

**Basic Building Blocks of Microwave Engineering**  
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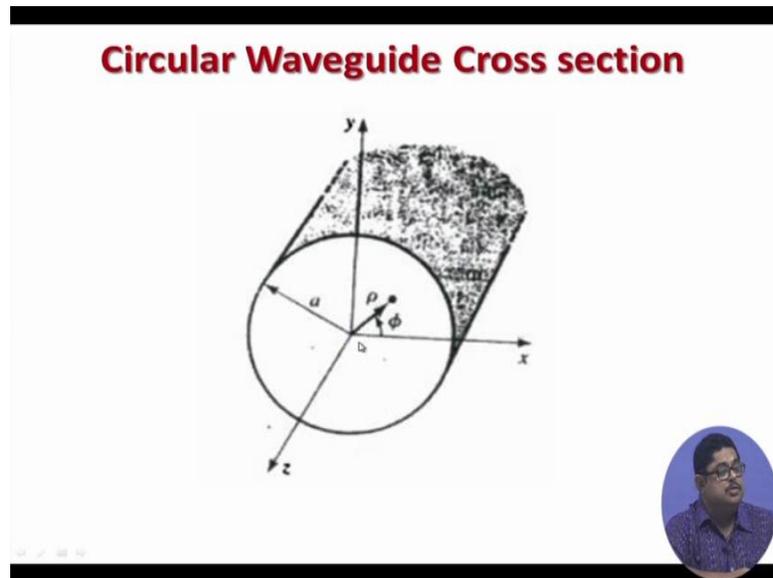
**Lecture – 08**  
**Circular Wave guide**

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So, in this lecture will start, we will see circular wave guide. Now this is the circular waveguide in our lab you see that flanges are still rectangular, but the wave guide inside that in instead of any rectangular pipe it is a circular pipe.

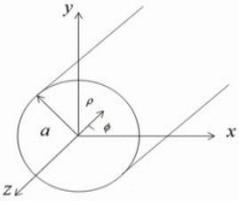
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Now, this is circular wave guide cross section, so the, this wall is metallic, and again since the structure is conformal with the cylindrical coordinates. So, we will switch over to row pi n z coordinate or cylindrical coordinate.

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### Circular Waveguide Modes



- A hollow circular metal pipe.
- Supports TE and TM modes, no TEM mode.
- Appropriate coordinate system  $\rightarrow$  cylindrical
- Wave propagation in  $z$  direction, spatial variation  $e^{-i\beta z}$

The diagram illustrates the cross-section of a circular waveguide. It shows a circular hole of radius  $a$  in a metallic wall. The coordinate system is defined with the  $z$ -axis along the direction of propagation, and the  $x$  and  $y$  axes in the plane of the cross-section. The radial distance from the center is  $\rho$ , and the angular position is  $\phi$ . A small circular inset in the bottom right corner shows a man with glasses speaking.

Now, again since it is a wave guide. This is a single conduct no TE m mode, and wave propagation in z direction TE modes.

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### TE Modes

$E_z = 0, H_z \neq 0$   
 $\nabla^2 \vec{H}_z + k^2 \vec{H}_z = 0$   
 $\vec{H}_z(\rho, \phi, z) = \tilde{h}_z(\rho, \phi) e^{-j\beta z}$

Reduced Helmholtz equation for  $\tilde{h}_z$  is,

$$\left( \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + k_c^2 \right) h_z(\rho, \phi) = 0$$

where  $k_c^2 = k^2 - \beta^2$

Using separation of variables,  $\tilde{h}_z(\rho, \phi) = f(\rho) g(\phi)$



So,  $e_z$  is zero, we will express the Helmholtz equation in terms of  $h_z$ . and this is the wave equation or Helmholtz equation. Wave equation and Helmholtz equation as same name, sometimes we interchange when we use them. Again here you apply separation of variables bracket into  $f$  row and  $g$  pi, already we have done that in case of field analysis by coax, when we saw the coax field analysis similar thing.

