of x t with this delayed version x t plus tau, where tau is the delay. So, we have a process and it is delayed version of the process, we are taking the mean of it - exhibited value of the x t into x t plus tau. We understood what is the meaning of wide-sense stationary process if x t is supposed to be a wide-sense stationary process, then this auto correlation function will be a function of only tau, you need not write t comma tau I can directly write it R x tau.

Now, as we understand that x t is having equal probability to achieve the value of plus 1 or minus 1; that means, symbols with the plus 1 value or symbol with the minus 1 value are equally probable to happen. Hence this autocorrelation value we can compute like the equation 1.3 in this way. We understand that it will be the combination of half of the probability that x t was transmitted 1, and we are receiving that delayed the version of that x transmitted x t equal to 1. From there, we have to subtract the probability half of the times the probability that I transmitted x t equal to 1, but I received the delayed replica to be minus 1. Also there is a probability half of the times that the probability is that I transmitted minus 1 and I received minus 1 delayed version is. And subtracted from that there is a probability that I transmitted minus 1, but wrongly I received plus 1.

So, for each correct probabilities, these are the correct probabilities of transmission of receiving correct transmitted symbol and these are the probabilities of the arrays. So, the total probability of all these is a auto correlation value that is defined. Now, remember that this probability x t plus tau given x t, these are the conditional probability were actually the probability of event a given the occurrence of b is called. From the theorem of the probability theory, we understand the total probability that whatever you have transmitted and you have received the correct one that means, the probability of the correct reception plus the probability of erroneous reception, the total probability are expected to be equal to 1. Here, in our case this, i will can be either plus 1 or minus 1. So, the first part is same the probability of the correct reception, and this is the probability of getting wrong reception. So, total probability of correct plus the wrong reception, it should be equal to1.

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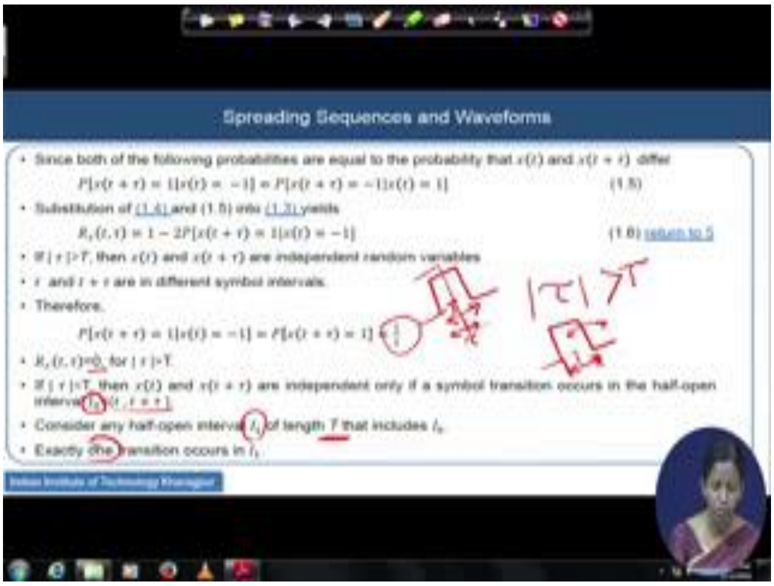

From this understanding, if we now have the both equal probabilities for x t and x t tau t plus tau, we understand that they will differ. I mean the both the following probabilities this probability that there is an erroneous reception because of minus 1 transmitted and received plus 1, or plus 1 transmitted received minus 1 both the probabilities are equal. Using this fact if I substitute this value into the equation what we saw earlier here. So, what I am substituting, I am substituting this probability is equal to probability of this is equal to probability of this. And also I am using the equation 1.4, if I substitute all this here in the R x tau expression, then I will converge to this equation 1.6, where the auto correlation value will be given by 1 minus 2 P x t plus tau 1 given x t minus 1. This, do this derivation by yourself and try to see whether you are converging here or not. It is the very easy substitution that you need to do to converge to this equation 1.6.

Now, there are several possibilities to occur in this situation. Let us consider a situation where the delay, let us consider a situation where this delay is happening in such a way that the delay tau wherever it is in the positive direction delay or the negative direction delay for whatever backed delay, but the mod value of the tau is becoming greater than the duration of the symbol capital T. If it happens, if it happens then it is true that your signal received signal if this is my duration of the transmission of a symbol and the delay is such that the delay it has delayed more than this period, basically the new symbol you are receiving a new completely new symbol, and there is no overlapping the existing symbol with the next received symbol.