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### Differential equation for $g(\phi)$

- Putting in reduced Helmholtz equation and multiplying by  $\frac{\rho^2}{fg}$

$$\frac{\rho^2}{f} \frac{\partial^2 f}{\partial \rho^2} + \frac{\rho}{f} \frac{\partial f}{\partial \rho} + k_c^2 \rho^2 = -\frac{1}{g} \frac{\partial^2 g}{\partial \phi^2}$$

- L.H.S. is a function  $\rho$ , R.H.S. is a function  $\phi$
- So, L.H.S. = R.H.S. = Constant =  $k_\phi^2$
- So,  $-\frac{1}{g} \frac{\partial^2 g}{\partial \phi^2} = k_\phi^2$

or  $\frac{\partial^2 g}{\partial \phi^2} + k_\phi^2 g = 0$



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### Solution of $g(\phi)$

- Solution is

$$g(\phi) = A \sin k_\phi \phi + B \cos k_\phi \phi$$

- $h_z(\rho, \phi)$  must be periodic in  $\phi$

$$h_z(\rho, \phi) = h_z(\rho, \phi \pm 2m\pi)$$

- $k_\phi$  should take discrete integer values

$$\therefore g(\phi) = A \sin n\phi + B \cos n\phi$$


So, to those things, and again that  $k_\pi$  and  $k_\rho$  square things, again the solution is similar. as I said that  $h_z$  must be periodic in  $\phi$ . So,  $h_z$  thing is if you change  $\phi$  the function, that time it was potential function, this time it was converts relate magnetic field pressure that should be same. So,  $k_\pi$  should take discrete values, discrete integer

values. So, that is why we give it a name instead of  $k\pi$ , we are calling it  $n\pi$  to remind us that it is integer values.

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**Differential equation for  $f(\rho)$**

- We also have, 
$$\frac{\rho^2}{f} \frac{\partial^2 f}{\partial \rho^2} + \frac{\rho}{f} \frac{\partial f}{\partial \rho} + k_c^2 \rho^2 = n^2$$

or

$$\frac{\rho^2}{\partial \rho^2} \frac{\partial^2 f}{\partial \rho^2} + \frac{\rho}{\partial \rho} \frac{\partial f}{\partial \rho} + (\rho^2 k_c^2 - n^2) = 0$$

This is Bessel's differential equation.  
So, the solution is  $f(\rho) = C J_n(k_c \rho) + D Y_n(k_c \rho)$   
where  $J_n(x)$  and  $Y_n(x)$  are the Bessel functions of 1<sup>st</sup> and 2<sup>nd</sup> kind, respectively.



Now, for other one, the row variation part or row function part you can write like this. Now this equation is an important equation in engineering mathematics, this is Bessel function. So, this is second order differential Bessel function, its solution was given by the scientist Bessel. So, that is. There are first kinds of Bessel function  $J_n$ , which generally denote by the first kind of Bessel function  $J_n$ , and these as second kind is  $Y_n$ .

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**Solution of longitudinal component**

- We want the fields to be finite inside the entire waveguide including the centre
- But  $Y_n(x)$  is infinite and  $J_n(x)$  is 1 at  $x=0$
- So, the constant D should be zero!

• Longitudinal component of field becomes

$$\tilde{h}_z(\rho, \phi) = (A \sin n\phi + B \cos n\phi) J_n(k\rho)$$


So, now we want the fields to be finite inside the entire wave guide including the center; that means, inside the whole circular wave guide we want the field to be finite, but the Bessel function of second kind  $y_n$  that has an infinity when at the center.

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**Eq. 15 for Cylindrical Coordinates**

- In Cartesian coordinates, we had expressed four transverse field components in terms of two longitudinal field components earlier and named that eq. 15.
- That equation we cannot use here in case of cylindrical coordinates.
- We can easily manipulate Maxwell's equations again and write the transverse field components in terms of longitudinal components.
- Consult notes for this manipulation



So,  $y_n x$  cannot be supported as a solution (Refer Time: 03:50). So; that means, these cannot be. So, this  $d$  then should go to zero. So, that this part does not come here. So, when  $d$  goes to zero we write like this; that means, only  $c$  is there, but  $c$  we are absorbed, because a constant. So,  $a$  and  $b$  has absorbed that  $c$ . So, now this is a general solution. This will subject to the boundary condition. So, this is the whole physical part we have enforced, and got one or you can say two way constant deduction; that is why we now need to come boundary condition find this  $a$   $b$ . So, as I said consult notes for this manipulation. Again in cylindrical coordinate will have to manipulate Maxwell's equations, so that you can express the transverse field components in terms of longitudinal components.

If you remember it equation 15 was that for condition coordinate, the similar thing we have not till now done, because though we have seen the coaxial line analysis, but that analysis was not in terms of these, because of the quasi static field structure in coax, in two conductor circular thing there we took the help of potential function, and took the solve only Laplace's equation, but here in TE m mode or in circular wave guide you do not have that luxury or that convenient; that is why we need to again manipulate Maxwell equations in the notes that will shown that how to write it in; that is why I am writing, or I should have written equation fifteen counterpart for cylindrical coordinates.

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**Eq. 15 for Cylindrical Coordinates**

$$E_\rho = \frac{-j}{k_c^2} \left[ \beta \frac{\partial E_z}{\partial \rho} + \frac{w\mu}{\rho} \frac{\partial H_z}{\partial \phi} \right]$$

$$E_\phi = \frac{-j}{k_c^2} \left[ \frac{\beta}{\rho} \frac{\partial E_z}{\partial \phi} + w\mu \frac{\partial H_z}{\partial \phi} \right]$$

$$H_\rho = \frac{j}{k_c^2} \left[ \frac{w\varepsilon}{\rho} \frac{\partial E_z}{\partial \phi} - \beta \frac{\partial H_z}{\partial \rho} \right]$$

$$H_\phi = \frac{-j}{k_c^2} \left[ w\varepsilon \frac{\partial E_z}{\partial \phi} + \frac{\beta}{\rho} \frac{\partial H_z}{\partial \phi} \right]$$


So, if you do that the four transverse components  $E_\rho, E_\phi, H_\rho, H_\phi$  can be written in terms of the longitudinal components instead in cylindrical coordinate; that is  $E_z$  and  $H_z$ . Now in case of TE modes you put that  $E_z = 0$  in case of TE modes you put  $H_z = 0$ . So, this 4 field gets expressed in terms of one longitudinal component only.

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**Boundary Condition for hollow metallic cylinder**

$E_{\tan} = 0 \Rightarrow E_\phi(\rho, \phi, z) = 0 \quad \text{at } \rho = a$

For TE modes

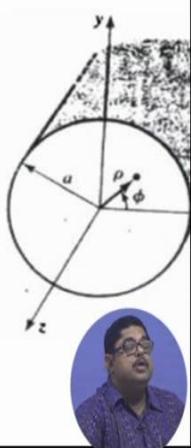
$$E_\phi(\rho, \phi, z) = \frac{-j}{k_c^2} \left[ -w\mu \frac{\partial H_z}{\partial \rho} \right] = \frac{jw\mu}{k_c^2} \frac{\partial H_z}{\partial \rho}$$

$$= \frac{jw\mu}{k_c} [(A \sin n\phi + B \cos n\phi) J'_n(k_c \rho)] e^{-j\beta z}$$

Boundary condition demands  $J'_n(k_c \rho) = 0$

Let  $p_{nm}'$  is the  $m$ th root of  $J'_n(k_c \rho) = 0$

Then,

$$k_{c \frac{nm}{nm}} = \frac{p_{nm}'}{a}$$


And boundary condition for hollow metallic cylinder what is that; obviously, the any metal or any conductor that tangential electric field is zero. What is tangential field here,  $E_\phi$ . So,  $E_\phi$  should go to zero at where on the metallic structure. So, that is why at  $\rho = a$ ; that means, from this whole outer surface it should go to zero, enforce that. So, for TE modes you get that  $E_\phi$  is like this we already knew the, is a thing that we just put that  $E_\phi = 0$ . So, boundary condition demands; finally, that this Bessel function.

This is actually the  $J'_n$  so; that means, the derivative of Bessel function, that function should be equal to zero. So, this is the demand. Now, this is an equation. So, there are number of roots to this equation. So, let the  $m$ th root is called  $p_{nm}'$ . Now remember that in case of rectangular wave guide, our structure was  $m$  and  $n$  modes were TE <sub>$m$ , $n$</sub> , where  $m$  stands for the number of variations along  $x$  direction. The way it was defined that  $k_x = m\pi/a$ . So, it was variation along  $x$  direction number

of variations how many (Refer Time: 08:40) variations we have along x, and n stands for along b direction or how many go (Refer Time: 08:50) variation, but here the order is different, these known that here n comes first m now. So, p n m dash is. Suppose it is a mth root of j n dash this equation. So, if that is a root; that means, we can say that k c into a is equal to p n m a, because this is a root; that means, if I put in this equation that value; that means, in place of row if I put that I am getting that.