So, what is should I say then that is no auto correlation kind of staff at because the time there are the time differences are there completely. And hence the auto correlation function for this kind of x t and x t plus tau will boiled down to 0. And what you will see that in such a situation, when tau is greater than capital T the probability that you are receiving an erroneous one, erroneous reception that is basically equal to the probability that you are having the x t plus tau is equal to correct reception. So, this probability will be basically given by half, and fundamentally there is no relation now between x t and x t power p plus tau because they are completely occurring in the two different timing interval and there is no common area between them. And if you to compute auto correlation which is basically finding out how much the two waveforms are related to each other will give 0, because there is no relation over the time of the interest.

Now, what will happen, if they value of this tau is less than capital T. When it is less than capital T, then your x t and x t plus tau, they are in can be independent. If and only if such something like this happen this was the time period of the capital T. And you are having a delay tau which is less than the capital T, but still I am saying that the x t transmitted symbol and T plus tau which is the delayed received symbol they are not same; it may happen only if there are some transition occurs. I mean the way the delay has happened by that time the transmitted symbol has got a transition like this. So, it was delayed; the delay was less than capital T, but it was delayed in such a fashion that by the time, there was a transition always the transition already the transition has happened for the symbol period. So, then in that situation x t and x t plus tau are again uncorrelated.

Let us think a situation, where the symbol transition occurs in the half open interval of I 0 where I 0 is given by t to t plus tau. And we are considering another half open interval I 1 who is considering who includes this I 0 and who is having a length of T. And suppose there is exactly only one transition occurring within that I 1.

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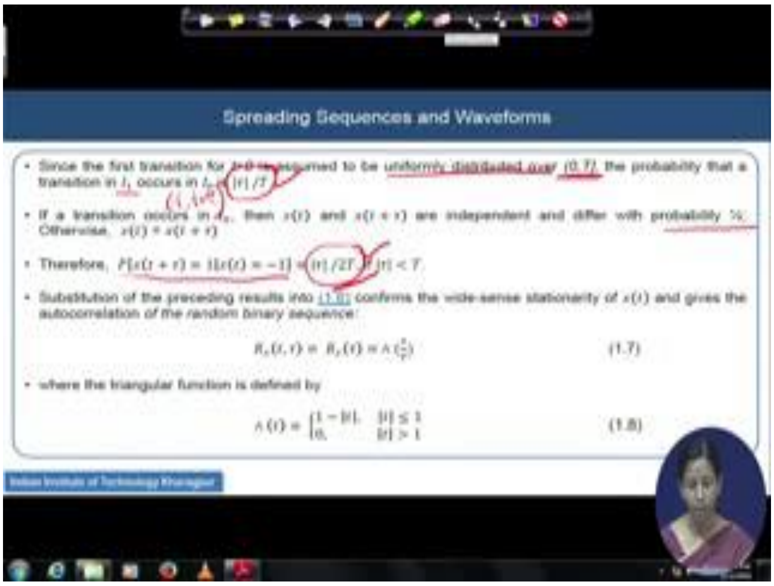

In such a situation, since the first transition for T greater than 0, we have already assumed that at will be uniformly distributed over this half open interval because it is a wide-sense stationary process. And the probability that the transition occurring inside I 1 is definitely which will be definitely in I 0. The transition is single transition occurring and that single transition that is occurring within I 1, what is the probability that it occurs within I 0 where the I 0 is having an open interval of t comma t plus tau, this probability will be given by mod tau divided by capital T.

Now, if the transition that is occurring in I 0 where actually x t and x t plus tau are independent. And they having a different they are hence if they are independent they will be considered as the different signal, and then probability will be equal to half. And therefore, the total probability that you will have this kind of the scenario, the next scenario that will be the probability that it will be having erroneous reception will be definitely governed by mod t by half of that, because it is getting multiplied with this probability of half.

So, mod t by tau getting multiplied with the probability half is giving you the mod tau divided by 2 of capital T, when their mod tau is less than T. So, this is the situation when actually your transition has happened and the transition is not within I 0; if it is within I 0, you are ending up with a probability here; if it is not exactly within I 0, it is beyond that. So, x t and x theta plus tau hence are becoming independent and hence therefore, the probability of that situation to occur is coming like this.