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### TE<sub>nm</sub> modes

- n<sub>b</sub> → refers to the no. of **circumferential** variation
- m → refers to the no. of **radial** variation.

m →	1	2	3
n	<i>P</i> <sub>n1</sub> '	<i>P</i> <sub>n2</sub> '	<i>P</i> <sub>n3</sub> '
0	3.832	7.016	10.17
1	1.841	5.331	8.536
2	3.054	6.706	9.970

- Phase constant of TE<sub>mn</sub> mode is

$$\beta_{nm} = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{p_{nm}'}{a}\right)^2}$$

- Cut off frequency

$$f_{c_{mn}} = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{p_{nm}'}{2\pi a\sqrt{\mu\epsilon}}$$


So, here I have written n refers to the number of circumferential variation; that means, azimuthal variation, and m refers to the number of radial variation. So, this, that Bessel function is derivative that equation the roots are well tabulated. So, if you see the root for various values of m and n if you see. So, and find out the cut off frequency. For cut off frequency first step will have to determine what this beta n m. So, k c already in the previous one - we have seen k c in the cut off wave number once you know that.

So, you can find beta, once you know beta you can c f c cut off frequency, and that will be given by this. So, you see cut off frequency is the, denominator is some constant, but the numerator that depends on the number of modes, the value of n m that you choose p n m. So; that means, the lowest value here that will be the lowest thing, so that will be the dominant mode. Now here if you see this stable it becomes clear that what are these,

these one is the lowest, and this is for what. this is for n is equal to 1 m is equal to 1; that means, I need a TE 1 1 mode TE 1 1 mode, that will give me the minimum cut off frequency amongst the modes, and so we can say that dominant circular wave guide mode is t e, till now TE mode, so that will be TE 1 1 mode.

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### Dominant TE mode

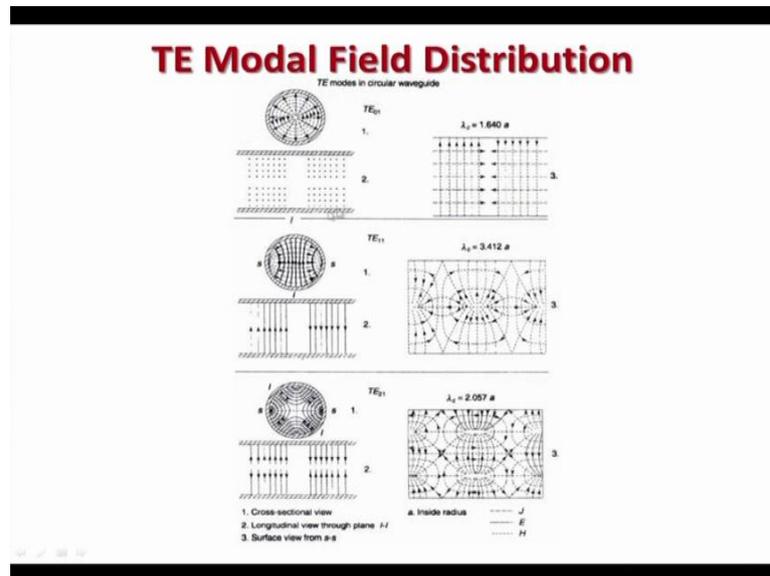
m→	1	2	3	
n	$p_{n1}'$	$p_{n2}'$	$p_{n3}'$	
0	3.832	7.016	10.17	$f_{c_{mn}} = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{p_{nm}'}{2\pi a\sqrt{\mu\epsilon}}$
1	1.841	5.331	8.536	
2	3.054	6.706	9.970	

- Dominant TE mode is clearly TE<sub>11</sub>
- For circular waveguide m is atleast 1. So, no TE<sub>n0</sub> mode and hence no TE<sub>10</sub> mode



So, dominant TE mode is this. The dominant TE mode is clearly TE 1 1, for circular wave guide m is at least one, because circular wave guide the solution requires that m is equal to zero is not possible. So, no t n zero mode, and hence no counter part of TE 1 0 mode here. Here it starts from TE 1 1. So, if you see the field distribution of TE 1 1 mode that you can see this is the TE 1 1 mode. this is the dominant mode.