Now, if I substitute slowly all these observation and the derivation into the equation, where the auto correlation was defined this equation 1 minus 2 P into x t plus tau. If I go back and define all that then were I will be ending up, I will be ending up with the equation of a triangular function. So, we understand that the substitution confirms that we are dealing with the wide-sense stationary process x t, and the auto correlation for these wide-sense stationary process x t will boiled down t to a triangular function. How we are reaching here we are reaching here by the substitution of all this understanding. And we are substituting the value of those probabilities that we are talking about and finally, we are ending up this. But this triangular function will be given by 1 minus mod t, when this mod t here it is tau by capital T is less than equal to 1; and it will give a 0 value, if the tau by t is greater than equal to 1.

So, finally, the main conclusion from this discussion is that if you are dealing with the random binary process, which is a basically stochastic process, and it is also confirming the property of a wide-sense stationarity. And in such a way that it is either is first symbol occurrence or the new symbol occurrence after t is equal to 0 is a random variable and that random variable is following a uniform distribution within an open interval of 0 comma T the which is where T is the symbol duration. Then we will end up with our auto correlation function where the auto correlation function will be basically a triangular function. This observation and this derivation we will utilize for a generation of a several sequences, and mainly the maximum length sequence or m l sequence. We will see that this triangular property how this triangular property is really very useful in case of any kind of the further analysis.

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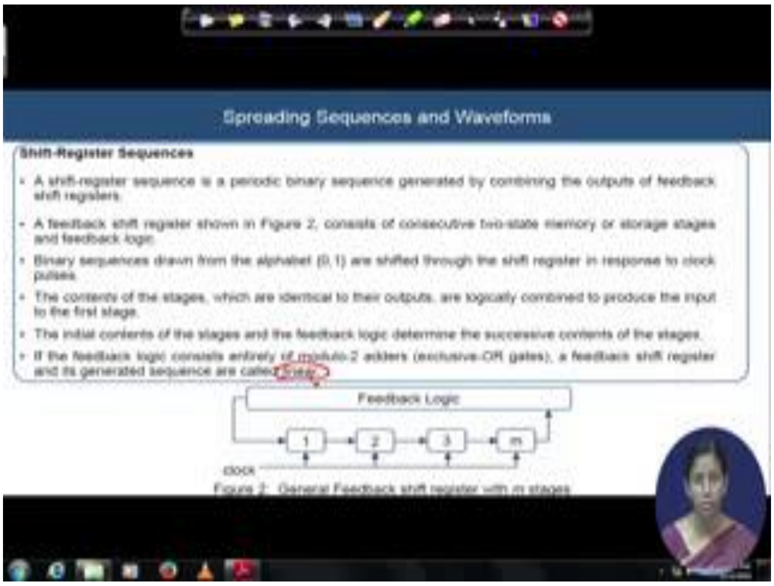

Now, true sense actually this random binary sequences are very important sequence for us. And given the chance everybody would like to utilize this random binary sequence for the system design because of it is good property it is nice auto correlation function. But problem is what in any practical system design, we would like to for synchronization purposes. We understood that for direct sequence spread spectrum as well as for frequency-hopping spread spectrum. Timing phase and frequency synchronization will be very, very critical because you are hopping involved in the frequency-hopping spread spectrum and direct sequence cases, you have to be well aligned in time with the incoming signal; otherwise it will be very hard to de-spread your signal and demodulate your signal.

We did all the analysis with the assumption that for synchronization is perfect in order to extract this information in practice in the receiver, we need actually the basic requirement is that you cannot send a random binary sequence like the way we have just now explained and discussed, we need actually some periodic waveform. So, that actually the measurements can be done over the several periods and decision can be done over the estimate of the offset values over the multiple periods.

So, once we are declaring and demanding some period x sequence, the concept of shift-register sequences coming to picture. Shift-register sequences, they are basically a binary sequence which we generate by combining the outputs of the feedback shift-registers.

We are showing here one general feedback shift-register, where if you see the structure, in the structure there will be different n number of the stages of the memories which are the two-state memory. Basically they can store if you are following the Galois field logic, Galois field logic two and then the filled values will be either in the Galois field two there will be two elements only either the value will be 0 or it will be 1.

So, all these memories can store either the value zero or the value one. And the output of this memory and stages they are combined logically. And the output of this logic circuit will be feedback as a input of a first memory of the first stage. And clock will control the proceeding of this values from the stage to stage. And we understand that if the feedback logic consist entirely of modulo-2 or exclusive-OR gates, if the logic is designed by only exclusive-OR gate then this kind of the feedback logic and the feedback shift-register will be called a linear shift-register. Why it is very important to understand is we will also declare some kind of the non-linear shift-register. And there are several advantages disadvantages between linear and non-linear shift-register, which will be discussed later. Currently, we will understand that this is the structure of a linear shift-register with exclusive-OR gates.