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But one problem if you see the field, you see the field is not symmetry. So, this side it is going like these, this side this is going like these, whereas you see this fields structure is symmetry. These are something like our coax field structure etcetera. So, one problem is generally we prefer this type of field structure. So, dominant mode field structure is known at least electric field structure, is not shown good. Now you can see other things field structure etcetera. So, that why sometimes in circular wave guide the dominant mode is not used. So, they would need to be able to design convert as to convert the dominant mode to derive that; obviously, some power is loss, but to have some other advantage, sometimes engineers do that.

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**Wave Impedance of TE mode**

$$Z_{TE_{nm}} = \frac{E_{\rho}}{H_{\phi}} = \frac{-E_{\phi}}{H_{\rho}} = \frac{\eta k}{\beta_{nm}}$$


And what is the wave impedance, again from that basic definition of e field by h field. Who carry power, e e row cross h pi that will carry power? So, that is why you are taking that ratio, not e row by h row, because they do not carry power that we have already explained earlier. So, this will be the things, depending on beta you can find that value, also conduct a loss is given by this and

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**Conductor loss for dominant TE<sub>11</sub> mode**

$$\alpha_c = \frac{P_l}{2P_0} = \frac{R_s (k_c^4 a^2 + \beta^2)}{\eta k \beta a (p_{11}^2 - 1)} = \frac{R_s}{\eta k \beta a} \left( k_c^2 + \frac{k^2}{p_{11}^2 - 1} \right)$$


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### TM Modes

$H_z = 0, E_z \neq 0$

- Reduced Helmholtz equation for  $\tilde{e}_z$  is,

$$\left( \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + k_c^2 \right) e_z(\rho, \phi) = 0$$

- General solution

$$\tilde{e}_z(\rho, \phi) = (A \sin n\phi + B \cos n\phi) J_n(k_c \rho)$$


### Boundary Condition for TM Modes

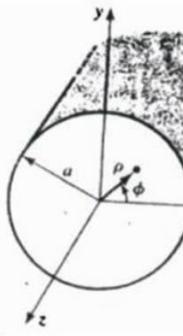
$$\tilde{e}_z(\rho, \phi) = (A \sin n\phi + B \cos n\phi) J_n(k_c \rho)$$

$$E_{\tan} = 0 \Rightarrow E_z(\rho, \phi) = 0 \quad \text{at } \rho = a$$

$$J_n(k_c \rho) = 0$$

$$k_{c_{nm}} = \frac{p_{nm}}{a}$$

where  $p_{nm}$  is the  $m$ th root of  $J_n(k_c \rho) = 0$



Now, t m modes h z is equal to zero, so the (Refer Time: 14:06) is e z and e z solution, something. Again you see Bessel function has come and sine and cosine variation as before. So, you put the boundary condition, tangential electric field is zero. So, in terms of here we can say that e z is zero; that mean this is also z, e z is also a transverse field, e z is also is a tangential field at tangential electric field. So, if you put that then you get the similar equation that characteristic equation that j n is this. So, you remember that, that time it was j n dash which is k c. the cut off wave number, if again we assume that p

n m that is mth root of this equation, then we got k c in terms of this constant, and this is a dimension that is also constant.

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### TM<sub>nm</sub> modes

*Values of P<sub>nm</sub> for TM modes of a circular guide*

n	P <sub>n1</sub>	P <sub>n2</sub>	P <sub>n3</sub>
0	2.405	5.520	8.654
1	3.832	7.016	10.174
2	5.135	8.417	11.620

- Phase constant of TM<sub>nm</sub> mode is
 
$$\beta_{nm} = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{P_{nm}}{a}\right)^2}$$
- Cut off frequency
 
$$f_{c_{mn}} = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{P_{nm}}{2\pi a\sqrt{\mu\epsilon}}$$

Now, from the table you can determine which is adding the lowest one. So, phase constant is given like this, and cut off frequency is like this. so; that means, again you can see the cut off frequency will be lowest, for the mode whose p n m is lowest, whose p n m is lowest let us see, that amongst these this is lowest, what is that mode; that zero and one.

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### Dominant TM mode

*Values of  $P_{nm}$  for TM modes of a circular guide*

n	$P_{n1}$	$P_{n2}$	$P_{n3}$
0	2.405	5.520	8.654
1	3.832	7.016	10.174
2	5.135	8.417	11.620

$$f_{c_{mn}} = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{P_{nm}}{2\pi a\sqrt{\mu\epsilon}}$$

- Dominant TM mode is clearly  $TM_{01}$



So, that  $TM_{01}$  mode will be the lowest mode. Dominant mode is  $TM_{01}$ . Now this thing, but then amongst TM together who is dominant.

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### Dominant Circular waveguide mode

- $p_{nm}$  for TE modes

m→	1	2	3
n	$P_{n1}$	$P_{n2}$	$P_{n3}$
0	3.832	7.016	10.17
1	1.841	5.331	8.536
2	3.054	6.706	9.970

$$f_{c_{TE_{11}}} = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{1.841}{2\pi a\sqrt{\mu\epsilon}}$$

*Values of  $P_{nm}$  for TM modes of a circular guide*

n	$P_{n1}$	$P_{n2}$	$P_{n3}$
0	2.405	5.520	8.654
1	3.832	7.016	10.174
2	5.135	8.417	11.620

$$f_{c_{TM_{01}}} = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{2.405}{2\pi a\sqrt{\mu\epsilon}}$$

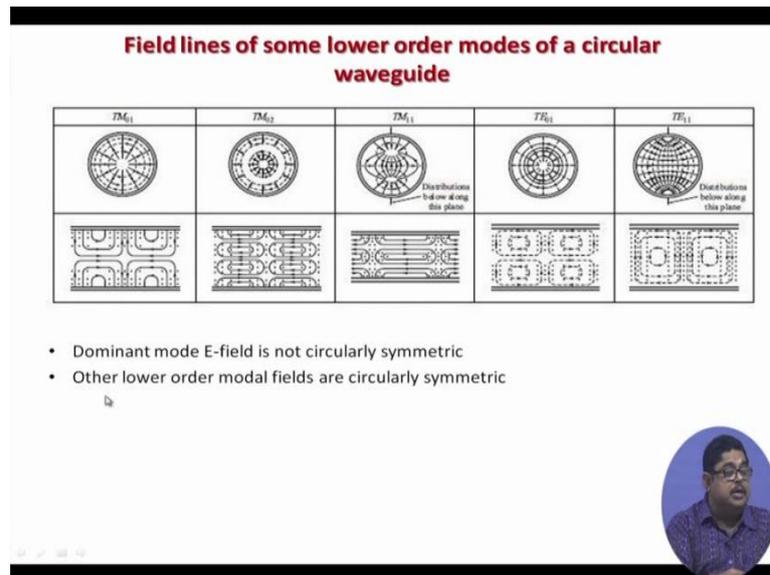
- So, dominant circular waveguide mode is clearly  $TE_{11}$



Then we will have to compare the dominant circular waveguide. So,  $f_c$  amongst TE this is dominant  $TE_{11}$ ; that is this is the expression for their  $f_c$ , this is the expression for t