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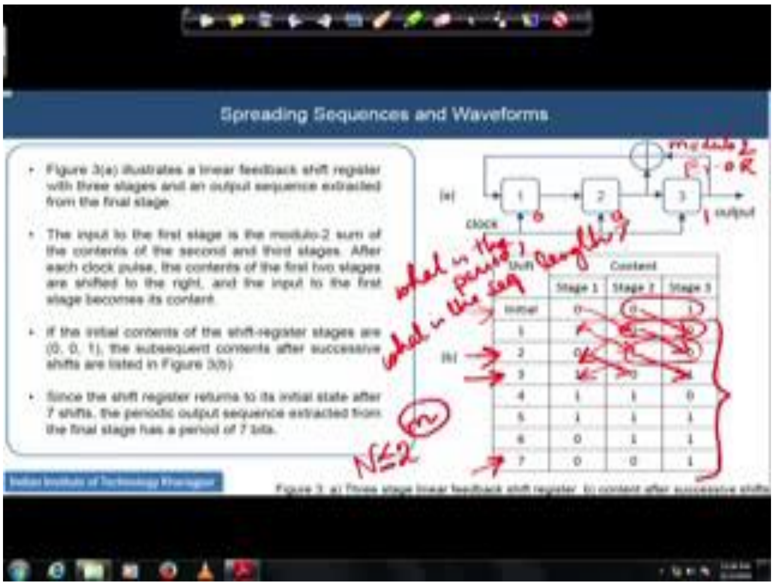

So, this is very simple example of generation of this kind of the sequence shift-register through the shift-register how to generate the spreading sequence. The example consists of only three stages the output of the stage number two and stage number three is added by modulo-2 operation which is basically an XOR operation going on. And the output of this XOR is feedback as the input to the first stage. When the clock is arriving actually the stage of the when the first clock pulse will arrive, the stages of the two and stages of one the value of stored in stages 2 and stages 1, they will be shifted to stage 2 and stage 1 respectively. And the last value whatever was stored in stage 2 and stage 3 will be combined, will be the input for the XOR gate and the output of that we will come back as a feedback to the stage 1. Remember all kind of such kind of shift is a sequences, they need to be stored with some or initialized with some initial value.

Let us take an example that we have initialized this architecture of the structure with the value of say 0 0 1, which is a initialization value stored, I am showed in this table. On the arrival of first clock pulse, what will arrive what will happen as I explained that the value stored in stage 2 and stage 1 will be shifted. So, you see the value of the stage 2 came to the stage 3, and the value of stage 1 came to the stage 2, and the input to this 0 of the output of the stage 2 and stage 3, the last clock whatever was stored in the last during the last clock pulse, they are XORed. And actually the XOR value will be is 1, so this XOR combination is now feedback and stored as a input to the stage 1.

When the second that was arrival of the first clock pulse, when the second clock pulse came now you are having the value 0 in the stage 2, so it is shifted to stage 3, the value of the stage 1 is shifted stage 2 and this 0 0, the XOR values of the both the 0s, it yields 0. So, now, it is stored here. This way it is going on the once more and the arrival of the clock pulse three we will be seen the same kind of information this will be followed here, and the 0th pulse will be coming back and 1 and 0 XOR combination of this two yielding one and getting stored in the first stage.

So, this way actually the slowly the value stored in 1, we requires actually it will be shifted slowly from one stage to the last stage. And the output is generated from the output of the last stage. So, the sequence that you are getting we will be starting like this, it was initially 1, so 1 0 0 1 0 1 1 is the sequence. And if you count it after the arrival of the first clock pulse, then it will be 0 0 1 0 1 1 1. Remember one thing actually when the clock pulse is coming one by one at the end of the seventh clock pulse here, we are seeing that the value stored in the stage in the memory that is the actually repeating the initial stage.

So, there is a term called the period of generation of a sequence the period is always given the by the value of the 2 to the power small m, which is always period will be less than equal to 2 to the power small m, where the small m is the number of the stages involved in the generation of the spreading sequence of shift-register based spreading sequence. And here in our situation the value of small m is equal to 3, so that is why we are getting seven period is equal to 7. So, after 7 clock pulse, the value is getting repeated. You will be seeing if you continue you will be generating the same table once again. So, I have a generated sequence 0 0 1 0 1 1 1, and it is having a period is equal to 7.

So, these are the two important parameter that we will always keep in mind while we are generating any spreading sequence, what is the period of the sequence, and what is the sequence length, what is the sequence length, so that is all for this module. We will continue in the next module and mostly focus on the generation mechanism in of the ML sequence using this linear feedback shift-registers.