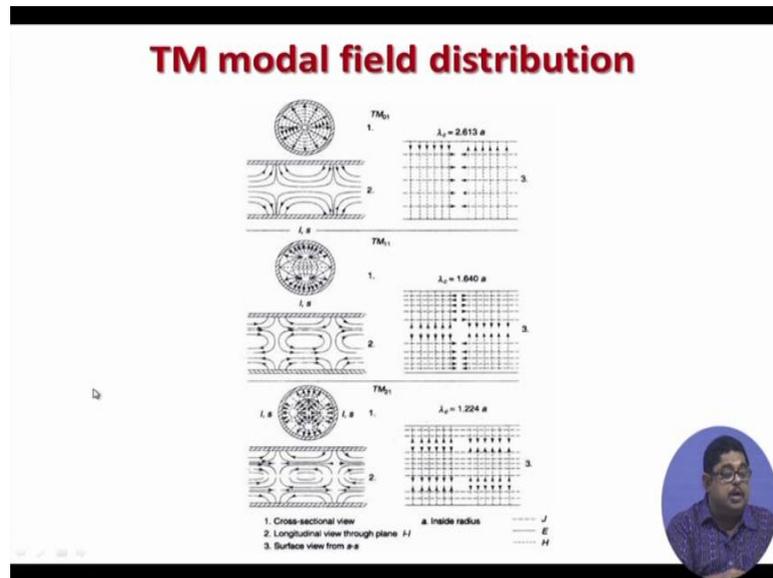
m's 1. So, this is this. So, you see clearly t 1 1 is a dominant mode, so t 1 1 is called the dominant circular wave guide mode. it is status is same as t 1 0 mode in rectangular wave guide.

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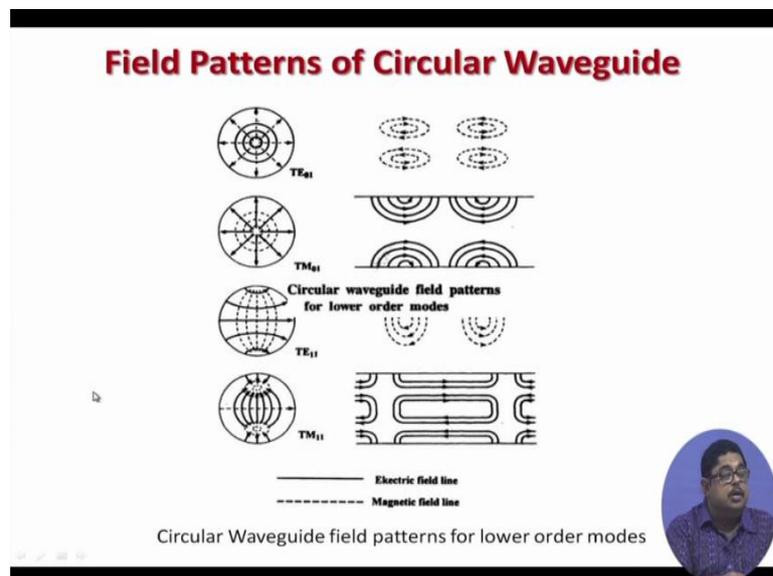


So, these are again field lines. So, you can see that, whatever I have said that sometimes this type of symmetric structures they are preferred over this. Where is t 1 1 you see this one. So, not very symmetric, where sometimes people have problem with this dominant mode e field is not circularly symmetric, other lower order modal fields are circularly symmetric, some other, not all other, some other.

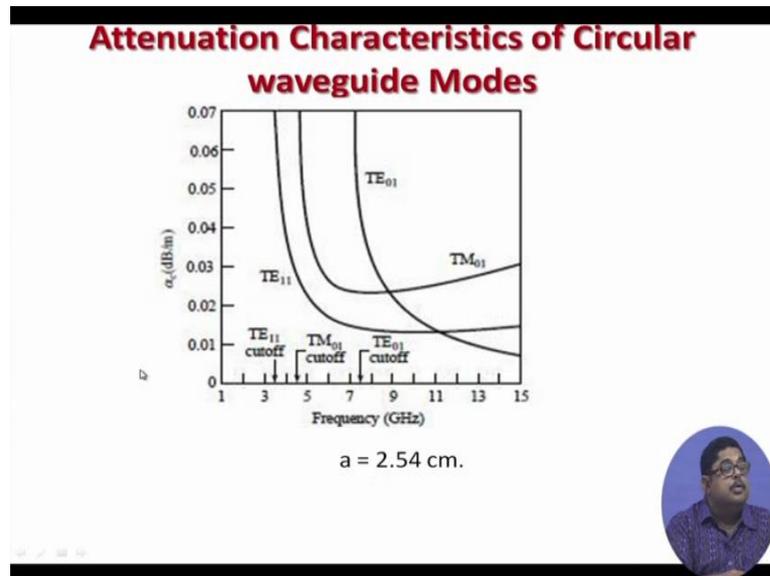
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TM modal field distribution, then attenuation characteristics you can see that TE 1 1 cut off etcetera, but the attenuation wise TE 1 1 has quite lower cut off after you have one of the certain frequency then this cut off is quite small.

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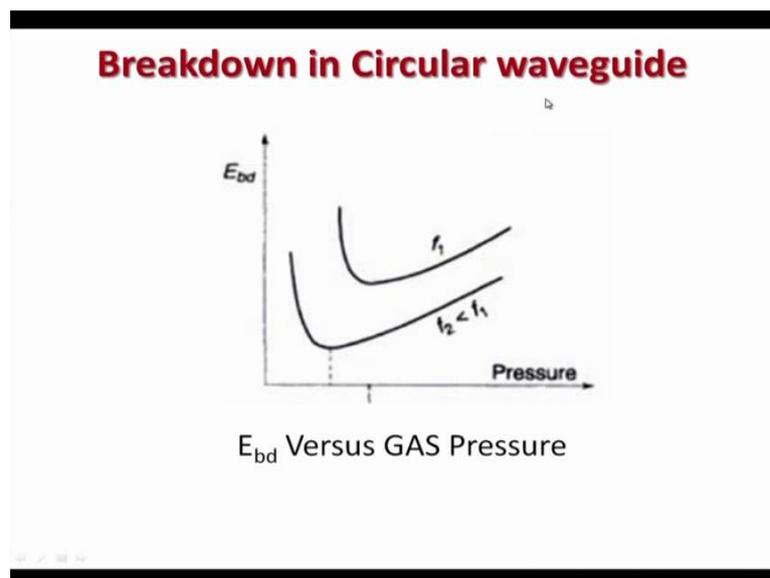
### Mode Converter

- TE<sub>01</sub> is of interest for very low loss
- Field structure of TE<sub>01</sub> is also symmetric
- In many applications circular waveguide is used as overmoded guide
- Mode converters used for converting dominant and other modes to the desired mode

Now as I said TE 0 1 is of interest for very low loss. So, you see from these, where that TE 0 1 you see that it is coming after a certain time, because it is cut off is per, but it has a sudden it goes to at higher frequency, it is slow, it is value is very small. So, it can give you very low loss propagation. So, for high power application where you want to, do not want to lose power it is preferred, but it is fields structure is also symmetric; that is fortunate that TE 0 1 if you locate, that t 0 1 you see, that it is a very, just like circular symmetric is there in its electric field; that is why these field was once very popular among the structures that how to extract that.

So, in many applications circular wave guide is used as over moded guide, because of that I want to extract TE 0 1, not the dominant t 1 1. So, that time it is used as over moded, and mode converters are used for converting dominant and other modes to the desired mode. As I said; obviously, that will give rise to certain amount of loss, but to get the benefit of the field structure circularly symmetric, we needed. Also if you do not have circular symmetric field structure, there are problems that at certain point's fields may be high. So, electric breakdown may occur. So, that is why in circular wave guide breakdown is a problem if you do not have a circular symmetric wave guide.

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Here we are showing that you know that, actually if you have a breakdown thing. Breakdown occurs because if you have inside a wave guide some air now some air. Now, air at eleven mega volt per meter it breaks down. So, if you give a field higher than that, then it will break down. So, there will be spark produced and the power cannot flow, because power will be then taken to the short at thing, short at plates. So, to avoid that, suppose if you were, nowadays there are applications coming up where people are trying to send huge power to wave guides. So, in that case the breakdown etcetera may come. So, to prevent that what they do if, you put a dielectric inside, generally in the form of gases it is put, then suppose you put the wave guide in a gas chamber, then depending on the dielectric if the, whatever the dielectric based on that square root of that factor it will be pushed up.

So, that is being shown that if you go on increasing the pressure, the electric break down value is coming down. So, this is also another thing that in high power application, these circular wave guide, you want to have circular symmetric thing, because then break down will not, the chances of occurrence of break down will be less, and then you can also would gas, generally c f 6 is used to do that in high power microwave people they use c f 6, to at least increase the break down values two to three times; that is circular wave guide. So, we have seen these two wave guides, and again. So; that means, we have seen how TE<sub>m</sub> mode propagates, how TE<sub>m</sub> mode propagates in rectangular wave guide and circular wave guide.

Thank you.